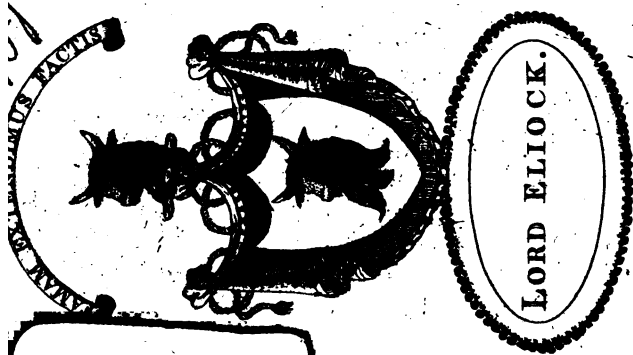


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A SHORT
TREATISE
James OF THE *Scotch*
General LAWS
23. OF *June*
MOTION
Amo. and centripetal 1724
FORCES:

WHEREIN,

By the by, Mr. GORDON's Remarks
on the *Newtonian* Philosophy are, in
a few *Corollaries* and *Scholies*, clearly
confuted.

By GEORGE PIRRIE, M. A.

EDINBURGH,
Printed for the AUTHOR, by WILLIAM
ADAMS *Junior*. MDCCXX.

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T O

The Right Honourable,

FRANCIS

Lord *Napier* of *Mer-*
chiston.

My LORD,

THERE is no moral
Principle more cer-
tain, plain, and in-
controvertible than this, that
it is a God-like Thing to do Good

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to Mankind; of which divine Quality your Lordship's noble Ancestors are honourable Examples and Instances.

WHEN I reflect on the great Services done to the learned World, by the most elaborate Performances of that very learned and worthy Gentleman *John Napier* Baron of *Merchiston*, whose Praises, for his most useful and wonderful Invention of the *Logarithms*, besides his other Improvements in solid Knowledge, deserve to be celebrated to all Ages; I cannot but think my self obliged to commemorate the Merits of so great a Man, and beg this small

Essay

Essay may obtain the Favour and Patronage of your Lordship his present lineal Representative : And I am the more encouraged to shelter this short Treatise under your Lordship's Protection, that I am assur'd of your Goodness, and that your Mind discovers noble Endowments superior to your Years.

YOUR Lordship's Family has not only adorn'd our Country with Men of eminent Learning and Knowledge in its Laws and Constitution, so that they were justly thought worthy to be Senators in our College of Justice; but has likewise afforded us noble Examples of true For-

titude, real Honour, and the
 most distinguish'd Loyalty, by
 boldly espousing the Rights and
 Privileges of their King and
 Country, when groining under
 the Pressure of the worst of
 Times: Particularly that wor-
 thy Patriot, *Archibald Lord Na-*
pier, suffered Imprisonment, and
 the greatest Hardships, for
 closely adhering to the Inte-
 rest of his Sovereign King
Charles I. of blessed Memory,
 and endeavouring with a true
 Christian Courage to support
 his sinking Country : And his
 Son *Archibald Lord Napier*
 (in no respect inferior to so
 great a Father) was forc'd at
 the end of an unsuccessful War,
 to

to retire to *Holland*, to shelter himself from the cruel Violence of the then prevailing Powers.

YOUR Lordship's Father, the worthy and honourable Sir *William Scot* of *Thirlstane*, Baronet, is so well known to be a Gentleman of such Probity, Love to his Country, polite Learning, and complete Accomplishments, and to have taken such extraordinary Care of your Lordship's Education; that his Memory will be always dear to all that have had the Happiness of his Acquaintance.

MY Lord, after what I have said of so great Examples, and

[viii]

so worthy of Imitation, I shall
only add my hearty Wishes you
may not only equal, but (if
possible) even exceed them ;
which cannot fail to render
you the Favourite of Heaven,
and the Darling of Mankind.
I am with the utmost Respect,

My LORD,

Your Lordship's.

most obedient and very
humble Servant,

GEORGE PIRRIE

THE

P R E F A C E.

THE ensuing Treatise is design'd to be a short, plain, and clear Introduction to the Laws of Motion and centripetal Forces for the Benefit of Youth, and my own private Use in Teaching. It is also partly intended for a Confutation of a small Book published last Year, which has lately receiv'd a second Edition, Intituled, Remarks upon the Newtonian Philosophy, by George Gordon. What this Gentleman the Author, or his Character in the World was, I could never certainly learn, till the second Edition of his Book came

came abroad with a large Preface, which the first entirely wanted: And the Truth is, I was apt to think with others, that the Name was counterfeit, though now I find the contrary. In his Preface he complains heavily of very hard Usage from a Set of Men, whose Interest (he says) it is to support the Newtonian Philosophy, and who have industriously endeavour'd to suppress his Book, and hinder every Body from buying or reading it, by decrying it in all Companies, and vilifying himself as an enterprizing Fool. And this, he thinks, they have done, because they are not able to answer his Arguments: For since Mr. Gordon has with no small Assurance and Confidence written his Book against the Newtonian System, it seems he is really perswaded, that it is impossible to confute what he has advanced to overthrow that System; and is therefore in his Preface (which, by the way, is not very civil to some learn'd Men) extremely pressing for an Answer. His Attempt is bold, and his Book manag'd with too little Respect towards two so great Men as Sir I.
Newton.

Newton and Dr. Gregory; therefore the Arguments he brings against them, and the incomparably best System of natural Philosophy that ever the World was blest with, had need be very strong and conclusive; and yet, in my Judgment, they are only the most cunning and subtil Piece of Sophistry that ever was formed on a Physico-mathematical Subject. His Way of reasoning does indeed, in several Places of his Book, carry a very specious Shew and Appearance of perfect Demonstration; and though stark naught at the Bottom, I doubt not but it has amused many, and even misled some into his Mistakes. I have therefore thought it proper to take his Work so far to task, as plainly to lay open the Weakness, Cheat, and Fallacy of its main Strength in a few Scholies and Corollaries of the following Tract, not reckoning it worth while to give a more large and direct Answer.

Mr. Gordon, by the Title Page of his Book, was obliged to shew the Fallacies of Sir I. Newton's and Dr. Gregory's mathematical Demonstrations, which he has quite

quite omitted to do. Had he shown the Sophistry but of some few of them, I dare venture to say, nay positively to affirm, that this ~~would have done him~~ vastly more Service than all his fine Demonstrations; and would have made his Book pass in the World, and procur'd its Sale, in Spite of all the Opposition it could have met with from any Party whatsoever. If he thinks fit, he may attempt this in a third Edition; and if he do any Thing this way to Purpose, I cannot doubt but he will be perswaded, that I have hinted a Method to him, effectually to ruine the Reputation of the Newtonian Philosophy, beyond any he has yet tried. But this, I believe, he will find to be the hardest Task he sever undertook in his Life.

I do not pretend that much of the ensuing Work is new except the Method and Order: But (yet besides the foresaid Scholies and Corollaries, and some Things that, I think, I have set in a better Light than any has done before) some small Part is my own, and perfectly new, as particularly Prop. 28, 29, 30, 31, 34; wherein, I
hope.

hope, I have clearly demonstrated the Possibility that a Body may, by a projectile and a centripetal Force conjoined, move in a Circle, or any other Curve that is concave to the Center of the centripetal Force: Which Possibility has ever hitherto been supposed by all, and contradicted by none I know of before Mr. Gordon, but never plainly demonstrated by any. At the End of this Tract, I have given a full and clear Demonstration of a short Way of arguing, often used by the incomparable Newton and his Followers, which, I believe, is new, and no where else to be found. I submit the whole to the Judgment of the candid Reader, hoping that he will easily pardon or overlook any light Slips and Oversight he may chance to meet with.

ADVERTISEMENT.

ALL Parts of the Mathematicks are taught by the Author at his House near the Cross in Edinburgh.

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A SHORT
T R E A T I S E

O F

The General Laws of Motion,
and centripetal Forces.

PART I.

Of the Laws of Motion,

DEFINITIONS.

1. **A**NY Portion or Quantity of Matter or material Substance, is called a *Physical* or *Natural Body*; and the Space contained by its Surface, is called its *Magnitude* or *Bulk*.

2. The Motion of a Body is its successive Change of Place.

A

3. *Celerity*

3. *Celerity* or *Velocity* is an Affection of Motion, whereby a Body runs through a certain Space in a certain Time.

4. *The Direction of a Body's Motion* is the straight Course, or Path, in which the Body tends.

5. *An equable or uniform Motion* is that, whose Celerity is neither increased nor diminished, but still continues the same.

6. *An accelerated Motion* is that, whose Velocity is still increasing.

7. *A retarded Motion* is that, whose Velocity is still decreasing.

8. *An equably or uniformly accelerated Motion* is that, to which in equal Times equal Degrees of Celerity are continually added.

9. *An equably retarded Motion* is that, whose Velocity in equal Times is always equally diminished.

10. *The Force, Power, or Quantity of a Body's Motion*, whereby it is able to produce such or such an Effect, is called its *Moment*; and so is the Force or Power of a Body, whereby it has a constant

stant Tendency to move, often called its *Moment*, though it be not in actual Motion.

Let it here be noted, that the Words, *Moment* and *Motion*, applied to a moved Body, are commonly taken in the same Sense.

11. That which resists, diminishes, or destroys Motion, is called an *Impediment*.

12. *Gravitation* or *Gravity* is that Force upon Bodies, whereby they are made to move or tend to the Center of the Earth.

13. *Centripetal Force* is that, whereby a Body is constantly urged, or made to tend to a certain Point as a Center.

Whence 'tis plain, that Gravity is a certain Sort of Centripetal Force.

14. A *regular Centripetal Force* is that, which always acts by one constant Rule or Law.

15. A *Centrifugal Force* is that, whereby a Body is continually urged from a certain Point.

16. The *Center of Gravity* of a Body is that Point thereof, by which if the

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so



so the Body *D* to the Bodies *A*. and *B* together ; then that Point *E* will be called the common Center of Gravity of those three Bodies *A*, *B*, *D*.

After the same Manner, may the common Center of Gravity of four or more Bodies be defined.

19. One Body is said to *hit* or *strike directly* against another, when the right Line in which the striking Body's Center of Gravity moves, passing through the Point in which the two Bodies touch one another when they meet, is perpendicular to the Surface of the Body that is struck : But to *hit* or *strike obliquely*, when the foresaid Line is oblique (and not perpendicular) to the Surface of the Body that is struck.

20. The relative Velocity of two Bodies is the Velocity, whereby the said Bodies approach to, or recede from one another : Which is the Sum of the Velocities, when the Bodies move towards contrary Parts; and the Difference, when towards the same Part.

So when two Bodies A and B (*Fig. 4.*) move both towards the same Part E, the Body A with a greater Velocity, and the Body B with a less; the Excess of the Velocity of A above the Velocity of B, is the Velocity whereby A and B approach together, and consequently is the relative Velocity of the Bodies A and B. If A move towards B at Rest, the relative Velocity of A and B is the same with the Velocity of A. If the Bodies A and B move towards contrary Parts, viz. A towards E, and B towards D, or A towards D, and B towards E; the relative Velocity whereby A and B approach to, or recede from one another, is the Sum of the Velocities of A and B.

21. In like Manner, the *relative Motion* or *Moment* of two Bodies, is the Motion whereby the said Bodies approach to or recede from one another: Which is the Sum of the Moments or Motions, when the Bodies move towards contrary Parts; and the Difference, when towards the same Part.

22. A

22. *A perfectly hard Body* is that, which yields not in the least to a Stroke, but keeps its Figure unaltered.

23. *A soft Body* is that, which changes its former Figure by a Stroke, and never of it self recovers it.

24. *An elastick Body* is that, which for some Time yields to a Stroke, but yet at last restores itself to its former Figure, at least nearly.

25. *Elasticity or elastick Force* is that Force, whereby a Body deprived of its former Figure, restores itself to the same Figure again.

26. *A perfectly elastick Body* is that, which restores itself to its former Figure, with a Force equal to that by which it was compress'd, and lost its Figure.

27. *Homogeneous Bodies* are Bodies of the very same Nature, Make, and Contexture of Parts; as two Pieces of Gold, or two Pieces of Lead.

28. *A void, free, or empty Space*, is a Space that is void of all Matter or material Substance.

29. If

29. If a Body A (Fig. 5.) moving in the right Line AB, strike against a Plane DE, and after the Stroke be reflected in the right Line BC; the Angle ABD is called *the Angle of Incidence*, and CDE *the Angle of Reflection*: But most commonly the Angle ABE (the right Line BE being perpendicular to the Plane DE) is called *the Angle of Incidence*, and CBE *the Angle of Reflection*.

30. *The absolute Quantity of a centripetal Force* is its Measure greater or less, according to the Efficacy of the Cause that propagates it around from the Center.

So the magnetical Virtue is greater in one Magnet or Load-Stone, and less in another.

31. *The accelerating Quantity of a centripetal Force*, is the Measure of the Velocity that the said Force generates in a given Time.

So the Virtue of the same Magnet is greater at a less Distance from it, and less at a greater Distance. Also at the same Distance from the Earth (the Resistance of the Air being removed) all
Bodies

Bodies descend with the same Degree of Velocity, and so their accelerating Forces are equal; but at unequal Distances they descend with unequal Velocities, and so their accelerating Forces are unequal.

M A X I M S.

T. EVERY Body or Piece of Matter will persevere in its State of Rest, or uniform Motion directly forwards; unless it be compell'd to change that State by some Force impress'd upon it.

2. Effects are proportional to their sole and adequate Causes.

So if two Forces, severally impress'd upon the same Body, be the adequate and complete Causes of two several Motions, the said Motions will be as the said Forces.

3. Equal Quantities of Matter or equal Bodies, carried with the same Velocity, have equal Moments or Quantities of Motion.

4. Equal

4. Equal and directly contrary Forces destroy one another.

5. Unequal and contrary Forces produce a Motion, that is equivalent to the Difference of the said Forces.

6. A motion that is produced by perfectly conspiring Forces (that is, such Forces as act according to the same Direction, and tend the very same Way) is equivalent to the Sum of the said Forces.

7. Homogeneous Bodies or their Quantities of Matter, are as the Bulks of the said Bodies.

8. If two Bodies strike directly the one against the other, the compressive Force or Magnitude of the Stroke arises from, and is equivalent to the Difference or Sum of the Moments, according as the Bodies move towards the same or contrary Parts, that is, to the relative Motion or Moment.

9. Action and Reaction are always contrary and equal: Or, the Actions of two Bodies upon one another are always equal, and have contrary Directions.

So

So if one Body press another, that first Body is equally repressed by this second, and the Directions of the Pressures are towards contrary Parts; and if one Body striking against another, by its Force make an alteration in this other Body's Motion, then that first Body's Motion will undergo an equal Alteration towards the contrary part, by Reason of the Equality of their mutual Pression. So if the Body A (Fig. 6.) strike directly against the Body B either at Rest, or moving more slowly towards E, the Body A will lose a Part of its Motion towards E, and the Body B will gain as much, *that is*, B will be urged by A towards E, and A will be just as much repressed or urged the contrary Way towards D by E. In like Manner, if A and B move towards one another, *viz.* A towards E, and B towards D; when they meet, they will be equally urged and pressed by one another towards contrary Parts. *Lastly*, if two Bodies A and B attract one another in Proportion to their Quantities of Matter, the Moments in both will

will be equal; for if A be double of B, A will have double the Influence upon B that B has upon A; and consequently B will move towards A with double the Celerity of A towards B, and the Bodies A and B will be reciprocally proportional to their Celerities. Whence (as will be proved in 6 Cor. 5 Prop.) the Moments of the Bodies will be equal; and in like Manner, in any other Proportion of A to B.

10. Physical or natural Causes are not to be multiplied without good Reason.

So if one Cause will produce a certain Effect, we are not to allow of two for that Effect: If two Causes will do as well as three, we are not to allow of three, and so forth. The Reason of this Maxim is, because Nature proceeds after the simplest Method; for surely the Greatness, Wisdom, and Glory of God is more conspicuous in producing great and wonderful Effects by simple than by manifold Means.

PROP.

PROPOSITION I

Theor. Fig. 7.

IN equable and uniform Motions of Bodies, if the Times be the same or equal; the Lengths or Spaces run through will be proportional to the Celerities.

Let a Body in a given Time run through the Space AB with a Celerity represented by c ; and in the same or an equal Time, let the same, or any other Body run through the Space DE , with a Celerity represented by c . I say the Line AB will be to the Line DE (both being run through with uniform Motions) as the Celerity c is to the Celerity c .

For if the Celerity c be double of the Celerity c , then the Space AB run through with the Celerity c , will be double of the Space DE run through in the same Time with the Celerity c . And if c be triple of c , then AB will be triple of DE : Also, if c be half of c ,

B

then

then will AB be half of DE . And universally, whatever Proportion c bears to c , the same Proportion does AB bear to DE . W. W. & D.

Corollaries.

F. HENCE, if the Times of Motion be unequal, the Spaces run through AB , DE will not be proportional to the Celerities c , c .

If this be denied, suppose there be as $AB : DE :: c : c$. Then since (by *hyp.*) the Times wherein the Spaces AB and DE are run through, are unequal: Let us suppose AB to be run through in a greater Time than DE , and some Part of AB , as AF , will be run through in the same Time with DE : Wherefore (by 1. *Prop.*) there will be as $AF : DE :: c : c$. But, as before, there is as $AB : DE :: c : c$. Whence as $AF : DE :: AB : DE$; and consequently $AF = AB$. W. I. A.

2. If the Spaces run through, be proportional to the Celerities, the Times of Motion will be the same or equal.

For

For if the Times be said to be unequal; then (by 1. Cor.) the Spaces run thro' will not be proportional to the Celerities, contrary to Hypoth.

PROPOSITION II.

Theor. Fig. 8.

IN uniform Motions, if the Celerities be equal; the Spaces run thro' will be proportional to the Times of Motion.

Suppose a Body run through the Space s in the Time τ , and another Body with equal Celerity run through the Space s in the Time t : I say there will be as $s : s :: \tau : t$.

For if τ be Double of t , then will s be double of s ; and if τ be triple, or an half of t , so will s be triple, or an half of s . And universally, whatever Proportion τ bears to t , the same Proportion will s bear to s . W. W. D.

A Corollary.

HENCE, if the Times be as the Spaces, the Celerities will be equal.

This may be interr'd from 2 *Prop.* after the same Manner as 2 *Cor.* 1 *Prop.* was inferred from 1 *Prop.* For we may easily prove, that, if the Velocities be unequal, the Spaces run thro' will not be proportional to the Times, which destroys the *Hypoth.*

PROPOSITION III.

Theor. Fig. 9.

IN compared Motions, if the Celerities be equal, the Moments or Quantities of Motion of the moved Bodies, will be as their Quantities of Matter or the Bodies themselves.

Let two Bodies *A* and *B* be both carried with the same Celerity *c*: I say that

the

the Moment of A is to the Moment of B, as the Body A is to the Body B.

For if the Body A be double of the Body B, the Body A may be divided into two equal Parts, which moved both with the Celerity c, have (by 3 Max.) equal Moments, each whereof is equal to the Moment of B moved with the Celerity c: And so the moment of A will be double of the Moment of B. In like Manner, if A be triple of B, may we prove the Moment of A to be triple of the Moment of B. So also, if A be half of B, we may prove the Moment of A to be half of the Moment of B. And universally, the Body A is to the Body B, as the Moment of A is to the Moment of B. W.W.D.

Corollaries.

FROM hence and 7 Max. if the Bodies be homogeneous; their Moments will be as their Bulks or Magnitudes.

2. If the Moments be as the Bodies, the Celerities will be equal.

This follows as Cor, 2 Prop.

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PROPOSITION IV.

Theor. Fig. 10.

IN compared Motions of the same or equal Bodies, the Moments will be proportional to the Velocities.

Let two equal Bodies *A* and *B* be moved, the former with the Celerity *c*, the latter with the Celerity *c*. I say, as the Moment of *A* is to the Moment of *B*, so is the Celerity *c*, to the Celerity *c*.

Suppose the Bodies *A* and *B* be impell'd by two several Forces, that cause them to move with the Celerities *c* and *c*. Since the Bodies *A* and *B* are equal in Matter, if the Force imprest on *A* be double the Force imprest on *B*; the Moment of *A* will (by 2 Max.) be double the Moment of *B*, and so will *c*

the

the Celerity of A be double of c the Celerity of B .

In like Manner, if the Force impress on A be half of the Force impress on B ; the Moment of A will be half of the Moment of B , and c half of c . And universally, as the Moment of A is to the Moment of B , so is the Celerity c to the Celerity c . w. w. D.

A Corollary.

IF the Moments be as the Celerities, the Bodies will be equal.

This follows as Cor. 2 Prop.

A Lemma.

IF there be any Number of Quantities of the same Kind, the Proportion of the first to the last is compounded of all the intermediate Proportions. So in three Quantities A , B , C , there is

$\frac{A}{C} = \frac{A}{B} \times \frac{B}{C}$; in four A , B , C , D , there is

$\frac{A}{D} = \frac{A}{B} \times \frac{B}{C} \times \frac{C}{D}$; and so forth.

This is a well known Principle in Geometry.

P R O

PROPOSITION V.

Theor. Fig. 111a

IN compared Motions, the Proportion of the Moments is compounded of the direct Proportion of the Bodies and their Celerities.

Suppose two Bodies B and b be carried, the former with the Celerity c , the latter with the Celerity c ; and let M denote the Moment of B , and m that of b . I say there will be $\frac{M}{m} = \frac{B}{b} \times \frac{C}{c}$.

Let G denote a third Body equal in Matter to the Body B ; but suppose it moved with the Celerity c of the Body b ; and let n denote its Moment. Now (by preced : Lemma) $\frac{M}{m}$ is $= \frac{M}{B} \times \frac{n}{m}$; but (by 4 Prop.) $\frac{M}{B}$ is $= \frac{C}{c}$, and (by 3 Prop.) $\frac{n}{m}$ is $= \frac{G}{b} = \frac{B}{b}$. Therefore is $\frac{M}{m} = \frac{B}{b} \times \frac{C}{c}$. w.
Q. D.

Corol.

Corollaries.

1. **HENCE**, ~~with the libration~~ the Proportion of the Celerities, is compounded of the direct Proportion of the Moments and the reciprocal Proportion of the Bodies:

[That is, $\frac{C}{c}$ is $= \frac{M}{m} \times \frac{b}{B}$.

For (by 5 Prop.) $\frac{M}{m}$ is $= \frac{B}{b} \times \frac{C}{c}$: wherefore dividing both Sides by $\frac{B}{b}$, you will have $\frac{C}{c} = \left(\frac{M}{m} \div \frac{B}{b} = \frac{M}{m} \times \frac{b}{B} = \right) \frac{M}{m} \times \frac{b}{B}$.

2. The Proportion of the Bodies, is compounded of the direct Proportion of the Moments and the reciprocal Proportion of the Celerities: That is, $\frac{B}{b}$ is $= \frac{M}{m} \times \frac{c}{C}$.

For (by 5 Prop.) $\frac{M}{m}$ is $= \frac{B}{b} \times \frac{C}{c}$: whence by dividing by $\frac{C}{c}$, there is $\frac{B}{b} = \left(\frac{M}{m} \div \frac{C}{c} = \frac{M}{m} \times \frac{c}{C} = \right) \frac{M}{m} \times \frac{c}{C}$.

3. If the Bodies be homogeneous; the Proportion of their Moments, will be

be compounded of the direct Proportion of their Bulks and Celerities. The Proportion of the Celerities, will be compounded of the direct Proportion of the Moments, and the reciprocal Proportion of the Bulks: And the Proportion of the Bulks, of the direct Proportion of the Moments, and the reciprocal Proportion of the Celerities. As is evident from 5 *Prop.* and 1 and 2 *Cor.* and 7 *Max.*

4. The Moments M, m of two Bodies B, b , are as the Rectangles or Products $B \times c, b \times c$ of the Bodies multiplied into their Celerities c, c : And if the Bodies be homogeneous, as the Products of their Bulks into their Celerities: As is evident from 5 *Prop.* Hence,

5. The Moment of any Body may be considered, as the Product of the said Body multiplied into its Celerity.

6. If the Moments of two Bodies be equal; the Bodies will be reciprocally proportional to their Celerities. And on the contrary.

For if $M \text{ be } = m$: Since (by 4 *Cor.*) as is $M : m :: B \times c : b \times c$, then is $B \times c = b \times c$,

$b \times c$, and consequently as $B : b :: c : C$.
 And on the contrary, if there be as $B : b :: c : C$, then is $B \times C = b \times c$, and consequently (by 4 Cor.) $M = m$.

7. The Velocities of Bodies are as the Moments applied to the Bodies, *that is*, as the Quotes resulting from the Moments divided by the Bodies.

For (by 1 Cor.) $\frac{C}{c}$ is $= \frac{M}{m} \times \frac{b}{B} = \frac{M}{B} \div \frac{m}{b}$.

and consequently as is $c : C :: \frac{M}{B} : \frac{m}{b}$.

Hence,

8. The Velocity of a Body may be considered as the Moment applied to, or divided by the Body.

9. Bodies are as their Moments applied to their Celerities.

For (by 2 Cor.) $\frac{B}{b}$ is $= \frac{M}{m} \times \frac{c}{C} = \frac{M}{C} \div \frac{m}{c}$.

and consequently as $B : b :: \frac{M}{C} : \frac{m}{c}$. Hence,

10. A Body may be considered as its Moment applied to its Velocity.

PROPOSITION VI.

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Theor. Fig. 12.

IN uniform Motions, the Spaces run through are in a compound Proportion of the Times and Celerities.

Let the Line s be a Space run thro' with the Celerity c in the Time τ , and the Line s another Space run thro' with the Celerity c in the Time t : I say there will be $\frac{s}{s} = \frac{\tau}{t} \times \frac{c}{c}$.

Let F be a Space run thro' with the Celerity c in the Time τ . Then (by Lem. 5 Prop.) is $\frac{s}{s} = \frac{s}{F} \times \frac{F}{s}$: but because s and F are run thro' in the same Time τ , therefore (by 1 Prop.) is $\frac{s}{F} = \frac{c}{c}$. Again, because F and s are run thro' with the same Celerity c , therefore (by 2 Prop.) is $\frac{F}{s} = \frac{\tau}{t}$; whence $\frac{s}{s}$ is $= \frac{c}{c} \times \frac{\tau}{t}$: w. w. d.

Corol.

Corollaries:

1. The Proportion of the Times, is compounded of the direct Proportion of the Spaces, and the reciprocal Proportion of the Velocities; that is, $\frac{T}{s}$ is $= \frac{s}{s} \times \frac{c}{c}$.

For (by 6 Prop.) $\frac{s}{s}$ is $= \frac{T}{t} \times \frac{c}{c}$.

Therefore, by dividing by $\frac{c}{c}$, is $\frac{T}{t} =$

$$\left(\frac{s}{s} \div \frac{c}{c} = \frac{s \times c}{s \times c} = \right) \frac{s}{s} \times \frac{c}{c}.$$

2. The Celerities are in the direct Proportion of the Spaces, and the reciprocal Proportion of the Times; that is, $\frac{c}{t}$ is $= \frac{s}{s} \times \frac{t}{t}$.

This is proved from 6 Prop. after the same Manner as 1 Cor.

3. The Spaces run thro' s, s , are as the Products $c \times t, c \times t$, of the Celerities and Times; as is evident from 6 Prop. And so any Space run thro' may be consider'd as the Product of the Celerity into the Time.

4. If the Spaces be equal, the Celerities will be reciprocally proportional to the Times; and on the contrary..

For if s be $= t$; then (by 3 Cor.) is $c \times t = c \times t$, and consequently as $c : c :: t : t$. And if there be as $c : c :: t : t$, then is $c \times t = c \times t$, and so $s = t$.

5. The Time is as the Space applied to the Celerity.

For (by 1 Cor.) $\frac{T}{t}$ is $= \frac{s}{s} \times \frac{c}{c} = \frac{s}{c} \div \frac{s}{c}$:

Whence as $t : t :: \frac{s}{c} : \frac{s}{c}$.

6. The Celerity is as the Space applied to the Time.

For (by 2 Cor.) $\frac{c}{c}$ is $= \frac{s}{s} \times \frac{t}{t} = \frac{s}{t} \div \frac{s}{t}$:

Whence as $c : c :: \frac{s}{t} : \frac{s}{t}$.

PROPOSITION VII.

Theor.

IN uniform Motions, the Moments are as the Products of the Bodies and Spaces applied to the Times; that is, (the Bodies, Moments, Spaces, and Times

Times being respectively denoted by $\bar{B}$, b ; $\bar{M}$, m ; $\bar{s}$, s ; $\bar{T}$, t ; as before) there is, as
 $M : m :: \frac{B \times S}{T} : \frac{b \times s}{t}$.

For (by 4 Cor. 5 Prop.) $\frac{M}{m}$ is $= \frac{B \times C}{b \times c} =$
 $\frac{B}{b} \times \frac{C}{c}$. But (by 6 Cor. 6 Prop.) as $c :$
 $c :: \frac{s}{T} : \frac{s}{t}$, and so $\frac{C}{c} = \frac{s}{T} \div \frac{s}{t} = \frac{s \times t}{T \times s}$.
 Therefore is $\frac{M}{m} = \frac{B}{b} \times \frac{s \times t}{T \times s} = \frac{B s}{b s} \times \frac{t}{T} =$
 $\frac{B s}{T} \div \frac{b s}{t}$. Whence as $M : m :: \frac{B s}{T} : \frac{b s}{t}$.
 W. W. D.

A Scholyn

THE same Thing may be more briefly expressed thus; the Moment is as the Product of the Body and Space applied to the Time, that is, M is as $\frac{B S}{T}$; and very compendiously demonstrated thus.

By 5 Cor. 5 Prop. M is as $B C$: But (by 6 Cor. 6 Prop.) c is as $\frac{s}{T}$. Therefore

fore ($\frac{S}{T}$ being put for c) M is as $B \times \frac{S}{T}$

Or $\frac{B \cdot S}{T}$:

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A Corollary

Hence, if the Times be equal, the Moments will be as the Products of the Bodies and Spaces.

PROPOSITION VIII.

[Theor. Fig. 13.]

IF there be, in a void Space, two soft or perfectly hard Bodies, free from the Action and Influence of all other Bodies; and the one strike directly against the other, whether that upon which the Stroke is made be at Rest, or moves more slowly towards the same Part, or lastly towards the contrary Part with a less Motion or Moment: After the Stroke, they will both move close together, with one and the same Degree of Velocity, towards that Part whither the striking Body tended.

Let

Let the Body B moving towards i strike directly upon the Body b , either at Rest, or moving towards E with less Celerity than that of B , or moving the contrary Way towards D with a smaller Quantity of Motion. Then the Body i when struck by the Body B , will move towards E . But b cannot move slower than B , by reason of B following it; nor can b move faster than B , there being (by Hyp.) no Elasticity, nor any other Cause to separate them when met. Therefore the Bodies B and b , after Collision, will both move close together towards E , with the same Degree of Velocity.

PROPOSITION IX.

Theor.

IF, in a void Space, two perfect elastic Bodies (free from the Influence of all other Bodies) strike directly the one against the other; their relative Velocity after the Stroke, will be equal to their relative

lative Velocity before the Stroke: And consequently the Bodies, after Collision, will recede from one another with the same Velocity wherewith they approach'd before.

For in any two Bodies when they meet, the compressive Force is, (by 8 Max.) equivalent to the relative Motion before the Stroke: But in perfectly elastick Bodies, the compressive Force is (by 26 Def.) equivalent to the Elasticity or restitutive Force. Therefore the relative Motion before the Stroke, is equivalent to the restitutive Force after the Stroke: And this relative Motion is the sole Cause of the relative Velocity before the Stroke. Now, if there was no restitutive Force, the Bodies, after the Stroke, would (by 8 Prop.) move close together with the same Velocity, without any relative Velocity: But the restitutive Force makes them separate, and recede from one another after the Stroke, and so creates a new relative Velocity, whereof the said restitutive Force is the sole Cause. Therefore the relative Motion before

Before the Stroke, and the restitutive Force after the Stroke, being, as before, equivalent, and the sole Causes of the relative Velocity before and after the Stroke, it is evident (from 2. Max.) that the said relative Velocities are equal. And consequently, the Bodies after Collision, will recede from one another just as fast as they approached before. *w. w. D.*

PROPOSITION X.

Theor. Fig. 13.

IF two Bodies, moving (in a free Space) either towards the same or contrary Parts, strike directly the one against the other ; the Sum of the Motions or Moments towards one and the same Part, will be the same, after the Bodies strike, that it was before.

Let *a* and *b* be two moving Bodies, and let *c* be the Celerity of *a*, and *c* that of *b*; also let *m* denote the Moment lost to *a*, and communicated by *a* to *b* by the Stroke.

II. Case.

1. *Case.* Suppose the Bodies B, b , both move towards the same Part E , before the Stroke. Then (by 5 Cor. 5 Prop.) BC is the Moment or Motion of B towards E , before the Stroke, and bc the Motion of b towards E ; whose Sum is $BC + bc$: But after the Stroke, the Motion of B towards E is $BC - m$, and that of b towards E is $bc + m$; whose Sum is $BC + bc$, as before.

2. *Case.* Suppose the Bodies B, b , before the Stroke, move towards contrary Parts, *viz.* B towards E , and b towards D : Then (by 5 Cor. 5 Prop.) BC is the Moment or Motion of B towards E , and (bc the Motion of b towards D , and so) $-bc$ the Motion of b towards E ; that is, $BC - bc$ is the Sum of the Motions towards E before the Stroke. But after the Stroke, the Motion of B towards E , is $BC - m$, and that of b towards E is $-bc + m$; whose Sum is $BC - bc$, as before.

A Scholy.

IF the Body b be at Rest before the Stroke, *that is*, if c be $= 0$, and consequently $bc = 0$; then the Motion of the Body B alone towards E , before the Stroke, is the whole Motion of both Bodies B and b towards E after the Stroke. Because, in that Case, $BC + bc$ is $= BC - m + bc + m = BC$.

PROPOSITION XL

Probl. Fig. 13.

TO determine the Celerities and Motions of soft and perfectly hard Bodies, after they strike directly one against another.

That two such Bodies, after the Stroke, will move with one common Velocity, is evident from 8. Prop. to determine which, suppose the Body B move with a greater Motion, and the Body b with a less, towards the same or contrary Parts;

Parts; and let c denote the Celerity of B , before the Stroke, and c that of b . Now if B and b move towards the same Part E , the Sum of their Motions towards that Part, before they strike, will be $Bc + bc$; but if towards contrary Parts, viz. B towards E , and b towards D , the Sum of the Motions towards E , before the Stroke, will be $Bc - bc$. But (by 10 Prop.) the Sum of the Motions towards the same Part, is the same before and after the Stroke. Therefore the whole Motion of the Bodies towards E , after the Stroke, is $Bc + bc$ or $Bc - bc$, according as they tended towards the same or contrary Parts before the Stroke. Therefore (by 8 Cor. 5 Prop.) $\frac{Bc + bc}{B + b}$, or $\frac{Bc - bc}{B + b}$ will give the common Velocity towards E , after the Stroke.

If the Body b be at Rest before the Stroke, that is, if $c = 0$, the common Velocity of the Bodies, after the Stroke, will be $\frac{Bc}{B + b}$.

If

If the Bodies B and b move towards contrary Parts, with equal Motions or Moments ; then will $\frac{B C - b c}{B + b}$ be $= 0$, that is, their common Celerity, after the Stroke, will be nothing, and so they will both rest.

Therefore, if the Bodies B, b , be given, and also their Celerities c, c , before the Stroke ; the common Celerity, after the Stroke, will easily be found. For Instance, suppose $B = 3, b = 2, c = 7, c = 5$; and suppose the Bodies move both towards E , before the Stroke ; then is $\frac{B C + b c}{B + b} = \frac{3 \cdot 7 + 2 \cdot 5}{5} = 6 \frac{1}{5}$ the Degrees (after the Stroke) of the common Velocity towards E . If B, b move contrary Ways before the Stroke ; then is $\frac{B C - b c}{B + b} = 2 \frac{1}{5}$ the common Velocity (after the Stroke) yet towards E , because $B c$ is more than $b c$.

The Moment of each Body, after the Stroke is had, by multiplying each into the common Velocity ; as is evident from 5 Cor. § Prop.

A Lem-

A Lemma. Fig. 14.

IF two Bodies B, b , move both towards the same Part E , B with a slower Motion, and b with a swifter; or the Body b towards E , and B towards the contrary Part D , with equal or unequal Motions: Then in both Cases, the relative Celerity of these Bodies added to the Celerity of the Body B , will give the Celerity of the Body b .

The first Case is evident; because the relative Celerity in that Case, is (by 20 Def.) the Difference of the two simple Celerities; and the Difference of any two Quantities added to the lesser gives the greater.

In the second Case let c denote the Celerity of the Body B , and c that of the Body b ; then (by 20 Def.) $c + c$ is their relative Celerity. Now if we consider the Celerity c , as a positive Quantity, the Celerity c being the direct contrary Way will be a negative one,

and

and so $c + c$ added to $-c$ gives c the Celerity of the Body b .

PROPOSITION XII.

Probl. Fig. 14.

TO determinè the Velocities of perfectly elastick Bodies, after they strike directly one against another.

1. Suppose two perfectly elastick Bodies B, b , move towards the same Part E before they strike, B with the Celerity c , and b with the Celerity c : Whence the relative Celerity, before the Stroke, will (by 20 Def.) be $c - c$. Therefore, since (by 9 Prop.) $c - c$ is also the relative Celerity after the Stroke, as well as before; if we put x for the Celerity of B , and z for that of b , after the Stroke, and leave x undetermined to the Sign $+$ or $-$, (because though b , after the Stroke, must certainly move towards E , yet B after the Stroke may move either towards E or D) we will have $x + c - c = z$, by pre-

D ceed

ceed. Lem. For Instance, if x be $= 3$, and $z = 5$; then $c - c$ the relative Velocity, after the Stroke, is either 2 or 8, according as the Body B , after the Stroke, moves towards E or D ; and $+ 3 + 2$ is $= 5$, also $- 3 + 8 = 5$.

Now the Motion of B towards E , after the Stroke (by 5 Cor. 5 Prop.) is Bx with an undetermin'd Sign; and the Motion of b towards E , after the Stroke, is $(bz =) bx + bc - bc$: And so the Sum of these two Motions, or the whole Motion of B and b towards E , after the Stroke, is $Bx + bx + bc - bc$: But the whole Motion towards E , before the Stroke, is $Bc + bc$. Therefore (by 10 Prop.) is $Bx + bx + bc - bc = Bc + bc$: Whence $Bx + bx = Bc + 2bc - bc$: And so $x = \frac{Bc + 2bc - bc}{B + b}$; which Celerity x (of the Body B after the Stroke) will be either positive, and so towards E , or else negative, and so towards D , according as $Bc + 2bc$ is more or less than bc .

Again,

Again, the Celerity of b , after the Stroke, viz. z is $(= x + c - c = \frac{BC + 2bc - bC}{B + b} + c - c) = \frac{2BC + bc - Bc}{B + b}$

Wherefore, if two perfectly elastick Bodies B, b , moving to the same Part z , be given, and also their Celerities c, c , before the Stroke; it will be easy to find their Celerities x, z , after the Stroke; and to what Parts they tend. For Instance, suppose $B = 3, b = 2, c = 7,$

$c = 5$; then is $x (= \frac{BC + 2bc - bC}{B + b} = \frac{21 + 20 - 14}{3 + 2}) = + 5\frac{2}{5}$; that is, the

Body B , after the Stroke, moves yet towards E (by Reason of the positive Sign)

with $5\frac{2}{5}$ Degrees of Celerity. Again,

z is $(= \frac{2BC + bc - Bc}{B + b} = \frac{42 + 10 - 15}{5}) =$

$+ 7\frac{2}{5}$; that is, the Body b , after the

Stroke, moves towards E with $7\frac{2}{5}$ Degrees of Celerity. And the relative

Velocity, after the Stroke, $7\frac{2}{5} - 5\frac{2}{5}$ is

$D 2$

equal

equal to $7 - 5$, the relative Velocity before the Stroke, as it should be.

If B be $= 2$, $b = 5$, $C = 13$, $c = 3$; then $x = \left(\frac{BC + 2bc - bC}{B + b} \right)$ is $= -$

$1 \frac{2}{7}$: Wherefore the Sign of $1 \frac{2}{7}$ or x being negative, the Body B , after the Stroke, moves backwards towards D with the Celerity $1 \frac{2}{7}$. Again, $z (= \frac{2BC + bc - Bc}{B + b})$

is $= + 8 \frac{5}{7}$: Wherefore the Body b , after the Stroke, moves towards E with the Celerity $8 \frac{5}{7}$.

2. If the elastick Bodies, B, b , before the Stroke, move with the Celerities C, c , towards contrary Parts, viz. B towards E , and b towards D ; their relative Celerity, before the Stroke, will be $C + c$. Therefore (by 9 Prop.) $C + c$ will also be their relative Celerity after the Stroke; and if we put x, z for their simple Celerities after the Stroke, leaving the Signs both of x and z undetermined, there will (by preceed. Lem.) be $x + C + c = z$, Now the Motion of B to-

towards E , after the Stroke, is Bx , and that of b towards E is $(bz =) bx + bc + bc$; whole Sum $Bx + bx + bc + bc$ is (by 10 Prop.) $= BC - bc$, the whole Motion of B and b towards E before the Stroke. Whence $Bx + bx = BC - bc - 2bc$; and x (the Celerity of B after the Stroke) $= \frac{BC - bc - 2bc}{B + b}$ positive or negative, according as BC is more or less than $bc + 2bc$.

Again, z (the Velocity of b after the Stroke) is $= x + c + c = \frac{BC - bc - 2bc}{B + b} + c + c = \frac{2BC + Bc - bc}{B + b}$.

Whence, in the Case of two perfectly elastick Bodies B, b , moving towards contrary Parts E, D ; the Bodies themselves, and their Celerities C, c , before the Stroke, being given; we may easily find their Celerities x, z , after the Stroke, and to what Parts the Bodies tend. For Example, suppose $B = 3$, $b = 2$, $C = 7$, $c = 5$; then is $x = \left(\frac{BC - bc - 2bc}{B + b} \right) = 2\frac{3}{5}$; Therefore

$D 3\frac{3}{5}$

the

the Body B before the Stroke, moving towards E with 7 Degrees of Velocity, will after the Stroke move towards the contrary Part D (because of the negative Sign before $2\frac{3}{5}$) with $2\frac{3}{5}$ Degrees of

Velocity. Again, z is $= \left(\frac{2BC + Bc - bc}{B + b} = \right)$

$+9\frac{2}{5}$; therefore the Body b , after the Stroke, will move towards E with $9\frac{2}{5}$ Degrees of Velocity.

Corollaries.

I. IF the elastick Body B move towards E before the Stroke, and the other Body b be at Rest; then is $c = 0$, and consequently $x = \frac{BC - bC}{B + b}$, and as $B + b : B - b :: c : x$. Again, since c is $= 0$, therefore is $z = \frac{2BC}{B + b}$, and as $B + b : 2B :: c : z$. Hence, if B be $= b$ at Rest, then is $x = \left(\frac{BC - BC}{2B} = \right) 0$, that is, B , after the Stroke, rests; and $z = \left(\frac{2BC}{2B} = \right) c$, that

c, that is, *b*, after the Stroke, moves towards *e* with the Celerity that *B* had before the Stroke.

2. If the ~~elastic~~ Bodies *B* and *b* be equal, and move towards contrary Parts before the Stroke: Then is $x = \left(\frac{-2Bc}{2B} = \right) -c$; and $z = \left(\frac{+2BC}{2B} = \right) +c$; that is, after the Stroke, the Bodies *B*, *b*, will move towards contrary Parts with interchanged Velocities, viz. *B*, after the Stroke, with the Velocity of *b* before the Stroke, and *b*, after the Stroke, with the Velocity of *B* before the Stroke.

3. If the Bodies *B*, *b*, be equal, and move towards the same Part before the Stroke, *b* going before with the Velocity *c*, and *B* following with a greater Velocity *c*; then is $x = \left(\frac{+2bc}{B+b} = \right) +c$, and $z = \left(\frac{+2BC}{B+b} = \right) +c$.

4. If the Bodies *B*, *b*, (whether equal or unequal) move towards contrary Parts

be-

before the Stroke, and there be as $B:b::c:C$; then is $BC = bc$, that is, the Moments or Motions are equal; and x ($= \frac{BC - bC - 2bc}{B + b} = \frac{-BC - bc}{B + b}$) is $= -C$, and z ($= \frac{2BC + Bc - bc}{B + b} = \frac{bc + Bc}{B + b}$) is $= +c$.

PROPOSITION XIII.

Theor. Fig. 15.

IF a Body A move uniformly in the right Line AB , whilst the said Line AB , moves uniformly always parallel to itself, keeping its Extremity A in the right Line AC ; and the Parallelogram $ABDC$ being completed, if the Velocity of the Body A in the Line AB , be to the Velocity of the Line AB itself, as AB to AC : Then the Body A by this compound Motion, will really describe the Diagonal AD in the same Time that it describes the Line AB , or that the End A of the Line AB describes the Line AC .

When

When the Line AB comes to any other Situation ab , let g be the Place of the Body A ; and draw gG parallel to AC . Now, since the Spaces run thro' in the same Time, by the Body A in the Line AB or ab , and by the Line AB itself, are AG or ag and Aa or Gg ; therefore (by 1 Prop.) as is AG to Aa , so is the Velocity of the Body A in the Line AB to the Velocity of the Line AB , that is, (by Hyp.) AB to AC . Whence the Parallelograms aG and CB are similar: Therefore the Point g is (by 26. 6. Eucl.) in the Diagonal AD ; and consequently (since the Point g was taken at Pleasure, ab being taken in any Situation parallel to AB) the Body A will always be in the Diagonal AD . And that it will describe the Diagonal AD in the same Time that it describes the Side AB , or that AB describes AC , is evident; because (by Hyp.) the Velocity of the Body A in the Line AB is to the Velocity of the Line AB as AB to AC , and consequently (by 2 Cor. 1 Prop.) in the same Time that the Body A , moving
in

in AB , describes AB , in the same Time AB describes AC , and at the End of the said Time coincides with CD .

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A Corollary.

HENCE it is evident, that, if a Body A be urged by two Forces together, the one acting in the Direction AB , and the other in the Direction AC ; the Force acting in the Direction AB will not at all hinder the Motion (by the other Force) of the Body A towards CD parallel to AB , nor will the Force in the Direction AC hinder the Motion of the Body A towards BD parallel to AC .

PROPOSITION XIV.

Theor. Fig. 15.

IF a Body A have two Forces impress upon it together in different Directions; by which Forces acting separately, it would uniformly describe the Sides AB , AC of a Parallelogram $ABDC$ in equal Times,
or

or (by 1 Prop.) with Velocities as AB , AC : I say, that the Body A with these two Forces united, will describe the Diagonal AD , in the same Time that it would describe AB , or AC with the one, or the other of the said Forces acting separately.

Suppose the Body A could describe the Side AB , in the Time τ , with a certain Force M ; and in the same or equal Time, the Side AC , with another Force N : The Force N acting in the Direction AC , is (by Cor. 13 Prop.) no Impediment to the Body's Motion towards BD , arising from the Force M ; the Body therefore will be carried to the Line BD , in the same Time τ , whether the Force N be imprest or not: Therefore at the End of the Time τ , it will be found somewhere in the Line BD . By the same Argument, it will at the End of the Time τ be found somewhere in CD ; and therefore of Necessity in the Course D of the said Lines BD and CD ; and so by the Forces M and N united, or the compound Force, moves in the Diagonal

Diagonal AD . Wherefore the Velocities in AB , AD , AC , will (by 1 *Prop.*) be as the Spaces AB , AD , AC .

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Corollaries.

1. **T**HE Forces M , N , and the compound Force resulting thence; also the Motions, and Velocities arising from these three Forces respectively, by which the Sides AB , AC , and the Diagonal AD may be described by the same Body in equal Times, are proportional to, and consequently may be expounded by the said Sides and Diagonal respectively.

For (by 1 and 14 *Prop.*) the Lines AB , AC , AD are proportional to the Velocities in AB , AC , AD ; and these Velocities are (by 4 *Prop.*) proportional to the Moments or Motions in AB , AC , AD ; and these Motions are (by 2 *Max.*) proportional to the Forces M , N , and the compound Force, acting in the Directions AB , AC , AD . Hence,

2. If

2. If the Diagonal AD and Side AB be equal; the Velocity in AD resulting from the compound Force, will be equal to the Velocity in AB resulting from the simple Force in AB .

3. Any Force or Motion, though in itself ever so simple, may be considered as compounded of other Forces or Motions. Thus the Motion as AD (Fig. 16.) in the Diagonal AD of the Parallelograms CB and FE , may be resolved into the Motions as AB and AC in the Sides AB and AC , and also into the Motions as AE and AF in the Sides AE and AF . And on the contrary, two Forces as AB and AC , urging in the Directions AB and AC , are united equivalent to one single Force as AD in the Direction AD .

PROPOSITION XV.

Theor. Fig. 17.

IF a Body in A be urged by a Force as AB in the Direction AB , by which in a certain Time it would uniformly describe

E scribe

scribe the Line AB ; but really in the same Time describes the right Line AD : The said Body is also impell'd in A , in the Direction AC parallel to the right Line BD that joins the Points B and D , by some other Force as AC , equal to BD .

For if AC be not the other Force in the Direction AC , whereby the Body in A is impell'd; it will be some other Force as AK , in the Direction AC or AK , or as AF in a different Direction AF . Complete the Parallelograms $BACD$, $BAKL$, $BAFE$, and draw the Diagonals AL , AE .

First, let the other Force be AK in that Direction; then AD (by *Hyp.*) is really describ'd by the Body, in the Time that AB would have been describ'd by the Force AB ; and AL (by 14 *Prop.*) is also really describ'd in the Time that AB would have been describ'd: Therefore AD and AL are both really describ'd by the same Body, in one and the same Time. *w. I. A.*

Secondly, if AF be the other Force in the Direction AF , by which the Body

in

in A is urged; the Diagonal AE will either be a different Line from the Diagonal AD , or that Line will fall upon this and be of different Length: Then just as before (by *Hyp.* and 14 *Prop.*) AD and AE will both really be describ'd by the Body, in one and the same Time, viz. in the Time that AB would have been describ'd by the Force AB . W. I. A.

A Scholy. Fig. 18.

LET AB and AS be two right Lines making any Angle at A , and let a Body in A be impell'd or urged, at the same Instant of Time, by two Forces in the Directions AB and AS ; then it is plain, that one of the said Forces may be so adjusted and proportioned to the other, as to make the Body move in any Direction AC between AB and AS .

PROPOSITION XVI.

Probl. Fig. 19.

TO determine the Directions and Cele-
rities, after the Stroke, of Bodies
striking one another obliquely.

Let two Bodies A, B, move in the
right Lines AC, BC, that incline to one
another; and let AC, BC express the
Proportion of the Motions of these Bo-
dies: Let the right Line ECM represent
a Plane which the Bodies touch in the
Point of Concourse c; to which Plane
demit from A and B the Perpendiculars
AE and BF, and complete the Re-
ctangles EG, FH.

The Motion as AC of the Body A in
the Direction AC, may (by 3 Cor. 14
Prop.) be resolved into other two Moti-
ons as AE and AG in the Directions AE
and AG; which three Motions are (by
4 Prop.) as the Velocities; and conse-
quently the Velocities are as AC, AE,
AG. Therefore the Velocity as AC in
the

the Direction AC , may be resolved into the Velocities as AE , AG , in the Directions AE , AG . For the same Reasons, the Velocity of the Body B as BC in the Direction BC , may be resolved into the Velocities as BF , BH in the Directions BF , BH . But AG and BH being parallel, the Velocities AG , BH , in the Directions AG , BH , do nothing to make the Bodies approach and strike, and so are not at all concerned with, nor altered by the Stroke. Therefore the Velocities, whereby the Bodies meet and strike one another, are only those that are as AE or GC , and BF or HC in the Directions GC and HC . Therefore the Bodies A and B striking directly against one another with the Velocities GC and HC , their Velocities, after the Stroke in the Line HC , may be determined by 11 *Prop.* if they be soft or perfectly hard Bodies, or by 12 *Prop.* if they be perfectly elastick

Suppose then CL was thus found to be as the Velocity, and consequently as the Force of the Body A , moving
E 3
from

from e towards g , after the Stroke; since, as before, the Force in the Body A , of moving in the Direction AG or EC with the Velocity AG , is not altered by the Stroke, produce EC till CM be $= EC$, and complete the Rectangle LM ; the Body A , after the Stroke, will (as is evident from 14 *Prop.*) move in the Diagonal EN , with a Velocity as EN . In like Manner, if (by 11 or 12 *Prop.*) we find the Velocity of the Body B , after the Stroke, to be as QC in the Line CH ; its true Velocity and Direction, after the Stroke, may be determined, by making $CS = EC$, and completing the Rectangle QS : For then the Body B , after the Stroke, will run in the Diagonal CR , with a Velocity as CR .

Example. Let two perfectly elastic Spherick Bodies A, B , as 2, 3, move in the oblique Directions AC, BC , with Velocities as AC, BC , or 7, 5; and let it be required to find the Velocities and Directions of the said Bodies, after they meet and strike in C . Bisect the Angle BCA by the right Line ECM , and

and ECM will represent the Plane which the Bodies A and B touch in C . Then (the Triangles ACE , BCF being similar) there will be as $AC:BC$ ($:: 7:5$)

$:: AE:BF :: GC:HC :: AG:BH ::$

$EC:FC ::$ (by making $CM = EC$, and $CS = FC$) $CM:CS$. Let x denote the

Velocity (after the Stroke) of the Body A referred to the Line HG , and z the

Velocity (after the Stroke) of the Body B referred to the same Line: Then since

the Bodies A and B are as 2 and 3, and the Velocities whereby they strike di-

rectly, the one against the other, are as GC and HC , or as 7 and 5; we will

find (by 2 Part 12 Prop.) $x = -7\frac{4}{5}$, and $z = +4\frac{6}{5}$. Therefore, after the

Stroke, the Bodies A and B will move towards contrary Parts, *viz.* A from C

towards G , with a Velocity as $7\frac{4}{5}$ refer'd to the Line HG , and B from C

towards H , with a Velocity as $4\frac{6}{5}$ refer'd to the same Line. Therefore, since

CM and CS are as 7 and 5, if we make $GL = 7\frac{4}{5}$, and $CQ = 4\frac{6}{5}$, and com-

plete the Rectangles LM and QS ; there
will

will be $CN = \sqrt{MN^2 + CM^2} =$
 $\sqrt{5476 + 49} = 10'18$, and $CR =$
 $\sqrt{RS^2 + CS^2} = \sqrt{2116 + 25} = 6'79$.
 Wherefore the Bodies A and B, after
 the Stroke, will really move, in the
 Diagonals CN and CR, with Velocities
 as 10'18 and 6'79.

PROPOSITION XVII.

Theor. Fig. 20.

A Stroke made by a Body A upon a firm
 and immoveable Plane EF, in an ob-
 lique Direction AC, is to one made in a
 perpendicular Direction AD, the Body
 moving separately in both Directions with
 the same Degree of Velocity, as AD is to
 AC, or as the Sine of the Angle of Incidence
 ACD is to the Radius.

The Rectangle ADCB being com-
 pleted, the Motion of the Body A, re-
 sulting from a Force as AC, in the Di-
 rection AC, is (by 3. Cor. 14. Prop.)
 equivalent to two other Motions in the
 Lines AD and AB, resulting from two
 other

other Forces as AD and AB . But the Force or Motion whose Direction is AB , is of no Significancy as to the Stroke upon the Plane EF ; because AB being parallel to EF , the Body moving in the Direction AB , would never meet with EF . Therefore the Force by which the Body moves in AC being as AC , that Force by which it strikes upon the Plane in that oblique Direction AC , is as AD . But if the Body mov'd in the perpendicular Direction AD , with a Force as AC , and consequently with the same Velocity as it moves in AC , the Quantity of the perpendicular Stroke would be equivalent to the Force as AC , because the said Force would be wholly destroy'd by the perpendicular Stroke. Therefore the Quantity of the oblique Stroke is to the Quantity of the perpendicular Stroke, as AD is to AC , that is, (AC being made Radius) as the Sine of the Angle of Incidence ACD is to the Radius. $W. W. D.$

PROPOSITION XVIII.

Theor. Fig. 21.

IF a perfectly-elastic Body A, moving in the right Line AB, strike obliquely against a firm and immoveable Plane HG: After the Stroke it will be reflected by that Plane with the same Force that it came with; and will move in such a right Line BC, that the Angle of Reflection CBE will be equal to the Angle of Incidence ABD.

Let AD represent the Force, or Motion of the Body A, in the Direction AB; then, AD being drawn perpendicular to the Plane HG, and the Rectangle DE completed, the Motion in AB is (by 3 Cor. 14 Prop.) equivalent to two Motions, in the Directions AD and AE, proportional to the Lines AD and AE. But since AE is parallel to HG, and AD perpendicular thereto; the Force by which the Body strikes against the Plane, is only that which is as AD. Make BE = DB,

DB , or AE , and complete the Rectangle EF , which will be every Way equal and similar to the Rectangle DE , and consequently $BC = AB$, and $\angle CBF = \angle ABD$. Since the Force as AE , acting in the Direction AE parallel to the Plane HG , is not diminished by the Stroke, the Force AE remains in the Body A after the Stroke, to move it in the Direction BF or AE . But from the Nature of a perfectly elastick Body it is evident, that the Body A , striking the immoveable Plane HG , in the perpendicular Direction AD or EB , will reflect with the same Force in the same Line of Direction: Therefore the Motion of the Body A , at the Point of Incidence B , is compounded of the Motions as BF and BE in the Directions BF and BE . Therefore (by 14 Prop.) the Body A , after the Stroke, will move in the Diagonal BC of the Rectangle EF , with a Force as BC equal to AB ; and the Angle of Reflection CBF is (as before) equal to the Angle of Incidence ABD . W. W. D.

PROPOSITION XIX.

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ALL Bodies near the Surface of the Earth, gravitate (in a free Space) in Proportion to their Quantity of Matter; that is, their *Weights* are proportional to the Bodies themselves.

For it is known by many Experiments, that all Bodies near the Earth's Surface, falling perpendicularly by the Force of Gravity, in a free Space, descend equal Spaces in equal Times: And therefore, at the End of any Time given, they acquire equal Velocities. Therefore the Motions or Moments acquired at the End of the said Times, being (by 4 Cor. 5 Prop.) as the Bodies multiplied into their equal Velocities, are as the Bodies themselves. But the Forces that generate these Moments, *that is*, the Gravitations or Weights of the Bodies, are (by 2 Max.) proportional to the said Moments: Therefore the Weights

Weights are proportional to the Bodies.

W. W. D.

Corol. Hence, a Body may be considered as its Weight.

PROPOSITION XX.

Theor.

THE Motion of a descending Body near the Surface of the Earth; falling from Rest, in a free Space, by the Force of its Gravity, is an equably accelerated Motion.

For the Gravity or Weight of a Body near the Earth, is not sensibly altered by a small Alteration of that Body's Distance from the Earth; or the Force of Gravity, at all small Distances from the Earth, acts equally on the same Body. Suppose then, the Time in which a heavy Body falls, to be divided into equal, but infinitely small Particles, and Gravitation acting in the first Particle of Time, to give the Body an Impulse towards the Center of the Earth, and make it acquire

quire a certain Degree of Velocity. Now, if, after that first Impulse, the Action of Gravity should cease, yet the Motion arising from the said Impulse would be continued, and the Body would (by 1 *Max.*) move uniformly, or with the same Velocity, towards the Centre of the Earth. But since Gravity acts in the second Particle of Time, with the same Force that it did in the first, the same Gravitation will give the Body another Impulse equal to the former; and so the whole Velocity, after these two Impulses, will be double of the first. If again the Action of Gravitation should cease, after the second Impulse, yet the Body would still move with two Degrees of Velocity. But since, in the third Particle of Time, the Body is yet urged by the same Force of Gravity as before, it will thereby acquire a third Degree of Velocity, equal to either of the other two. And in like Manner, in the fourth Particle of Time, it will acquire a fourth Degree; and so forth. Therefore, the heavy Body will (by 8 *Def.*) descend with

with an uniformly accelerated Motion;
W. W. D.

Corol. Hence, the Velocity acquired by the Fall of a Body from Rest, is always as the Time of the Fall.

Scholies.

1. **I**T may, in like Manner, from the same Principles be demonstrated, that, if a Body be forced directly upwards, it will move with an equably retarded Motion :— Because the Force of Gravity still acting equally, contrary to the Body's Motion upwards, will in equal Times equally diminish that Motion, till it be totally destroy'd.

2. Though in the preceeding *Proposition* we have, to render the Demonstration the clearer, suppos'd the Body's Fall to begin from Rest ; yet the Motion downwards will be an uniformly accelerated Motion, though we suppose its Fall to begin from any Degree of Velocity ; by reason of the continual equal Impulses of Gravity.

PROPOSITION XXI.

Theor. Fig. 22.

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IF one side AB of a Triangle ABC represent the Time, in which a Body falls from Rest in a free Space, and another Side BC the Velocity acquired at the end of that Time; and through any Point D of AB there be drawn a right Line DE parallel to BC : This DE will represent the Velocity acquired at the end of the Time represented by AD .

For (by reason of the similar Triangles ABC , ADE) as is $AB : AD :: BC : DE$. But BC represents the Velocity at the end of the Time AB : Therefore, since (by Cor. 26. Prop.) the Velocities are as the Times, DE will represent the Velocity at the end of the Time AD .
W. W. D.

PROP.

PROPOSITION XXII.

Theor. Fig. 23.

THE Space run through in a certain Time, by a heavy Body falling from Rest, in a free Space, near the Surface of the Earth; is the half of that Space it would run through in the same Time, with the Velocity acquired in the last Instant of that Time:

Let AB be as the Time in which a heavy Body falls from Rest, and BC as the Velocity acquired at the end of it; complete the right-angled Triangle ABC . Suppose the Time AB divided into an infinite Number of equal Parts $Ar, re, ei, im, mp, \&c.$ and draw $ro, ef, ik, mn, \&c.$ parallel to the Base BC . Then (by 21. Prop.) $ro, ef, ik, mn, \&c.$ will represent the Velocities in the Particles of Time $re, ei, im, mp, \&c.$ And (by 1. Prop.) the Spaces run thro' in these Particles of Time, are as the said Velocities $ro, ef, ik, \&c.$ or (by 1. 6. Encl.)

as the Parallelograms eo , if , mk , &c. Therefore the whole Space run thro', by the Fall of the heavy Body, in the Time AB , is as the Triangle ABC compos'd of all the infinitely little Parallelograms eo , if , mk , pn , &c. But if the Body was carried with the Celerity BC during the whole Time AB , the Space run thro' in that Time, would (by 3. Cor. 6. Prop.) be as the Rectangle $AB \times BC$, which is double of the Triangle ABC . Therefore the Space run thro', by the Fall of a heavy Body from Rest, in the Time AB , is half the Space that it would have run thro' in that Time, with the Celerity acquir'd at the end of that Time. *w. w. d.*

P. Corollaries.

HENCE, as the Space descended from the Beginning of the Fall, in the Time AB , is represented by the Triangle ABC ; so the Spaces descended, in the Times Ap , Am , may be represented by the Triangles Apq , Amn . Hence,

2. The

2. The Spaces descended from the Beginning of the Fall, are in a duplicate Proportion of the Times of Descent. For the Spaces descended in the Times AB, Ap , are as the Triangles ABC, Apq , which being similar are in a duplicate Proportion of the Sides or Times AB, Ap . Hence,

3. If the Times of a heavy Body's Fall from Rest be as 1, 2, 3, 4, 5, &c. the Spaces descended in those Times will be as 1, 4, 9, 16, 25, &c. the Squares of those Numbers. Hence,

4. If the Times of Descent Ar, Ae, Ai, Am , &c. be as 1, 2, 3, 4, &c. the Spaces descended in the Times Ar, re, ei, im , &c. will be as the odd Numbers 1, 3, 5, 7, 9, &c.

5. Since the Velocities acquired by the Falls from Rest, are (by Cor. 20 Prop.), as the Times; the Spaces descended from Rest will (by 2 Cor. 22 Prop.) be in a duplicate Proportion of the Velocities acquired at the Ends of the Times.

A Scholy.

WE may, just as in 20. *Prop.* prove, that if a Body be constantly urged in any Direction, by a Force acting equally; its Motion will be an equably accelerated one: And consequently, as in 22 *Prop.* that the Space it runs through from the Beginning of the Motion, is half the Space it would run thro', in the same Time, with the Velocity acquired in the last Instant. From whence we may infer (after the same Manner as we inferred 2 *Cor.* from 22 *Prop.*) that the Spaces run thro', from the Beginning of the Motion, are in a duplicate Proportion of the Times.

PROPOSITION XXIII.*Theor. Fig. 24.*

IF a Body *A* be held immovable by three Powers, or Forces urging it according to the Directions ΔB , ΔC , ΔE ; these Powers

Powers will be to one another as three right Lines AD , AC , CD , making a Triangle, whose first Side AD is a Part of the first Direction AB produc'd, the second Side AC the same with the second Direction AC , and the third Side CD parallel to the third Direction AE .

For if the Body A be held immovable by two Powers, or Forces, urging it in contrary Directions AB and AD , these two Forces will be equal; and so each of them may be represented by one and the same determinate Line AD . But (by Hyp.) the Body A is kept immovable by three Forces urging it in the Directions AB , AC , AE : Therefore the joint Force of the latter two, must be equivalent to the first alone which is as AD , and must united urge the Body in the Direction AD with a Force as AD . But (by 3 Cor. 14 Prop.) two Forces which are as AC and AE , are united equivalent to the Force as AD urging in the Direction AD , and consequently to the Force as AD urging in the Direction AB . Therefore the three Forces

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Sine of the Angle of Inclination of that Plane to the Horizon is to the Radius.

Let CD represent the Horizon, and the Angle ACD will be the Inclination of the Plane; from A and B to CD demit the Perpendiculars AD and BE ; through B draw a right Line HBF perpendicular to the Plane AC , and from F raise FG perpendicular to CD . The Body B is urged by three Powers, and kept immoveable by them: The First is the Body's absolute Force of Gravity, acting according to the Direction BE perpendicular to the Horizon CD ; the Second is the Power, or Force R , urging the Body in the Direction BR parallel to the Plane; and the Resistance of the Plane, urging the Body according to the Direction BH , supplies the Place of the third Power. Therefore FG , QG , QF being respectively parallel to the Directions of these three Powers; there is (by 23 *Prop.* and its *Cor.*) as the Power R to the Body's absolute Gravity, so QG to FG :: (by 2 *Cor.* 8. 6. *Euc.*) $FG : GC ::$ (by 4. 6. *Euc.*) $AD : AC ::$

$AC :: \text{Sine of Inclination to the Radius.}$
 W. W. D.

www.bibliolibrary.cn *A Corollary.*

SINCE the Power R hinders the Descent of the Body B on the Plane AC , and is equivalent to the Body's Momentum whereby it endeavours to descend; it is manifest, that a Body's Force to descend on an inclining Plane is to its absolute Force of Gravity, whereby it endeavours to descend in the Perpendicular to the Horizon, as the Sine of the Plane's Inclination is to the Radius.

PROPOSITION XXV.

Theor. Fig. 26.

THE Descent of a heavy Body upon an inclining Plane, is an equably accelerated Motion.

Let B be a Body descending on the inclining Plane AD . Then (by *Cor. 24 Prop.*) the Force, whereby B endeavours

vours to descend on the Plane AD , is to its absolute Force of Gravity, as the Sine of the Plane's Inclination ADC is to the Radius; which is a constant and invariable Proportion: And consequently, the absolute Force of Gravity being still the same in the Body B , its Force, whereby it endeavours to descend upon the inclining Plane, remains also still the same. Therefore this last Force will always act equally on the Body B ; and consequently its Descent on the inclining Plane AD , will easily be proved to be an uniformly accelerated Motion, by the same way of Reasoning that was used in 20 *Prop.*

Corollaries.

1. **H**ENCE, the Velocity acquired by the Descent of a heavy Body from Rest on an inclining Plane, is always as the Time of Descent.

2. From 25 *Prop.* and 1 *Cor.* it is evident, that whatever has been demonstrated in 22 *Prop.* and its *Corollaries*, of
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heavy Bodies falling perpendicularly, is after the same Manner demonstrable of their Descent upon inclining Planes. Namely, that the Space run thro' in a given Time by a heavy Body on an inclining Plane, computed from the Beginning of the Motion, is half the Space that it would uniformly run thro' in the said Time with the Velocity last acquired. Also that the Spaces run thro' from the Beginning of the Motion, are in a duplicate Proportion of the Times, and also of the last acquired Celerities.

3. The Ascent of a heavy Body upon an inclining Plane, is an uniformly retarded Motion.

PROPOSITION XXVI.

Theor. Fig. 26.

THE Velocity that a heavy Body acquires descending from Rest on an inclining Plane AD , in any given Time, is to the Velocity that it would acquire in the same Time, falling from Rest perpendicularly

cularly in AC , as the Plane's Height AC is to its Length AD .

For the Increments of Velocity of the Body B , descending from A , in the Perpendicular AC , and on the inclining Plane AD ; produced in an infinitely small Particle of Time, are to one another as the Forces whereby they are produced: But these Forces are (by *Cor. 24 Prop.*) in the constant Proportion of the Radius to the Sine of the Plane's Inclination ADC ; or as the Plane's Length AD to its Height AC : Therefore the Increments of the Velocities are in the Proportion of AD to AC . Therefore (by *12. 5. Eucl.*) the Sum of the Increments in the Perpendicular AC is to the Sum of the Increments in the Plane, as AD is to AC ; that is, the Velocity of the heavy Body, falling in the Perpendicular, is to the Velocity it would acquire, descending in the same Time on the inclining Plane, as the Length of the Plane is to its Height. *W. W. D.*

PROPOSITION XXVII.

Theor. Fig. 27.

IF AB represent an inclining Plane, BC an horizontal Line, and AC a Perpendicular hereto; then CD being drawn perpendicular to AB , I say, in the Time that a heavy Body descends, on the inclining Plane from A to D , that in the same Time the same or any other heavy Body would descend in the Perpendicular from A to C .

If this be denied: Let AE be the Space run through on the inclining Plane, while AC is run through in the Perpendicular AC . Then because (by 22 Prop.) in the Time that the heavy Body descends from A to C , or from A to E , twice the Length of AC would be run through with an uniform Velocity, equal to that which is acquired in C by the perpendicular Descent, and so (by 2 Cor. 25 Prop.) would twice the Length of AE be run through with the Velocity acquir'd in E : The Velocity acquir'd in C will (by 1 Prop.) be to the Velocity acquir'd in E , as twice AC to twice AE ,
OR

or as $\overline{AC}$ to $\overline{AE}$. But since $\overline{AC}$ and $\overline{AE}$ are Spaces descended in the same Time; therefore (by 26 Prop.) the Velocity in c is to the Velocity in e , as $\overline{AB}$ is to $\overline{AC}$. Wherefore (by 11. 5. Eucl.) as is $\overline{AB} :: \overline{AC} :: \overline{AE} :: \overline{AD}$. But (by 2 Cor. 8. 6. Eucl.) as is $\overline{AB} : \overline{AC} :: \overline{AC} : \overline{AD}$. Therefore as $\overline{AC} : \overline{AE} :: \overline{AE} : \overline{AD}$; and consequently $\overline{AE}$ is $= \overline{AD}$. W. I. A.

Corollaries.

1. FROM this Demonstration it is evident, that the Velocity acquired, in the same Time, by heavy Bodies descending from Rest in the Perpendicular and on the inclining Plane, are as the Spaces run through by them.

2. Hence is found the Space through which a heavy Body falls in the Perpendicular, in the same Time that another Body descends a given Length $\overline{AD}$ on the inclined Plane $\overline{AB}$; namely, if from the Point p there be raised to $\overline{AB}$ a Perpendicular $\overline{DC}$, meeting the Perpendicular $\overline{AC}$ to the Horizon in c , $\overline{AC}$ will be the Space sought.

PART II.

Of Centripetal Forces.

PROPOSITION XXVIII.

Lem. Fig. 28.

IF two right Lines AB and DB meet in B , and touch a Circle in A and D ; then SB being drawn from the Center S , also $DC \parallel SB$, and AB be produc'd till it meet with DC in C : I say, that BD is $= BC$.

For SA and SD being drawn, SA is $= SD$; and (by 2 Cor. 36. 3. *Eucl.*) BA is $= BD$; also SB is a common Side of the Triangles ASB and BSD : Therefore (by 8. 1. *Eucl.*) is $\angle ABS = \angle SBD$: But (by 29. 1. *Eucl.*) $\angle SBD$ is $= \angle BDC$, and $\angle ABS$ is $= \angle C$: Therefore is $\angle BDC = \angle C$; and consequently (by 6. 1. *Eucl.*) $BD = BC$. W.W.D.

Corol.

Corol. Hence $e > ABS = SBD$ is $= \frac{1}{2} ABD$.

PROPOSITION XXIX.

Lem. Fig. 29.

LET BEHIK be a regular Polygon described about a Circle, that is, a Polygon of equal Angles, and equal Sides whereof every one touches the Circle: And let a Body in A be impelled in the tangential Direction AC by one single Impulse, and move on uniformly till it come to B; where, let it receive another single Impulse in the Direction BS from a centripetal Force tending to the Circle's Center s, such as being conjoined with the foresaid tangential or projectile Force in B, may turn the Body from the Tangent AC and make it move along the Tangent BE. When the Body is come to E, let it there receive a second Impulse from the centripetal Force directed to s: I say, it will then move along the third Tangent EH. When it comes

to H , let it receive a third Impulse from the centripetal Force : Then will it move along the fourth Tangent HI : And so forth : And the Body's Velocity through every Tangent will still be the same.

For DC being drawn parallel to SB , BD is (by 28 Prop.) $= BC$, and (by 2 Cor. 14 Prop.) the Body's Velocity in BD (or BE) is equal to the Velocity it would have had in BC before the first centripetal Impulse in B , or the Velocity it had in AB . When the Body comes to E , it receives the second centripetal Impulse, which is equal to the former, because SE is $= SB$: For (by Cor. 28 Prop.) $\triangleright SBD$ is $= \frac{1}{2} ABD = (\text{by Hyp.}) \frac{1}{2} DEG = (\text{by Cor. 28 Prop.}) SED$; and (by 18: 3. Eucl.) $\triangleright BDS$ is $= a \text{ Right} = SDE$, and SD common : Therefore (by 26. 1. Eucl.) is $SB = SE$; in like Manner is $SE = SH$. Therefore, since the Body's Velocity along the Tangent BE , is the same with its Velocity along the Tangent AB , and the second centripetal

tal.

tal Impulse at E is equal to the first centripetal Impulse at B ; the second centripetal Impulse must draw it just as much aside from the Tangent BE or DE , as the first did from the Tangent AB ; that is, the Line of Direction, after the second Impulse, must make an Angle with BE equal to the Angle ABE ; but (by Hyp.) $\angle BEH$ is $= \angle ABE$: Therefore the Tangent EH must be the Body's Direction after the second centripetal Impulse; for this Direction must be between ES and EF : And GF being drawh parallel to ES , EG will (by 28 Prop.) be equal to EF ; and (by 2 Cor. 14 Prop.) the Velocity in EG or EH will be the same with what it was in DE or BE and AB . In like Manner, after the third Impulse of the centripetal Force in H (which is equal to either of the former in B or E , by reason that SH is $= SE = SB$) the next Direction will be in the Tangent HI with the same Velocity as before. And so on in the rest of the Tangents. w. D.

A Scholy.

IN the preceeding Demonstration we only suppose, that the Center of the centripetal Force is the Center of a Circle, and that the Impulses of the said Force at equal Distances from the said Center are equal; which is a very possible and simple Hypothesis.

PROPOSITION XXX.*Theor.*

A Body may move in the Periphery of a Circle by a projectile Force once impress'd, and an uniform centripetal Force constantly impress'd.

For a Body moved by a projectile Force once impress'd, and a centripetal Force acting equally by an infinite Number of Impulses one after another, may (as is evident from 29 *Prop.*) run in the Perimeter of a regular Polygon described about a Circle. But the said Polygon's Peri-

Perimeter, when its Sides are infinite in Number, coincides with, and is nothing different from the Periphery of the Circle; and the said centripetal Impulses, when infinite in Number, are the same with an uniform centripetal Force acting constantly without Intermission. Wherefore it is evident, that a Body once impell'd by a projectile Force, and a centripetal Force acting constantly, may move in the Periphery of a Circle. w. w. d.

PROPOSITION XXXI.

Theor.

A Body moved in the Periphery of a Circle by a projectile Force once impress'd, and an uniform centripetal Force constantly impress'd, is constantly carried with an equable Motion; or its Velocity is still the same.

This is evident from 29 Prop. since the said centripetal Force is all one with a centripetal Force acting by an infinite Number of equal Impulses, and the Velocity

locity in the Perimeter of the circumscribed regular Polygon of an infinite Number of Sides (which is the same with the Circle it self) is by that *Prop.* still the same.

A Corollary.

From 30 *Prop.* it is evident, that Mr. Gordon's first *Theorem* (in his *Remarks upon the Newtonian Philosophy* Pag. 27.) whereby he pretends to prove, that a Body impell'd by a projectile and a centripetal Force, constantly approaches to, or constantly recedes from the Center of the centripetal Force, is false and groundless: But this shall be farther proved below. It is also evident from 31 *Prop.* that all he says in *Pag.* 86 and 87 of his *Remarks*, about the constant Increase of Velocity of a Body moving in a Circle, is perfectly precarious and absurd.

Scholy

Scholy: I.

IF our ~~Remarker's~~ ~~fore said~~ ~~first~~ ~~Theorem~~ was true, it would indeed overturn the main Foundation of a great part of the *Newtonian Philosophy*: Therefore, that the *Remarker's* Mistake or Sophistry, and the Falshood of his Theorem may the better appear, we shall here set it down with his Demonstration; and plainly shew the Demonstration to be so faulty and inconsistent, that instead of proving the pretended Theorem, it proves nothing at all. His Theorem then is as follows in his own Words. See Fig. 30.

A Body turned from a streight lined Motion into a Curve that lies in a Plan, and describes, round any Point s, Areas proportionable to the Times, by a continuing centripetal Force directed to s; must either move constantly away from s, or come constantly nearer to it, in a spiral Line.

Let it here be observed, before we proceed further, that that Clause of this Theorem, which mentions the propor-

tionality of the Areas to the Times, is superfluous, since he makes not the least use of it in his Demonstration. So that the said Theorem should be expressed thus: *A Body turned, by a continuing centripetal Force directed to s , from a streight lined Motion into a Curve that lies in a Plan; must either move constantly away from s , or come constantly nearer to it, in a spiral Line.*

His Demonstration, in his own Words also, is this.

Let the Body's streight-lined Motion be the Line ABF , let the centripetal Force directed to s , be supposed to act upon the Body at the Point B ; the Line BF in which the Body at B would run out, must make with BS , the Line of the Direction of the centripetal Force acting in B , either a Right, an Acute, or an obtuse Angle. If the Angle SBF be a right or an acute Angle, then is the Angle SBG (a Part of that Angle) less than a right Angle. Draw out BC in infinitum towards Q and P , let us with our Authors, conceive the Point c to be the
next

next Point in which the centripetal Force acts; which being supposed to act continually, or in every Point, the Point c must be the next Point to b in the Line bq . Drop a Perpendicular from s , upon the Line pq , it will fall somewhere betwixt b and q ; either upon c the next Point to b in that Line, or upon some other Point betwixt c and q ; suppose upon the Point n . If the Perpendicular fall upon c , then is c the nearest Point in the Line pq to the Point s , nearer, to wit, than the Point b ; if the Perpendicular fall upon n , then is c nearer to s than b , because nearer to n : So that the centripetal Force meeting with the Line in which the Body would run out at right Angles, or less than right Angles, must necessarily force the Body to move from a Point more distant to a Point less distant from the Center s . The Line cq is that Line in which the Body at c would run out next, and the Point c is the next Point in which the centripetal Force is supposed to act; but a Perpendicular from s upon pq fell, as above,

either upon c , and then the Force directed to s , acting in the Point c , is at right Angles with cq , or upon n , and then is this Force at less than right Angles with cq . And so by the same reasoning, if the Force directed to the Center be at any Time at right, or less than right Angles with the Line in which the Body would run out, it must still continue to be so; and while it is so, the Body must move every Time the Force acts, or continually, from a Point more distant from s , to a Point less distant from s . But if a Curve be such, that the Lines in which the Body would run out, viz. the Tangents of the Curve, make with the Radius constantly more than right Angles; then is the Body that moves in that Curve, moving constantly away from s : Or if the Tangent and Radius of this Curve at any Time be at right Angles, or less than right Angles, then must the moving Body come constantly nearer to the Center s in a spiral Line.

Q. E. D.

This

This Demonstration is pretty cunning-
 ly contriv'd, but Fallacy all over : I shall
 discuss the first Part, wherein its main
 Strength lies, and the second, which is on-
 ly hinted, will fall of Course. The said
 Demonstration then, supposes the centri-
 petal Force to act upon the Body at the
 Point *B*, and then at the Point *c* next to
B ; now if the Points *B* and *c* be next to
 one another, either there is no Distance
 between them, or there is some Distance.
 If there be no Distance between *B* and *c*,
 then *B* and *c* coincide and really make but
 one Point, and so are equally distant from
s, and consequently $SC = SB$. But
 if there be some Distance (though ever
 so small) between the Points *B* and *c*,
 as the Demonstration plainly implies; for
 it supposes *BC* to be a right Line, *SCB* an
 acute Angle, and *SCB* a right one, from
 whence it infers the Point *c* to be nearer
 the Point *s* than the Point *B* is, which
 is indeed a very just Inference from the
 said Supposition, as is evident from 19.
 1. *Eucl.* I say, if there be some Distance
 between the Points *B* and *c*, then *c* is

H 3

not

not the next Point to B in which the centripetal Force acts, since it acts continually, and consequently has acted in an Infinity of Points in the Time that the Body has moved from B to C , and the Body's Path has not been a right Line, as the said pretended Demonstration supposes, but a Portion of a Curve (as being produced by a projectile and constant centripetal Force) which we may represent by the Line BC , and allow it, being extremely small, to be very little different indeed from a right Line, yet still a Curve. Now since BC is really a Curve, though we suppose SC to be perpendicular thereto, we cannot therefore certainly infer that SC is shorter than SB , and consequently that C is nearer S than B is : For though SC be perpendicular to the Curve BC , *that is*, to the Tangent at the Point C , it may for all that be shorter or longer than, or equal to SB , according to the Nature of the Curve BC . For, for Instance, if BC be a Portion of a Circle whose Center is S , then SC perpendicular to BC is equal.

qual to SB ; if BC be a Portion of an Ellipse whose Center is s , and SC half the shorter Ax , which certainly is perpendicular to the Curve, then SC is shorter than SB ; and if SC be half the longer Ax , which also is Perpendicular to the elliptick Curve, then SC is longer than SB .

It is easy also to shew, that though SC be not perpendicular to the Curve BC , yet for all that, SC may be shorter or longer than SB . For if we suppose BC to be a very small Portion of an Ellipse, whose Center is s , and SB to be half the longer Ax which therefore is perpendicular to BC , but SC is not perpendicular thereto, but oblique, because oblique to the Tangent in c ; in this Case, SC is shorter than SB : Neither is it material here, whether we call the Angle SBc a right Angle or an acute one, it being a mixtilinear and not a rectilinear Angle. Again, if SB be half the shorter Ax , and so also perpendicular to BC , SC is oblique to BC , and yet SC is longer than SB . Therefore that part of the preceeding Demonstration,

stration, where the Perpendicular is supposed to fall upon N , on purpose that it may be oblique to BC , is of no Force or Moment imaginable. From all which (though we should add no more) it is abundantly evident, that the said Demonstration is so far from proving Mr. Gordon's Theorem, that it really proves nothing at all. Besides, it is inconsistent with itself, one Supposition destroying another.

But lastly, perhaps it will be said, that, since a Curve may be considered as compos'd of an infinite Number of right Lines, the Line BC , the Body's Path from B to C , is an infinitely little right Line: This indeed we can easily allow, supposing the centripetal Force not to act at all between the Points B and C , but not otherwise; for if it act between B and C , it must either act constantly or by Starts; if constantly, then BC must be a perfect Curve, as above, though infinitely little different from a right Line, and so the pretended Demonstration is already overthrown; if by Starts, then BC must be an inflected Line, and not

a streight one, though yet infinitely little different from a streight one. Wherefore it is plain, that, if BC be a streight Line; as the Demonstration plainly supposes, the centripetal Force acting upon the Body in the Point B , does not act upon it again till it comes to the Point C . Now it is asserted, that a Perpendicular from S to PQ , or BC produc'd, must either fall upon C , or upon some other Point between C and Q : But I wonder how Mr. Gordon is sure of this, since, I believe, any Body but himself will grant, that the Perpendicular from S to PQ may possibly fall upon some Point between B and C , as well as upon C , or between C and Q : And if it fall between B and C , as certainly it may, because, be the Line BC ever so short, the Angle CBF may be so extremely little, or BC may ly so extremely close to BF , that the Angle SCB may be acute, as well as the Angle BCS ; then SC may either be equal to, or longer or shorter than SB ; which again spoils the fine Demonstration.

Scholy

Scholy II.

AN Infinity of Kinds of centripetal Forces may be conceived, or are possible, each constantly acting in a certain regular Manner, or by a constant Rule or Law: Though the Law whereby one acts, be different from the Law whereby another acts.

As for Instance, there may be one kind of centripetal Force that may act upon a Body in such Manner, that its Impulses may constantly be directly as the Distances of the Body from the Center of the centripetal Force: Another Kind of centripetal Force may so act, that its Impulses may constantly be, as the Distances reciprocally: A third may so act, that its Impulses may constantly be as the Squares of the Distances reciprocally; and so forth. Of the third Kind it is highly probable, that there are a great many in Nature.

If v be put for the Energy, Efficacy, or Impulse of the centripetal Force, D for

for the impelled or attracted Body's Distance from the Center of the centripetal Force; then the Law of the first mentioned Kind of ~~centripetal~~ Force is, that v is every where as d directly: The Law of the second Kind is, that v is still as d reciprocally: And the Law of the third is, that v is every where as d^2 reciprocally. There may be a fourth Kind of centripetal Force conceiv'd, that acts equally or alike at all Distances: And so of others to Infinity.

Now 'tis plain, that an Infinity of Kinds of centripetal Forces is possible, each acting by one fixt and constant Law, the Conception of this involving no Impossibility or Contradiction.

PROPOSITION XXXII.

Theor. Fig. 3 L.

IF a Body once impell'd by a projectile Force in any right Line af , be turned from it, by a constant centripetal Force directed to any fixt Point s , not lying

lying in the Line af ; the Body will move in a Curve lying in the Plane that passes through s , and the Length of af ; the Curve will also be concave or hollow towards s , and the Body will describe rounds Areas proportional to the Times of describing.

Suppose the Body, in any Particle of Time, to describe by the projectile Force, the Line ab ; it would, if nothing hindered it, in an equal Time, describe $bf = ab$: And if it be urged in b , by the centripetal Force, in the Direction bs , it will (by 14 Prop.) in an equal Time, describe the Diagonal bc of the Parallelogram $bgsf$. The $\triangle asb$ is $= \triangle bsf$, by 38. 1. Eucl. because ab is $= bf$; also $\triangle bsc$ is $= \triangle bsf$, by 37. 1. Eucl. because fc is $\parallel bs$: Therefore is $\triangle bsc = \triangle asb$. Now these Triangles bsc , asb are Areas described in equal Times; and by the same Way of Reasoning, the centripetal Force directed to s acting again at the Point c , must oblige the Body to describe the Area csd equal to the Area bse , in a third Time equal to

to either of those in which it described the Area asb or bsc ; and so of the Area dse , or any Number of such Areas. And if any two Numbers of such Areas be taken, their Sums will be to one another as the Sums of their Times; or if these Areas be proportionally diminished, they will still be to one another as before. Let us therefore now suppose the centripetal Force not to act by Starts, but constantly without Intermission; and consequently that the Lines ab , bc , cd , &c. and the Areas comprehended within these Lines are diminish'd to Infinity, so as to bring the Points b , c , d , e , &c. in which the centripetal Force is supposed to act, next to one another; then will the Line $abcde$ turn into a Curve, and the Areas that the Body will describe, round s in this Curve, will be proportional to the Times of Description, as above. And 'tis evident also, that the Curve will lie in a Plane passing through s and ab , and will be concave towards $s.w.w.d.$

A Scholy.

THE preceeding Prop. is Sir Isaac
Newton's 1 Prop. 1 Lib. Princip.
 and Dr. *Gregory's* 11 Prop. 1 Lib. Astron.
 and I have expressed and demonstrated
 it very near in Mr. *Gordon's* Words, the
 Demonstration, as he gives it, being just
 enough. But here Mr. *Gordon* very un-
 reasonably finds Fault with those two
 great Men (see Pag. 25, 26, 27 of his
Remarks) because they have not deter-
 mined the Species of the Curve, that
 the projectile and centripetal Forces ob-
 lige the Body to describe, though he
 Thews not (as I'm confident he cannot)
 the least Flaw in the Demonstration. It
 is true indeed, that neither *Newton* nor
Gregory determines the Kind and Nature
 of the Curve, for in this Place they could
 not, the Thing being absolutely impossi-
 ble; since they suppose only any Kind
 in general of a centripetal Force acting,
 and not any particular Species of centri-
 petal Force acting by one constant and
 par-

particular Law: So that the Curve may be any one amongst an Infinity of Curves. And the Nature of the Curve must depend upon the Nature of the centripetal Force, and the Quantity of the projectile Force, whereby it is described. Gordon may just as reasonably quarrel with a Person treating only of the Properties of a Parallelogram in general, without designing to descend to any particular Sort of Parallelogram; that he did not determine the particular Sort of the general Parallelogram, with which alone he was at that Time concerned. Now how unreasonable and absurd this would be, we leave every one to judge.

PROPOSITION XXXIII.

Theor. Fig. 31.

EVERY Body moving in a Curve, and describing round another Body Areas proportional to the Times of Description, is forced into such a Curve by a projectile Force once imprest, and a centripetal Force

constantly directed to that Body, round which the Areas are describ'd.

Any Curve may be conceived as made up of an Infinity of right Lines; suppose the revolving Body to move in the inflected Line $abcde$, and let this Line be compos'd of such infinitely small right Lines $ab, bc, cd, &c.$ as the Body describes in equal Particles of Time. Let the single projectile Impulse be made at a , in the Direction ab ; and let s be the Point or Body at Rest, about which the other Body revolves. Produce ab to f , till bf be $= ab$, and draw $as, bs, fs, cs, &c.$ The Body in a , in any certain Time, describing by the projectile Force the Line ab , would, if nothing hindered it, describe, in an equal Time, $bf = ab$; and since it describes not bf , but bc , it is turned from moving in bf by some Force acting in b , different from the projectile Force. The $\triangle asb$ is (by 38. 1. *Eucl.*) $= \triangle bsf$; the $\triangle bsc$ is (by *Hyp.*) $= \triangle asb$, because these are Areas described in equal Times: Therefore is $\triangle bsc = \triangle bsf$: Whence (by 39.

39. 1. *Euch.*) fc is $\parallel bs$. Therefore the Direction of the Force that turn'd the Body from moving in the Line bf , to move in the Line bc , is (by 15 Prop.) parallel to cf , and consequently is the Line bs . By the same way of Reasoning it is plain, that the Force (that turns the Body from a straight Course) acting in all the other Points $c, d, e, \&c.$ may also be proved to be directed to s .

The same Demonstration holds good, when both the Body s and the revolving Body, are urged by equal accelerating Forces according to parallel right Lines, *that is*, when both are urged with equal Velocity in a straight Course; as will be evident from the first *Scholy* following.

Scholy I.

IF Bodies be moved anywise among themselves, and afterwards be urged by equal accelerating Forces, according to parallel right Lines, or (which is all one) if their common Center of Gravity move in a right Line: They will all continue

to move after the same Manner as at first, in respect of one another, as if they were not urged by the said Forces.

For the said equal accelerating Forces, acting in parallel Directions, will move all the Bodies with equal Velocities in the said Directions; and will therefore never change the Positions and Motions of the Bodies in respect of one another; and consequently they will continue to move in respect of one another, after the same Manner as before.

Scholy II. Fig. 31.

THE last Prop. is Sir Isaac Newton's 2 Prop. 1 Lib. and Dr. Gregory's 12 Prop. 1 Lib. All that Mr. Gordon says to it, in Pag. 14, 15, 16, 17, 18, 19, 20, 21, 22 of his Remarks, that is any-wise to Purpose, is, that bog being a Parallelogram, the Force in $b.g$ directed to s may be resolved into the Forces bo and bx , neither of which is directed to s , which indeed is very true; and so may the said Force be resolved into a thousand

and other Forces; and in that Case, the Sense of the Proposition is, that the Force, that is compounded of all these Forces, is directed to the Point *s*, as Sir Isaac Newton himself acknowledges and affirms in *Sch.* of his forementioned Prop. in these Words, *Urgeri potest corpus a vi centripeta composita ex pluribus viribus; in hoc casu sensus Propositionis est, quod vis illa quæ ex omnibus componitur, tendit ad punctum s.*

But now, when we apply the preceding Proposition to the real Phænomena of Nature, and not to artificial Things, as a Ship carried by a Stream, or animate Things, as a Man walking, which are Instances the Remarker very impertinently brings in 15 *Pag.* is it not far more reasonable (since, according to 10 *Max.* Nature acts by the simplest Methods) to suppose, that the Force directed to *s* is the single one in *bg*, than two in *bo*, and *br* compounding the Force in *bge*. The last Case of the two Forces is indeed very possible, but the first of the single Force is far more probable: Since

tis

'tis highly improbable that Nature uses a compound way, where a simple way will do as well. If then the Planets revolve round the Sun in Curves, and describe Areas proportional to the Times, as is well known by a Multitude of astronomical Observations; must we not conclude, that each of them is urged by a simple centripetal Force (and not a compound one) directed to the Sun, though not with absolute mathematical Certainty? If we suppose the direct centripetal Force to be compounded of other Forces, in different Directions running quite away from the Sun, in one Place of a planetary Orbit, we have the same Reason to suppose the like in all other Places thereof. Now what a needless, foolish, and unreasonable Composition of Forces does this seem to be? And what imaginable Ground can there be for such an absurd Fancy, though this be not simply and absolutely impossible?

PROPO.

PROPOSITION XXXIV.

Theor. Fig. 22.

A Body impell'd by a projectile Force once impress'd, and a centripetal Force constantly impress'd, may move in any Curve that is concave to the Center of the centripetal Force.

This, I think, is pretty near the Reverse of, and almost quite contrary to our Remarker's first Theorem, which pretends the Curve to be a spiral Line only.

Let any Curve, that is concave to the Center of the centripetal Force, be circumscribed by many Tangents AB , BC , CD , DE , &c. produc'd to H , I , K , &c. and let s be the Center of the centripetal Force: Suppose a Body in A be impelled by one single projectile Impulse, in the Direction of the Tangent AB ; and after it has run from the Contact A to B , let it receive, in B , a first single Impulse from the centripetal Force directed to

to s , such as may turn it from the first Tangent AB or AH into the Direction of the second Tangent BC : For (by *Sch. 15 Prop.*) we may conceive a Force tending to s , in the Direction BS , so adjusted and proportioned to the Force in AH or BH , as may make the Body move in any middle Direction BC between BH and BS . When the Body has run in the second Tangent BC beyond the Contact F to the Point C , let it get a second Impulse from the centripetal Force tending to s , and that such as may (by *Sch. 15 Prop.*) turn it into the Direction of the third Tangent CD between CI and CS : When it has run beyond the Contact G to some other Point D , let a third centripetal Impulse turn it into the Direction of a fourth Tangent DE ; and so on. Now suppose the Number of centripetal Impulses, and the Number of Tangents to be each multiplied to Infinity; then the infinite Number of Impulses of the centripetal Force, will be equivalent to a constantly continuing Impulse, or a centripetal Force acting incessantly; and the
infinite

infinite Number of Tangents (which will all be infinitely small, because the centripetal Impulses are infinite in Number) will degenerate into, ~~and be all one~~ with the Curve it self. From whence it is manifest, that a Body impell'd by a projectile Force once impress'd, and a centripetal Force constantly impress'd, may move in any Curve which is concave to the Center of the centripetal Force.
W. W. D.

A Scholy.

WE have here supposed any centripetal Force in general acting at Pleasure, without determining any particular Law by which it acts; because we have determined no particular Species of Curves. This we were obliged to do, that the Demonstration might be the more general; and it is most evident, that the said Demonstration agrees to the Ellipse; any Point within it being the Center of the centripetal Force; since the Ellipse will then be one Species of such Curves.
But

But if the Center of the centripetal Force be the Center of the Ellipse, or one of the umbilick Points, it is demonstrated by Sir *Is. Newton*, that the centripetal Force will be a regular one, or such as acts by one constant Law: In the former Case, his Law of the centripetal Force is, that its Impulses are directly as the Distances of the revolving Body from the Center; see 10th *Prop.* 1 *Lib. Princip.* In the latter Case, when one Focus of the Ellipse is the Center of the centripetal Force, his Law is, that the centripetal Impulses are as the Squares of the Distances reciprocally, or (as that great Man expresses it) the centripetal Force is reciprocally as the Square of the Distance; see 11th *Prop.* 1 *Lib. Princip.* Sir *Is. Newton* also determines the particular Laws of the centripetal Force in several other Curves; all which he deduces from his general Law delivered and demonstrated in his 6th *Prop.* 1 *Lib.* The Proof of this general Law, on which the rest depend, also the Proof of the two particular Laws in the two forementioned

tioned Cases, we shall here largely deliver in the five following Propositions, viz. The general Law, and that of the latter Case, after Dr. Gregory's full and clear Method; but that of the former Case (which Gregory has not) after Newton's own Method enlarged. Newton's own Demonstration of his general Law we shall refer to the end of this Tract.

PROPOSITION XXXV.

Lem. Fig. 33, 34, 35.

THE nascent, or evanescent Subtense of the Angle of Contact in a Circle, is in a duplicate Proportion of the conterminal Arch.

This is Doctor Gregory's 24 Prop. Astron.

Let ADC be a Circle, AB a Tangent in A , and so the Angle BAD , made by the Tangent AB and Arch AD , the Angle of Contact. I say, that any Subtense thereof, infinitely near the Point A of Contact, as BD , is as the Square of the

K

the

the Arch AD ; that is, this Subtense BD is to another Subtense bd infinitely near A , as the Square of the Arch AD is to the Square of the Arch Ad ; provided BD and bd be parallel as in *Fig. 33* and *34*, or, being produced, meet in some Point O remote from A , as in *Fig. 35*, so that they be infinitely near parallel.

Draw the Diameter AC , which (by *18. 3. Eucl.*) will be perpendicular to the Tangent AB .

1 Case. Let the Subtenses DB , db be (in *Fig. 33.*) perpendicular to AB .

Draw DE , de parallel to AB ; draw also CD , cd , and the Chords AD , Ad . The Arch AD being infinitely little or nascent, does infinitely near coincide with its Chord AD ; and consequently is infinitely little different from, and so is to be considered as the same with the said Chord or right Line AD : In like Manner does the Arch Ad coincide with its Chord Ad . But (by *31. 3. Eucl.*) ADC is a right Angle; therefore (by *2 Cor. 8. 6. Eucl.*) as is AC : Chord AD :: Chord AD : AE or BD . Therefore is Chord

$\Delta Dq = BD \times AC$; and consequently
 $\frac{\text{Chord } ADq}{AC} = BD$. Just so is Chord Adq

$= bd \times AC$, and $\frac{\text{Chord } Adq}{AC} = bd$. There-

fore is $\frac{\text{Chord } ADq}{AG} : \frac{\text{Chord } Adq}{AC} :: BD : bd$,

and consequently Chord ADq : Chord $Adq :: BD : bd$. Therefore, since the Arch AD coincides with, and so is infinitely near equal to, the Chord AD , and the Arch Ad coincides with, and so is infinitely near equal to the Chord Ad ; there will be as $BD : bd :: \text{Arch } ADq : \text{Arch } Adq$. W. W. D. I.

2. Case. Let the Subtenses BD , bd (Fig. 34.) of the Angle of Contact be yet parallel; but not perpendicular to the Tangent AB .

Draw DF , df perpendicular to AB ; then (by reason of the equiangular Triangles DFB , dfb) there will be as $BD : bd :: DF : df$. But (by 1 Case) as $DF : df :: ADq : Adq$. Therefore as $BD : bd :: ADq : Adq$. W. W. D. 2.

3 Case. Let the Subtenses BD , bd (Fig. 35.) converge towards some remote

note Point o , but so that they may be infinitely near parallel.

Draw DF , df perpendicular to the Tangent AB . Since BD , bd are supposed to be parallel, the $\angle FBD$ is $\equiv \angle fbd$, and consequently the Triangles FBD , fbd are equiangular. Therefore as BD : bd :: DF : df . But (by 1 Case) as DF : df :: $\triangle Dq$: $\triangle dq$. Therefore as BD : bd :: $\triangle Dq$: $\triangle dq$. W. W. D. 3.

A Corollary.

THE nascent or evanescent Subtense of the Angle of Contact, is also in a duplicate Proportion of the conterminal Arch, in any other Curve to which there may be described an equicurve Circle; or a Circle whose Curvature is the same with (or infinitely little different from) the Curvature of a small Portion of that Curve; such as are the Conic Sections, and many other Curves. For if the Circle AD (Fig. 34.) be of the same Curvature with a small Portion AD of the Curve ADC , the Points D and

d

d will both be in the Periphery of that Circle, and also in this Curve : Therefore the nascent or evanescent Subtenses BD , bd of the common Angle of Contact, will be in a duplicate Proportion of the conterminal Arches AD , Ad of the Curve ADG , as well as of the Circle ADC .

PROPOSITION XXXVI.

Theor. Fig. 36.

IF a Body be projected according to the Direction of any right Line PR , and at the same Time be urged by a centripetal Force constantly tending to a Center s , so that by the compound Motion it describe the Curve APP ; if also, the right Line PR touching the Curve in any Point P , from another Point B in the Curve infinitely near P there be drawn the right Line BD perpendicular to the right Line SP ; and BR parallel to SP : And the like Construction be made at any other Point p of the Curve, pr being a Tangent in p ,

b infinitely near *p*, *rb* parallel and *bd* perpendicular to *sp*. Then the centripetal Force in *p* will be to the centripetal Force in *p* as $\frac{SPq \times bdq}{BR}$ is to $\frac{SPq \times BDq}{BR}$; or the centripetal Force in any Point *p* will be reciprocally as the solid $\frac{SPq \times BDq}{BR}$; when the Figure *PRBD* is infinitely little.

This is the great Newton's general Law of centripetal Forces, which the deservedly famous Dr. Gregory demonstrates after the following Manner.

Let the Force or Impulse in *p* tending to the Point *s*, be called *v*; and let the Time wherein the infinitely little Arch *pb* is run thro' by the Body with the compound Force, or whereby the Body, by the projectile or natural Force alone, would run thro' the infinitely little Tangent *pr*, be called *t*.

Let the centripetal Force in *p* be called *v*; and the Time, in which the infinitely little Arch *pb* is run thro', or in which the Tangent *pr* would be run thro';

thro', be called t . Let $p c$ be an Arch run thro' in a Time equal to the Time t , in which the Arch $p b$ is run through; and draw $c f$ parallel to $s r$.

Then (by *Lem. 5 Prop.*) is $\frac{B R}{b r} =$

$\frac{B R}{c f} \times \frac{c f}{b r}$: But (by 35 *Prop.* and its *Cor.*)

$\frac{B R}{c f}$ is $= \frac{P B q}{P c q}$; and these Arches $P B$, $P c$

being infinitely little are (by 1. 6. *Eucl.*)

as the Triangles $B S P$, $c S P$; that is (by

32 *Prop.*) as the Times in which they

are described, or (by *Constr.*) as the

Times in which the Arches $P B$, $p b$ are

described, or as τ , t : Consequently

$\frac{B R}{c f} = \frac{P B q}{P c q}$ is $= \frac{\tau^2}{t^2}$. Again (by 2 *Max.*)

the little Line $c f$ is to the little Line $b r$

as the Causes that produce them, that

is, as the centripetal Force in P to the

centripetal Force in p , or as v to v : Con-

sequently $\frac{c f}{b r}$ is $= \frac{v}{v}$. Therefore is $\frac{B R}{b r} =$

$\frac{\tau^2}{t^2} \times \frac{v}{v}$. Whence $\frac{v}{v}$ is $= \frac{B R}{b r} \div \frac{\tau^2}{t^2} =$

$\frac{B R \times t^2}{b r \times \tau^2} = \frac{B R}{b r} \times \frac{t^2}{\tau^2}$. But (by 32 *Prop.*)

as τ to t so is Area $s B P$ to Area $s b p$, or

twice

twice SBP to twice sbp , or (by 41. *Eucl.*) as $SP \times BD$ to $sp \times bd$. Therefore is $\frac{t^2}{T^2} = \frac{Spq \times bdq}{SPq \times BDq}$; and consequently $\frac{V}{v} = \frac{BR}{br} \times \frac{t^2}{T^2}$ is $= \frac{BR}{br} \times \frac{Spq \times bdq}{SPq \times BDq}$; whence y is to v (as $BR \times Spq \times bdq$ is to $br \times SPq \times BDq$, or as $\frac{BR \times Spq \times bdq}{BR \times br}$ is to $\frac{br \times SPq \times BDq}{BR \times br}$ or) as $\frac{Spq \times bdq}{br}$ is to $\frac{SPq \times BDq}{BR}$. Therefore the centripetal Force in P (tending to s) is to the centripetal Force in p , as $\frac{Spq \times bdq}{br}$ is to $\frac{SPq \times BDq}{BR}$. Or (to express the Thing shorter) the centripetal Force in P , is reciprocally proportional to the nascent or evanescent solid $\frac{SPq \times BDq}{BR}$.

W. W. D.

A Corollary.

HENCE, if any Curve APP be given, and a Point s to which the centripetal Force tends; the Value of the Solid $\frac{SPq \times BDq}{BR}$ may be determined from the Nature of the Curve; and consequently the Law of the centripetal Force, which is reciprocally as the said Solid, may be found. PRO.

PROPOSITION XXXVII.

Levi. Fig. 37.

I F a right Line RPZ touch, in any Point P , an Ellipse API , whose umbilick Points or Foci are s and F ; and through the Ellipse's Center c there be drawn a Diameter IK parallel to RZ ; Then, a right Line being drawn from s to P , that Part thereof EP , which is intercepted by the Parallels IK and RZ , is equal to CA half the longer Ax.

Draw FP , and $FH \parallel RZ$ or IK . Since (by 3. 4. of Milnes's Conic Sections) the Angles FPZ , HPR are equal; also the Angles PFH , PHF alternate to these, are equal; whence PH is $= PF$. Again, since s and F are the Foci, and c the Center, sc is $= cf$: Therefore (by 2. 6. Eucl.) sh is $= eh$. But sh is the Difference of Ps and PH : Therefore sh is also the Difference of Ps and PF , and eh the Half-difference of the same. Wherefore EP (made up of the lesser Quantity PF or PH ,

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Let APM be the Ellipse in which the Body revolves, and let s be that Focus of the Ellipse to which the centripetal Force is directed. Draw the conjugate Axes TA , GY , crossing one another in the Center c . And at any Point P of the elliptick Curve draw a Tangent HPZ , draw also the Diameter PM , and ICK a conjugate thereto or parallel to the Tangent HZ , and to IK demit the Perpendicular PN . Complete the Parallelogram $PCIH$, and IH will touch the Ellipse in i . Joyn SP cutting IK in E . From the Point B infinitely near P draw BR parallel to SP , and Bx to RZ , meeting SP in x , and MP in o . Draw also BD perpendicular to SP .

The centripetal Force tending to s is (by 36 Prop.) reciprocally as the Solid $\frac{SP^3 \times BD^3}{BR}$. This then must be computed from the Nature of the Ellipse. To which Purpose let L denote the Parameter of the longer Ax TA : Then (by Lem. 5 Prop.) is $\frac{L \times BR}{BD^3} = \frac{L \times BR}{L \times P^2}$ $\times$

$\frac{L \times P o}{M o \times P o} \times \frac{M o \times P o}{B o q} \times \frac{B o q}{B x q} \times \frac{B x q}{B D q}$. But
 (by 1. 6. *Eucl.*) $\frac{L \times B R}{L \times P o}$ is $= \frac{B R}{P o} = \frac{P x}{P o}$
 (by 2. 6. *Eucl.*) $\frac{P E}{P C}$, because $x o$ is $\parallel E C$
 and (by 37 *Prop.*) $P E$ is $= A C$
 Therefore is $\frac{L \times B R}{L \times P o} = \frac{A C}{P C}$. Again, $\frac{L \times P o}{M o \times P o}$
 is $= \frac{L}{M o}$. And (by 2 *Cor. 20. 1. Miln. Con-*
nic. and Permut.) $\frac{M o \times P o}{B o q}$ is $= \frac{M C \times C P}{I C q} =$
 $\frac{C P q}{I C q}$. And, in the present Case, $B o$ is $=$
 $B x$, the Point B being infinitely near the
 Point P : Therefore is $\frac{B o q}{B x q} = 1$, and so
 $\frac{B o q}{B x q}$ goes out of the first Equation. Far-
 ther $\frac{B x q}{B D q}$ is $= \frac{P E q}{P N q}$; because the Tri-
 angles $B D x$, $P N E$ are equiangular, for
 the Angles at D and N are Right, and the
 alternate Angles $B x D$, $P E N$ of the
 two Parallels $B x$, $E N$ are equal. And
 $P E q$ is $= C A q$, because, as before,
 $P E$ is $= C A$: Therefore is $\frac{B x q}{B D q} = \frac{C A q}{P N q}$.
 But (by 11. 2. *Miln. Conic.*) the Rect-
 angle

angle $GC \times CA$ is equal to the Parallelogram $PCIH$ or (by 35. 1. *Eucl.*) the Rectangle $IC \times PN$; therefore (by 16. 6. *Eucl.*) as $CA : PN :: IC : GC$; whence as $CA_q : PN_q :: IC_q : GC_q$, therefore is $\frac{B \times q}{BD_q} = \frac{IC_q}{GC_q}$. Therefore since, as before, the Proportion of $L \times BR$ to BD_q is compounded of the Proportions $L \times BR$ to $L \times PO$, $L \times PO$ to $MO \times PO$, $MO \times PO$ to BO_q , (BO_q to $B \times q$ which goes off) and $B \times q$ to BD_q ; it must also be compounded of the Proportions respectively

equal to these; that is, $\frac{L \times BR}{BD_q}$ is $= \frac{AC}{PC} \times$

$$\frac{L}{M_o} \times \frac{CP_q}{IC_q} \times \frac{IC_q}{GC_q} = \frac{AC \times L \times CP_q \times IC_q}{PC \times M_o \times GC_q \times IC_q} = \frac{AC \times L \times CP}{M_o \times GC_q}.$$

But (by 4 *Cor.* 24. 1. *Milnes Conic.*) $AC \times L$ is $= 2 GC_q$. There-

fore is $\frac{L \times BR}{BD_q} = \frac{2 GC_q \times CP}{M_o \times GC_q} = \frac{2 CP}{M_o}$. But

since the Points B and P are infinitely near one another, MO is $(= MP) = 2 CP$.

Therefore is $BD_q = L \times BR$; and consequently $\frac{SP_q \times BD_q}{BR} = \left(\frac{SP_q \times L \times BR}{BR} \right) =$

$SP_q \times L$; Wherefore the centripetal Force

L

Force

Force in P is reciprocally proportional to $spq \times L$, as is evident from 36 Prop. or, because L is a constant and invariable Quantity, the centripetal Force is reciprocally as spq . Therefore the centripetal Force tending to one of the Foci of an Ellipse, is reciprocally as the Square of the revolving Body's Distance from that Focus. *W. W. D.*

A Scholy.

At the End of this Demonstration it is proved, that the centripetal Force is reciprocally as $spq \times L$; that's to say, the centripetal Force in the Point P is to the centripetal Force in any other Point p (when the Force is directed to a Focus s of the Ellipse) as $spq \times L$ is to $spq \times L$, or as spq is to spq ; which is all one as to say, that the centripetal Impulses or Forces in the Points P and p are reciprocally as the Squares of the revolving Body's Distances from the said Focus s .

PROPOSITION XXXIX.

Theor. Fig. 39.

IF a Body once impell'd by a projectile Force, and constantly urg'd by a centripetal Force, revolve in an Ellipse, the centripetal Force tending to the Center of the Ellipse: The Law of the said centripetal Force will be such, that its Impulses on the revolving Body, will always be directly as the Distances of the said Body from the said Center.

Let sA , sG be the Semi-axes of the Ellipse, whose Center s is also the Center of the centripetal Force; MP , IK two conjugate Diameters, PN , BD Perpendiculars to the said Diameters; Bo an Ordinate to the Diameter MP , and consequently parallel to PR the Tangent at the Vertex P and to IK ; complete the Parallelogram $BoPR$, and suppose the Point B to be infinitely near the Point P .

Then (by 2. Cor. 20. 1. Miln. Conic.) as is $PO \times OM$: Boq :: SPq : SIq ; and

L. 2.

(by

by reason of the similar Triangles $\triangle B O q$,
 $\triangle P N q$) as $B O q : B D q :: S P q : P N q$.

Therefore is $\frac{P o \times O M}{B o q} = \frac{S P q}{S I q}$, and $\frac{B o q}{B D q} =$
 $\frac{S P q}{P N q}$. Whence $\frac{P o \times O M}{B o q} \times \frac{B o q}{B D q} =$ (by

Lem. 5 Prop.) $\frac{P o \times O M}{B D q}$ is $= \frac{S P q}{S I q} \times \frac{S P q}{P N q} =$
 $\frac{S P q \times S P q}{S I q \times P N q}$. Therefore as is $P o \times O M$:

$B D q :: S P q \times S P q : S I q \times P N q$; and
 consequently $B D q \times S P q \times S P q = P o$
 $\times O M \times S I q \times P N q$. Wherefore
 $\frac{B D q \times S P q \times S P q}{P o} (= \frac{B D q}{P o} \times S P q \times S P q)$

is $= O M \times S I q \times P N q$: And consequent-
 ly as $O M : \frac{B D q}{P o} :: (S P q \times S P q : S I q \times$
 $P N q ::) S P q :: \frac{S I q \times P N q}{S P q}$. Complete

the Parallelogram $S P H I$, and for $P o$
 put its Equal $B R$, and for $S I \times P N$ (or
 $S P H I$) put $S G \times S A$ equal thereto by
 I. 2. *Miln. Conic.* also the Points B and
 I coming close together, for $O M$ put
 $S P$: Then, multiplying the Extremes
 and Middle of the last Analogy, you
 will get $\frac{B D q \times S P q}{B R} = \left(\frac{S P \times S G \times S A q}{S P q} = \right)$

$\frac{2SGq \times SAq}{SP}$. But (by 36. Prop.) the centripetal Force in R is reciprocally as the nascent Solid $\frac{SPq \times BDq}{BR}$. Therefore it is also reciprocally as $\frac{2SGq \times SAq}{SP}$. Now $2SGq \times SAq$ is a given or constant Quantity; therefore is $\frac{2SGq \times SAq}{SP}$ as $\frac{1}{SP}$. Whence the centripetal Force is reciprocally as $\frac{1}{SP}$, and consequently directly as the Distance SP . W. W. D.

A Scholy.

IN the former Demonstration it is proved, that $\frac{SPq \times BDq}{BR}$ is $= \frac{2SGq \times SAq}{SP}$.

Now, if pr be a Tangent in p , rb parallel to sp , and bd perpendicular to sp , the Points b and p being infinitely near one another; it may be proved, after the very same Manner as above, that $\frac{Spq \times bdq}{br}$ is $= \frac{2SGq \times SAq}{Sp}$. But (by 36. Prop.) the centripetal Force in r is to the centripetal Force in p , as $\frac{Spq \times bdq}{br}$ is to $\frac{SPq \times BDq}{BR}$; that is (when the said Force

is directed to the Center of the Ellipse) as $\frac{2SGq \times SAq}{Sp}$ is to $\frac{2SGq \times SAq}{SP}$, or as $\frac{1}{Sp}$ is to $\frac{1}{SP}$, because $2SGq \times SAq$ is a given or constant Quantity. Therefore the centripetal Force in p is to the centripetal Force in P (as $\frac{1}{Sp}$ is to $\frac{1}{SP}$ or) as sp is to SP the Distances directly; because Fractions of the same Numerator, are reciprocally proportional to their Denominators.

PROPOSITION XL.

Lem. Fig. 40.

LET BGA be an Ellipse, BA its longer Ax , c the Center, cG half the shorter Ax , and F one of the Foci; then, if from F there be drawn an Ordinate FE to the longer Ax BA , I say that FE is more than FA .

Put BC or $CA = a$, $CG = c$, $CF = e$; then is $BF = a + e$.

By 4 Cor. 2. 4. Milu. Conic. FE is equal to half the Parameter of the Ax

AB .

AB. And (by 4 Cor. 24. 1. Miln. Conic.)
 $FE \times CA$ or $FE \times a$ is $= CG_q = c^2$. Whence
 FE is $= \frac{c^2}{a}$. Also (by 2 Cor. 20. 1. Miln.
 Conic.) $as BF \times FA : BC \times CA :: FE_q :$
 CG_q , that is, $asa + e \times FA : a^2 :: \frac{c^4}{a^2} : c^2$.

Whence there is $ac^2 + ec^2 \times FA = (a^2$
 $\times \frac{c^4}{a^2} =) c^4$: And consequently $FA =$
 $(\frac{c^4}{ac^2 + ec^2} =) \frac{c^2}{a+e}$. But, as before, FE is
 $= \frac{c^2}{a}$; and 'tis evident that $\frac{c^2}{a}$ is more
 than $\frac{c^2}{a+e}$. Therefore is FE more than FA .
 W. W. D.

Corollaries.

1. **T**HE Semi-parameter or Ordinate
 FE drawn from the Focus F , is
 longer than any right Line FD drawn
 from the said Focus to any Point D of
 the elliptick Curve between A and E . This
 will be evident by describing a Circle
 from F as a Center at the Distance FE .

2. It is evident, that any right Line
 FK drawn from the Focus F to any Point
 of

of the elliptick Curve between F and B , is longer than FE and shorter than FB .

3. It is evident, that the Line FI lying nearer FE , is longer than FD lying further from FE . And that any Line FK lying nearer FB , is longer than FG lying further from FB .

4. It is evident, that FA the Part of the longer Ax lying between the Focus F and the nearest Vertex A , is the shortest of all right Lines that can be drawn from the said Focus to any Point of the elliptick Curve: And that the remaining Part FB , is the longest of all.

PROPOSITION XLI.

Lem. Fig. 40.

THE Distance FG between either Focus F of an Ellipse and the End G of the shorter Ax , is a mean arithmetical Proportional between the Segments BF and FA of the longer Ax , made by the same Focus F .

For

For $\overline{PG}$ being drawn from the other Focus P , there is (by 5. 4. Miln. Conic.) $FG + PG$ or $2 FG = AB$, and consequently $FG = \frac{1}{2} AB$ or the half Sum of BF and FA : Wherefore FG is a mean arithmetical Proportional between BF and FA . W. W. D.

PROPOSITION XLII.

Theor. Fig. 41.

IF a Body move in an Ellipse by a projectile Force, and a centripetal Force constantly tending to a Focus of the Ellipse: Its Velocity in any Point in its second Revolution, will be the same it was in that Point in its first Revolution: And so in any other Revolution.

Let s be the Center of a centripetal Force which joined with a projectile, makes a Body describe the Ellipse $AKPN$ whose longer Axis is AP and shorter BF ; suppose also s to be one Focus of the Ellipse. Then (by 32 Prop.) the Body will

will describe Areas proportional to the Times of Describing. Let the infinitely little Areas $\triangle SC$, $CS D$, DSE , ESH , HSI , &c. be equal; and consequently the Times in which the Body describes the infinitely small Parts AC , CD , DE , EH , HI , &c. of the Ellipse, will be equal. Therefore (by 1 *Prop.*) the Velocities of the Body in these Parts will be in the same Proportion as the said Parts; for the Motions in these infinitely little Parts may every one be considered as uniform: And so the Velocity in AC will be as AC , the Velocity in CD (though greater than that in AC) will be as CD , the Velocity in DE as DE , and so on. But it is plain that AT is $\equiv AC$, $TR \equiv CD$, $RN \equiv DE$, and so on; and consequently the Velocities in these Parts are the same respectively. From whence it is evident, that the Velocity in A in the second Revolution, is the same it was in A in the first Revolution; and so in any other Point of the Ellipse. What is proved of a second Revolution, holds good, after the very same

same Manner, in a third, fourth, &c.
W. W. D.

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Corollaries.

1. FROM this *Prop.* and 40 *Prop.* with its *Corollaries* it is evident, that the Body's Velocity (the Center of the centripetal Force being the Focus *s* of the Ellipse) is still increasing from *A* through *K* to *P*, and again decreasing from *P* through *B* to *A*; so that the Velocity in *A* is least, and in *P* greatest. It is also evident from 41 *Prop.* that the Velocity in *B* and *F*, the Ends of the shorter *Ax*, is a Mean between the least Velocity in *A* and the greatest in *P*.

2. Hence it is plain, that all that Mr. Gordon advances in *Pag.* 87, 88, 89 of his *Remarks*, concerning the constant Increase of Velocity of a Body revolving in an Ellipse, about one of the Foci as the Center of the centripetal Force, is false, and his Banter groundless. For from what we have just now proved it is evident, that though the Velocity increases in one
Half

Half of the Ellipse, it gradually decreases as much in the other; and that, after a complete Revolution, the Velocity is the same it was at first.

PROPOSITION XLIII.

Lem. Fig. 42.

THE versed Sine of an indefinitely small Arch of a Circle, is equal (at least extremely near) to the Square of the said Arch divided by the Diameter: That is, if AB be a very small Arch of a Circle whose Diameter is AG , the versed Sine AC is (very nearly) equal to the Square of the Arch AB divided by the Diameter AG .

This already has been virtually proved in 35 Prop. but we shall here explicitly demonstrate it.

Draw the Chord AB , the Sine CB , and the Line GB . Then (by 2 Cor. 8. 6. Eucl.) as is $AG : AB :: AB : AC$; whence AC is $= \frac{AB^2}{AG}$, that is, the versed Sine AC of the Arch AB is equal to the Square of

of the Chord AB divided by the Diameter AG . But an indefinitely small Arch and its Chord coincide. Therefore the versed Sine AC , is equal to the Square of the Arch AB divided by the Diameter AG . W. W. D.

A Scholy.

MR. Gordon in his *Remarks* begins at the Foot of *Pag.* 38. to quarrel hard with *Sir Isaac Newton* and *Dr. Gregory*, for their affirming that the Force by which the Moon is hindred from running out in straight Lines and kept in her Orbit, is the same with that Force by which heavy Things fall to the Ground; and endeavours to confute them. In order to which he affirms in *Pag.* 42. that it very clearly appears, that the real Motion of the Moon, is (by *Newton*) compared with the apparent Part of falling Bodies only. Now suppose this be granted him; though I see no good Reason that it should, since the Fall of a Body very near the Surface of the Earth, is

M found

found by many Experiments to be about $15 \frac{1}{12}$ Paris Feet, in a Second of Time, in a free Space; so that whether the Earth roll about its Axis or not, the Fall of $15 \frac{1}{12}$ such Feet in a Second of Time is determined by Mr. *Hugens*, Sir *Is. Newton*, and their Followers, to be the full and real (and not apparent) Effect of Gravity near the Earth: Yet the odds of falling in a Second, a Space more than $15 \frac{1}{12}$ Feet, viz. such a Space more as is equal to the versed Sine of $17''$ (as *Gordon* would have it, or rather $15''$, so much as any Point of the Earth's Surface runs, by its diurnal Motion, in a Second of Time) of the Earth's Circumference, besides the said $15 \frac{1}{12}$ Feet or 181 Inches, which at most is but $\frac{3}{4}$ of an Inch, is so small in respect of 181 Inches, that in such nice Experiments it need not be considered.

That

That the versed Sine of an Arch of $15''$ of a great Circle on the Earth is not full $\frac{3}{4}$ of an Inch, will thus appear. Such a great Circle's Periphery is, by late accurate Observations, found to be 123249600 Paris Feet, and consequently the Diameter to be 39231600 such Feet. Now as 1296000 the Seconds in the whole Periphery is to 123249600 the Feet in the whole Periphery, so is 15 Seconds of the Periphery to 1426'5 Feet, answering to 15 Seconds. This Arch 1426'5 Feet squared, and its Square 2034902'25 divided by the Diameter, the Quote '052 will (by 43 Prop.) give the versed Sine of $15''$ sought: And this Decimal '052 of a Foot is not full $\frac{3}{4}$ of an Inch. From whence it is plain, that the *Remarker* here makes a Noise to little Purpose, especially since the mean Diameter of the Moon's Orbit is not yet completely determined, though it be determined to be about 60 Diameters of the Earth.

PROPOSITION XLIV.

Theor.

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THE Force whereby the Moon tends to the Center of the Earth, and is kept in her Orbit; is the same with the Force of Gravity, whereby terrestrial Bodies tend to the said Center.

The Time of the Moon's Revolution in her Orbit is 27 Days, 7 Hours, 43 Minutes, or 2360580 Seconds of Time. In 360° there are 129600".

And as 2360580 Seconds of Time is to 1296000 Seconds in the Periphery, so is 1 Second of Time to $\frac{129600}{236058}$ of a Second in the Periphery, equal to $\frac{1}{549017}$ of a Second, equal to the Arch of the Moon's Orbit that is described in 1 Second of Time.

The Moon's Orbit is 7394976000 Paris Feet; and the Diameter of her Orbit is 2353896000 Paris Feet.

Then as 1296000, the Seconds in the whole Periphery, is to 7394976000, the Feet

Feet in the whole Periphery, or as 1296 is to 7394976, so is '549017" of the Periphery to 3132'691 Feet, answering to '549017". So then the Arch of the Moon's Orbit, run in 1 Second of Time, is 3132'691 Feet.

This Arch of 3132'691 Feet being squared, and the Square 9813752'901481 divided by the Diameter 2353896000, the Quotient '004169 of a Foot will (by 43 Prop.) give the versed Sine of the Arch the Moon runs in a Second, which is the Measure (at least extremely near) of the Moon's centripetal Force in her Orbit. But the said Force is as the Square of the Distance reciprocally. Therefore, if the Moon was brought to the Surface of the Earth, or 60 Times nearer the Center than she is, her centripetal Force, would be 60×60 , or 3600 Times '004169 of a Foot, or 15'0084 Feet equivalent to 180'1 Inches. So then the Moon near the Surface of the Earth being let fall, would descend by her centripetal Force in a Second of Time, 180'1 Inches. But it is known by many Experiments, that

heavy Bodies near the Surface of the Earth fall in a Second of Time $15 \frac{1}{12}$ Feet or 181 Inches : And these Numbers 180'1 and 181 differ but very little from one another. Therefore the Moon's centripetal Force is all one with Gravitation, or that Force whereby heavy Bodies near the Surface of the Earth tend to the Earth's Center.

This Calcul is founded upon the Supposition, that the Diameter of the Moon's Orbit is 60 Diameters of the Earth : But if we suppose the Diameter of the Moon's Orbit to be somewhat more (as probably it is) viz. $60 \frac{1}{4}$ Diameters of the Earth, and renew the Calcul, we will find that the Moon near the Surface of the Earth, would descend by her centripetal Force about 182 Inches, in a Second of Time : And this will answer all that Mr. Gordon can demand about his real and apparent Gravity, as is plain from *Sch. 43 Prop.*

A Scholy. Fig. 43.

MR. Gordon in *Pag. 52* of his *Remarks* delivers a *Theorem*, from which in *Pag. 55* he infers a *Corollary*, design'd not only to overturn the preceeding *Proposition* about the Gravitation of the Moon, but also all Measures and Estimates of centripetal Forces by versed Sines. His *Theorem*, in his own Words, is as follows.

A Body describing an Arch ad , really falls from the Tangent of every Point of that Arch a certain Space, and the Sum of all those Spaces is, in respect of the verse Sine of ad , infinitely little.

This *Theorem* looks very like a Paradox, and yet if the Arch ad be supposed infinitely little, it will perhaps be found to agree better with a Principle delivered by Sir *Is. Newton*, Dr. *Gregory*, and Mr. *Whiston*, than any Thing in all Mr. *Gordon's Remarks*. The said Principle is *Newton's 1 Cor. 11 Lem. 1 Lib. Princip.* *Gregory's Schol. 24 Prop. 1 Lib. Astron.* and

and Whiston's 4 Cor. 2 Prop. *Prælect. Physico-mathem.* which we have before laid down in Cor. 35 Prop. Gordon's Demonstration of his Theorem being all bare Assertion, without any Reasons assigned, I shall not be at the Pains to examine it; but shall only more fully express the true Meaning of his Theorem, and after I have endeavoured to give a more clear, succinct, and accurate Demonstration, deliver his Corollary: Which done, I believe it will easily appear, that the said Corollary has either no Connection at all with the said Theorem, or else that the Connection is so obscure, that the Remarker is oblig'd to shew it. The true Meaning then of the Theorem I take to be this; Suppose an infinitely little Arch $a d$ of a Curve to be divided into an infinite Number of Parts; and Tangents drawn to all the Points of Division; a centripetal Force constantly directed to a certain Point s , will make a Body describing the Arch $a d$ fall from all the Tangents certain Spaces, and the Sum of all these Spaces together will be infinite-

ly little in respect of $k d$ the versed Sine of the Arch $a d$, or right Line drawn parallel to $s a$ from d to $a l$ the Tangent in a . Which I thus demonstrate.

Suppose the infinitely little Arch $a d$ divided into two equal Parts $a c$, $c d$: When the Body has come from a to c , the centripetal Force tending to s has made it fall from the Tangent of the Point a the Space $e c$, and when it is come to d , the said Force has made it fall from the Tangent of the Point c another Space equal to $e c$, at least infinitely near so; the Sum of which two Spaces, viz. $2 e c$ is $= \frac{1}{2} k d$, because (by 35 Prop. and its Cor.) as is $e c : k d :: a c q : a d q :: 1 : 4$. If, again, we suppose the Arch $a d$ divided into three equal Parts $a b$, $b c$, $c d$, the Body, by the Force tending to s , will fall from the Tangent of a the Space $f b$, from the Tangent of b as much, and from the Tangent of c also as much, that is, it will fall from the three Tangents of a , b , c , three Spaces, every one of which is equal to $f b$; the Sum

Sum whereof, viz. $3fb$ is $= \frac{1}{3} kd$, because (by Cor. 35 Prop.) as is $fb : kd :: abq : adq :: 1^3 : 3^3 :: 1 : 9$. In like Manner, if we suppose the Arch ad divided into four equal Parts, the Sum of the Spaces fallen from the Tangents of the Point a and the next three Points of Division, will be equal to $\frac{1}{4} kd$: And so forth. Therefore, since the Sum of the Spaces fallen from the Tangents constantly decreases, as the Number of the Points of Contact increases; it is evident, that, when the Points of Contact are infinite in Number, the Sum of all the Spaces fallen from the Tangents, while the Body is describing the Arch ad , is infinitely little in respect of the versed Sine kd of that Arch. w. w. d.

The Corollary our Remarker pretends to draw from the fore said Theorem, is what follows.

Hence it appears, that all Estimates of the Quantity of a Force turning a Body from straight Lines into a Curve, if they measure the Force that produces any as-

sign-

Signable Arch by the *verse Sine* of that Arch, or compare the Forces that produce any two Arches, one whereof is greater than another, by the *verse Sine* of those Arches, are erroneous.

We shall leave the Remarker to make good his *Corollary*, either from his *Theorem*, or any other Way he shall think fit. In the mean Time, whatever becomes of his *Theorem*, and whether his *Corollary* be by a just Consequence deducible from his *Theorem* or not, I think we can prove his *Corollary* to be false. For suppose ad and dz to be infinitely small Particles of the Curve adz , whether equal or not, described by a Body in very small, but equal Particles of Time: Let al be a Tangent at a , and dn a Tangent at d ; also let dk be parallel to sa , and zn parallel to sd : While the Body is moving from a to d , in the infinitely little or nascent Arch ad , the centripetal Force tending to s is infinitely little altered, which is much the same Thing as to say, it is not altered at all, but continues the same in every Point of the

the Arch ad , till it come to d . In like Manner, while the Body is moving from d to z , in the infinitely little Arch dz , though the centripetal Force in dz may be somewhat less or more than it was in ad , yet it must be supposed to continue the same unaltered thro' whole dz , till it come to z . Now kd is the Effect of the centripetal Force, when the Body has moved from a to d , and nz the Effect of the said Force, when the Body has moved, in an equal Particle of Time, from d to z ; and the said Forces are the sole and adequate Causes of the said Effects: Therefore the infinitely little or nascent Lines kd and nz , must (by 2 Max.) be proportional (at least infinitely near) to the centripetal Forces in the Points a and d , or in the Arches ad and dz . Therefore kd is the (infinitely near) Measure or Estimate of the Quantity of the centripetal Force at a , and nz the Measure of the said Force at n ; notwithstanding that the preceeding Corollary affirms the contrary, by pronouncing all

Esti-

Estimates of centripetal Forces by versed Sines erroneous.

Note, That it is not necessary that the Arches ad and dz should be contiguous, although they be so in the present Figure.

Again suppose ad to be an infinitely small Arch of the Curve adz , and consequently the centripetal Force to continue the same from a to d ; suppose also ab to be a Part of the Arch ad : Then, notwithstanding the preceeding Corollary, fb is the Measure of the centripetal Force in a , with respect to the Time wherein the Arch ab is described; and kd the Measure of the same Force in a , with respect to the Time wherein the Arch ad is described.

Note, that, since ad is supposed an infinitely little or nascent Arch, it is all one upon the matter, whether the infinitely little or nascent Line kd be perfectly and rigorously parallel to the right Line sa , or be a Particle of the right Line sd produced, and consequently infinitely near parallel to sa . And, in like manner, it is

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all

ask one whether the nascent Line nz be perfectly parallel to sd , or be a Particle of sz produced.

It will not, I think, be improper in this Place to observe, that though the projectile and centripetal Forces in a Body moving in a Curve, may be very unequal; yet the centripetal and centrifugal Forces (taken in a strict and rigorous Sense, the former as urging directly to a Center, and the latter directly from it) are in every Point of the Curve exactly equal. For suppose, as before, ad to be an infinitely small or nascent Particle of the Curve adz , al a Tangent in the Point a , s the Center of a centripetal Force, which joined with a Projectile Force, in the Direction of the Tangent al , obliges a Body to describe the Curve adz : Draw the right Lines sa and sd , and dk will be infinitely near parallel to sa . Then, as above, the nascent Line kd will be the full Effect of the centripetal Force, while the Body is moving in the Curve from a to d , and also the Measure of the same in the Point

Point *a*. But while the Body is urged by the centripetal Force directly towards the Center *s*, though, by reason of the projectile Force conjoined, it cannot really move directly towards the Center, but must move in the Curve; the Body's natural Tendency being (by 1 Max.) according to the Direction of the Tangent *al*, if (the projectile or natural Force remaining) the centripetal Force in *a* was destroyed, the Body would come to *k*, in the same Instant of Time, that it came to *d* by the Action of the centripetal Force, and consequently would be removed from the Curve, the nascent Space *dk*. Now the Force that would cause the Body to move the Space *dk* from the Curve, in the Direction of the Line *s d k*, viz. from *d* to *k*, in the same Particle of Time, that the Body really moves in the Curve from *a* to *d*, is what is properly called *the centrifugal Force*, which has its Origin indeed from the projectile or tangential Force. Therefore, since the Line *dk* would be the true and full Effect of the centrifugal

N 2

Force,

Force, while the Body is moving from a to d , and consequently the Estimate and Measure of the said Force in a ; as it really is the true and full Effect of the centripetal Force, and Measure of the same in a ; it is evident, that the centrifugal and centripetal Forces, in the said Point a , are equal. And so in any other Point of the Curve.

PROPOSITION XLV.

Theor.

EVERY Body A which, a Radius being drawn to the Center of another Body B howsoever moved, describes Areas, about that Center, proportional to the Times of describing; is urged by a Force compounded of the centripetal Force tending to this second Body B , and of all the accelerating Force whereby the said Body A is urged: That is, the first Body A will, at the same Time, be urged by the said two Forces jointly.

This

This seems pretty evident, for else the Proportionality of the Areas to the Times could not be constantly observed, which would overturn the *Hypothesis*.

Otherwise thus : If both Bodies be urged in parallel Lines, by a new accelerating Force equal and contrary to that whereby the second Body *B* is urged ; all the accelerating Force in *B*, and as much accelerating Force in *A*, will be destroy'd : Yet the Body *A* will (by 1. *Sch.* 33 *Prop.*) continue to move with its remaining Force, after the same manner, as at first, in respect of *B*, and so will describe about *B* Areas proportional to the Times. Then, since *A*, by its remaining Force, describes round *B*, Areas proportional to the Times, the said Force in *A* (by 33 *Prop.*) tends to *B*. But when at first *A* described about *B* Areas proportional to the Times, *A* was urged by the accelerating Force that was destroyed in it, or all the accelerating Force in *B*, and the remaining Force in *A*. Therefore *A* is urged not only by the said remaining Force, or Force tending

60 ¹, but also by all the accelerating Force whereby B is urged. w. w. d.

A Scholy.

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OUR Remarker, in *Pag.* 35, 36, 37, 38, runs out at a strange Rate, quibbling against this *Proposition* to no Purpose, but speaking with a mighty Air of Assurance and Vanity; and feigns, at the End of *Pag.* 37 and Beginning of *Pag.* 38, a gross Absurdity he pretends will follow from the said *Proposition*, from our Authers (Sir Is. Newton and Dr. Gregory) taking very good Care, as he says, not to mention that *Halt* of the Cause, which consists in the Endeavour of the Body to run out in Tangents; which (continues he) if they had not done, the Absurdity of supposing the Cause of any Body's Motion to be an Endeavour in the Body, at every Instant of Time, to run out in two different Lines (as the Tangents of those two different Curves are) at the same Time, is so very gross and evident, that it could never have been

been swallowed by any Man of common Sense.

But here our *Remarker* seems to talk after so ridiculous and extravagant a Manner, that one would be tempted to think, that either he at this Time was not in his right Senses, or else that he was resolved to say Something or other, right or wrong, Sense or Nonsense, against the said *Proposition* and its Authors, rather than let it pass for Truth, and allow them to be in the Right. For what Occasion was there, I would fain know, to mention, in the preceeding *Proposition*, the projectile or tangential Force of the Body *A*, though it is not excluded, since the Body *A* (by 33 *Prop.*) is obliged to move; as the last *Proposition* expresses, by a projectile Force, and a centripetal Force tending to the Body *B*. Suppose the Body *A* had in that *Proposition* been said to describe about *B*, Areas proportional to the Times by a projectile Force once impress'd, and a constant centripetal Force tending to *B*; would that Expression have altered the Sense of the *Proposition*?

no,

no, it would not in the least have affected it, nor its Demonstration neither. The said *Proposition* is also as little or less concerned to mention a tangential Force in the Body B, because it may or may not have one.

PROPOSITION XLVI.

Theor. Fig. 44.

THE Force or Efficacy of a Vertue which is propagated to or from a Center in streight Lines, every way round in a circular Space, is at different Distances from the Center, as the said Distances reciprocally. And the Force or Efficacy of a Vertue which is propagated to or from a center in streight Lines, every way round in a spherick Space, is, at different Distances from the Center, reciprocally as the Squares of the said Distances.

1. Let s be the Center, to or from which the Vertue is propagated in a circular Space, about which describe two circular Peripheries cdm , efn , at any
Distances

Distances sd , sf : I say, that the Force or Efficacy of the Vertue, at the Distance sf , will be to the Force of the same, at the Distance sd , reciprocally as sd is to sf .

For the same Quantity of the Vertue that is equally diffused through the Arch cd , is also equally diffused through the similar Arch ef : Then if ef be double of cd , or eb (the Half of ef) equal to cd , the Quantity of the Vertue in eb will be just half the Quantity of it in cd ; and so the Force of the Vertue in ef will be half its Force in cd ; that is, the Force in ef will be to the Force in cd , as cd to ef , or as sd to sf , for similar Arches of Circles are as the Semi-Diameters. In like manner, if ef be $= 3 cd$, the Efficacy of the Vertue diffused thro ef , will be only $\frac{1}{3}$ of the Efficacy thereof diffused thro' cd , because the Vertue in ef will be three Times sparser than in cd ; consequently its Efficacy or Force in ef will be to its Efficacy in cd , as cd to ef , or as sd to sf . And so universally, whatever Proportion ef bear to cd , the Efficacy

Efficacy of the Vertue in ef is to its Efficacy in cd , reciprocally as the Distance sd is to the Distance sf .

2. Suppose the Vertue be diffused or propagated in streight Lines from the Center s every way round in a Sphere, about which Center describe two spherick Surfaces cdm , efn : Lay, that the Efficacy of the Vertue, at the Distance sf , will be to the Efficacy of the same, at the Distance sd , reciprocally as sd to sf .

Let ef and cd represent like Parts of the spherick Surfaces efn and cdm ; and so there is as $ef : cd :: efn : cdm$. Now we can prove, just as in the first Part, that the Efficacy of the Vertue in the Surface ef is to its Efficacy in the Surface cd , reciprocally as the Surface cd is to the Surface ef , or as the whole spherick Surface cdm is to the whole spherick Surface efn . But, by *Archimedes's* Doctrine of the *Sphere* and *Cylinder*, spherick Surfaces are as the Squares of the Semi-diameters; and consequently the spherick Surface cdm is to the spherick

rick Surface efn , and so the Surface cd to the Surface ef , as sdq to sfq . Therefore the Efficacy of the Vertue in the Surface ef , at the Distance sf , is to its Efficacy in the Surface cd , at the Distance sd , reciprocally as sdq is to sfq .

A Scholy.

BY the preceeding Demonstration it is solidly proved, that the Efficacy of a Vertue diffused to or from a Center, in streight Lines, in a circular Space, and acting upon an Arch, is reciprocally as the Distance of the Arch from the Center; and that the Efficacy of a Vertue diffused in streight Lines to or from a Center in a Sphere, and acting upon a spherick Surface, is reciprocally as the Square of the Distance of that Surface from the Center. From whence Mr. Gordon pretends by a Parity of Reason to prove, in his third *Theorem Pag. 75*, that the Force or Efficacy of any Vertue, that spreads itself in streight Lines through all the surrounding Space, equally to or from a Center, and acts

acts upon the solid Content or trine Dimension of Bodies, must, in different Distances from that Center, be as the Cubes of those Distances (we must suppose, though he does not express so much, that he means) reciprocally.

Now in order, to prove that his said third *Theorem* is false and inconsistent with Reason, and consequently that his pretended parity of Reason (on which alone it is founded) does here quite fail him; let the following *Principle* be carefully observed, viz. that if the same Quantity of a Vertue be diffused through a greater Space and a lesser, whether these Spaces be both linear, or both superficial, or both solid, the Efficacy or Force of the Vertue in the greater Space will be less than in the lesser Space (because the Vertue is more sparse, being more scattered, in the greater than in the lesser Space) and that in the same Proportion that the lesser Space bears to the greater; that is, *the Efficacy of the Vertue in the greater Space will be to its Efficacy in the lesser, reciprocally as the lesser Space*

Space is to the greater. So the Force or Efficacy of a Vertue diffused thro' a double Space, will only be half the Force of the same Quantity of the Vertue diffused thro' the single Space; because the Vertue will be twice as sparse in the double Space as it is in the single Space. In like manner, the Efficacy in a triple Space, will be a third Part of the Efficacy of the same Quantity in the single Space; because the Vertue will be three Times sparser in the triple Space than in the single.

Suppose now two similar Bodies or Solids, arising from the Revolution of the plane Surfaces $crt d$, $aefb$ (in *Fig. 44.*) about the Radius sk , whose respective Distances from the Center s let be st , sf . Then these similar Solids will be in a triplicate Proportion of the Arches rt , ef , or the Lines td , fb , their homologous Sides. Let both Solids be conceiv'd as made up of an indefinite Number of concentrick spherick Surfaces (not mathematical but physical) of indefinitely small but equal Thickness: Then, if the Arch

rt be as 1, and the Arch ef as 2; and consequently also td (the Thickness of the Solid $crt d$) as 1, and fb (the Thickness of the Solid $aefb$) as 2; the Solid $crt d$ will be as 1, and the Solid $aefb$ as 8: And there will be twice the Number of spherick Surfaces in the Solid $aefb$, that there are in the Solid $crt d$, because fb is $= 2 td$. Now the Quantity of the Vertue, propagated in right Lines around alike, thro' the spherick Space, to or from the Center s , is the same in every one of the said spherick Surfaces rt , cd , ef , ab , and all the rest of the intermediate ones: Therefore the Quantity of the said Vertue diffused thro' the Solid $aefb$, is double of the Quantity diffused thro' the Solid $crt d$. But the Solid $aefb$ is 8 Times bigger than the Solid $crt d$, as before. Therefore, twice the Quantity of the Vertue being diffused thro' 8 Times the Space, or (which is all one) the same Quantity of it being diffused thro' 4 Times the Space; the Efficacy of the Vertue in the Solid $aefb$, is (by the Principle laid down

down before) to the Efficacy of it in the Solid $crt d$, reciprocally as the Solid $crt d$ is to 4 Times the same Solid, or half the Solid $ae f b$, that is, as 1 is to 4. But since, as before, the Arch rt is $= 1$; and the similar Arch $ef = 2$: Therefore is $st : sf :: 1 : 2$, and consequently $st q : sf q :: 1 : 4$. And since the Solid $crt d$ is to half the Solid $ae f b$ as 1 is to 4, or as $st q$ is to $sf q$. Therefore the Efficacy of the Vertue in the Solid $ae f b$; is to the Efficacy of it in the Solid $crt d$ as $st q$ to $sf q$, that is, reciprocally as the Squares of the Distances of the said Solids from the Center s .

If the Arches rt and ef be as 1 and 3; and consequently also td and fb as 1 and 3; the similar Solids $crt d$ and $ae f b$ will be as 1 and 27, the Cubes of 1 and 3; and there will be thrice the Number of spherick Surfaces in the Solid $ae f b$, that there are in the Solid $crt d$; and consequently thrice the Quantity of the Vertue will be diffused thro' the Solid $ae f b$ that is diffused thro' the Solid $crt d$, or the same Quantity of the Vertue will be

diffused thro' a third Part of the Solid $aefb$, or 9 Times the Solid $crt d$, that is diffused thro' the single Solid $crt d$. Therefore the same Quantity of the Vertue being diffused thro' 9 Times the Space, the Efficacy of the Vertue in the Solid $aefb$ will be to its Efficacy in the Solid $crt d$, reciprocally as the Solid $crt d$ is to 9 Times the Solid $crt d$, or as 1 to 9. But, as above, the Arches rt and ef are as 1 and 3; therefore is $st : sf :: 1 : 3$, and consequently $stq : sfq :: 1 : 9$. Therefore the Efficacy of the Vertue in the Solid $aefb$ is to its Efficacy in the Solid $crt d$ as stq to sfq , that is yet, reciprocally as the Squares of the Distances of the said Solids from the Center s . And, in like manner, in any other Proportion of rt to ef , or of st to sf .

But, after all, it is not necessary to prove, that a Vertue, such as a centripetal Force, propagated in right Lines around in a Sphere, acting upon the trine Dimension or Substance of a Body, is at different Distances of the Body from the

the Center reciprocally as the Squarēs of the Distances; since the Distance of a Body from the Center of a Sphere, is as various as the Distances of the several Particles, of which the Body is composed; are, which are innumerable; though the Distances of all the Particles of a spherick Surface from the Center, is every where the same. So then, though the centripetal Force acts on the Substance and internal Parts of Body, or (which is all one) on all the physical Surfaces of which the Body is compos'd, from its inmost to its outmost Extremity, and not on its external Surface only (as Mr. Gordon would have it) yet it cannot act alike on every Part of the Body, but must (by 2 Part 46 Prop.) so act, that its Efficacy must still be reciprocally as the Square of the different Distance of the Parts from the Center.

PROPOSITION XLVII.

Theor. Fig. 45.

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THE Flux and Reflux of the Sea proceed from the Attractions, or centripetal Forces of the Sun and Moon, but especially of the Moon.

This may be proved in respect of the Moon, thus: Let M represent the Moon; Z and N the Earth, c its Center, z the Place where the Moon is in the Zenith, n where in the Nadir, EF the Horizon. Now 'tis evident, that the Water in z , being nearer the Moon M , is more attracted by her, than the Center of the Earth c (or any Parts about the Horizon EF) and this again more than the Water in n . So then the greatest Attraction of the Parts of the Earth towards the Moon is at z , the least at n , and the mean Attraction at c or EF ; that is, since c tends faster towards M than n does, and z faster than c , we may consider the Attraction at c as none (or at least almost none) at all, and the

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Attractions at z and n as tending contrary ways, viz. the Attraction at z tending directly to the Moon, and that at n directly from her; that is yet the same Thing, as if the Parts about c were at Rest, and those about z and n were moving contrary ways. From all which it will be evident, that the Water of the Earth about the Horizon EF will gravitate and be pressed more strongly towards the Center c , than the Water about z and n where the Moon is vertical. And hence it plainly appears; that the Water at z and n will rise and swell, and that the Water about EF will settle down and run towards z and n ; and so the Earth will put on an oval Figure. This certainly would be the Consequence, if the Earth was covered round with Water to any considerable Depth: And even in its present State, it will imitate that Form as much as the dry Land will allow, and by the Moon's Motion occasion the Ebbing and Flowing of the Sea.

The like is to be understood of the Sun's Attraction, though in a much smaller

ler Degree, upon account of his vast Distance.

Scholy I. Fig. 45.

IT will not be amiss here, to assign the Proportion that there is between the Difference of the Moon's Attractions at z and c , and the Difference of her Attractions at c and n ; which is pretty easily done thus. Let the Attraction of the Earth's Center c , towards the Moon m , be 3600, the Square of 60 Semi-diameters of the Earth, or the Moon's middle Distance from the Center of the Earth; for that Attraction may be expressed by any Number we please: Then, since the Moon's Attractions, at different Distances, are reciprocally proportional to the Squares of the Distances, her Attractions at z , c , n must be (as MN^2 , MC^2 , MZ^2 , or as 61 $\times$ 61, 60 $\times$ 60, 59 $\times$ 59, or) as 3721, 3600, 3481 in Order: Therefore the Difference of the Attractions at z and c is 121, and the Difference at c and n is 119, which two Differences are pretty near

near equal; so that the Water at z can be but insensibly higher than the Water at n .

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A Corollary.

FROM hence it is plain, that M^s Gordon's 4th Theorem, in Pag. 105 and 106 of his *Remarks*, is false; wherein he pretends to prove, that the Protuberance of Water which is under the Moon, is considerably higher than the opposite Protuberance, *that is*, that the Water at z (in our *Fig. 45.*) is considerably higher than that at n .

The Substance of his Demonstration is as follows. The Water at n is pressed towards the Earth's Center c by the strongest Attraction, *viz.* the Attraction of the Earth and the Attraction of the Moon m together; whereas the Water at z is pressed to the Earth's Center by the least Attraction, *viz.* the Attraction of the Earth diminished by the Attraction of the Moon, and so gravitates less than the Water

Water at N . The Waters also at E and F are drawn towards z , and not towards N ; and the Water at N , being most pressed, will run towards these low Places E and F ; but the Water at z , being least pressed, will not. So that the Water at N , E , and F will run towards z , and make a considerably greater Protuberance at z than at N .

The Fault of this Demonstration in short is, that though it considers the Attractions or Tendencies of the Parts of the Earth at z , N , E , and F towards the Moon, yet it does not compare these Attractions together; and (which is the greatest Defect) entirely neglects the Tendency of the Center c and Parts about it towards the Moon.

Scholy. II.

BY this Time, I hope, the Reader sees, that I have demolished all *Gordon's* main Forts, from which he has endeavoured to destroy the *Newtonian Philosophy*. And as to his smaller Batteries which at last

last he runs to, they are of so small Moment, that the least Attack in the World will soon ruine them. These last are chiefly the Absurdities, he thinks will follow from the Laws of universal Gravitation, and the Resistance the celestial Bodies meet with from the Fluid of Light scattered through the celestial Regions. As to the First of these, I suppose it will easily be allowed, that an omnipotent Being can impose any Laws upon Matter, that are not inconsistent with its Nature, and involve no Contradiction, as the universal Law of Gravitation, 'tis plain, does not. So that all the Absurdities our *Remarker* deduces from hence, are mere Rove-ries of his own Brain, without any real Foundation. And as to the Second, the Rays of Light are so extremely fine, and so scattered, that the Quantity of the whole Fluid of Light bears hardly any Proportion at all, to the Quantity of the immense Space through which this Fluid is diffused; and consequently the Resistance that the Planets and other celestial Bodies meet with from hence, is almost

nothing at all. So that the said Bodies may, for all this, continue many Thousands of Years without any sensible Alteration in their Motions.

We shall now leave Mr. *Gordon* to his second Thoughts, and give a distinct Account of the Foundation, on which is built a short Way of Argumentation often used by the Great *Newton* and his Followers, to prove one Proportion to be compounded of other two. In order whereto we shall premise the following *Proposition* or *Lemma*.

PROPOSITION XLVIII.

Lem. Fig. 46, 47.

IN rectangular Parallelograms R, r , the Proportion of two Sides A, a , is compounded of the direct Proportion of the Rectangles themselves R, r , and the reciprocal Proportion of the other two adjoining Sides B, b : that is, $\frac{A}{a}$ is $= \frac{R}{r} \times \frac{b}{B}$.

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For (by 23.6. Eucl.) $\frac{R}{r}$ is $= \frac{A}{a} \times \frac{B}{b}$.

Therefore, if we divide by $\frac{B}{b}$, there will
 be $\frac{A}{a} (= \frac{R}{r} \div \frac{B}{b}) = \frac{R}{r} \times \frac{b}{B}$. W. W. D.

PROPOSITION XLIX.

[Theor. Fig. 46, 47.]

IF there be six Quantities so related to one another, that when the third and fourth are the same or equal, the first is to the second as the fifth to the sixth; and when the fifth and sixth are the same or equal, the first is to the second as the third to the fourth: Then, when neither the third is equal to the fourth, nor the fifth to the sixth, the Proportion of the first to the second is compounded of the Proportions of the third to the fourth, and the fifth to the sixth.

This Theorem in the Newtonian Style would be expressed thus. If three Quantities (that is really, three Proportions of six Quantities) be so qualified, that, the
 P second

second being given (*that is* really, the second Proportion being a Proportion of Equality) the first is directly as the third and the third being given, the first is directly as the second : Then, neither the second nor the third being given, the first is as the second and third conjunctly or directly as the second and directly as the third.

The said Theorem, in the first Expression, is demonstrated thus. The first and second Quantities will be represented by two Rectangles R and r , the third and fourth by the Bases B and b , and the fifth and sixth by the Altitudes A and a : Because (by *Sch. 1. 6. Eucl.*) when B is $= b$, there is as $R : r :: A : a$; and (by *1. 6. Eucl.*) when A is $= a$, there is as $R : r :: B : b$. Therefore then, since the said six Quantities in order are just so related as, and represented by R, r, B, b, A, a ; and also since (by *23. 6. Eucl.*) the Proportion of R to r is compounded of the Proportions of B to b and A to a ; it is evident that the Proportion of the first of the forelaid Quantities to the second, is

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compounded of the Proportions of the third to the fourth and the fifth to the sixth. W. W. D.

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PROPOSITION L.

Theor. Fig. 46, 47.

IF there be six Quantities so related, that when the third is equal to the fourth, the first to the second is reciprocally as the sixth to the fifth; and when the fifth is equal to the sixth, the first is to the second directly as the third to the fourth: Then, when neither the third is equal to the fourth, nor the fifth to the sixth, the Proportion of the first to the second is compounded of the direct Proportion of the third and fourth, and the reciprocal Proportion of the fifth and sixth.

This Theorem in the Newtonian Style would be expressed thus. If three Quantities (that is really, three Proportions) be so qualified, that, the Second being given (or really, being a Proportion of equality) the first is reciprocally as the

third ; and the third being given, the first is directly as the second : Then, neither the second nor the third being given, the first is directly as the second, and reciprocally as the third.

The said Theorem expressed the first way, is demonstrated thus. The first and second Quantities may be represented by the Altitudes Λ, a of two Rectangles R, r , the third and fourth by the Rectangles themselves R, r , and the fifth and sixth by the Bases b, B , in a contrary Order : Because (by 14. 6. *Eucl.*) when R is $= r$, there is reciprocally as $\Lambda : a :: b : B$; and (by *Sch.* 1. 6. *Eucl.*) when B is $= b$, there is directly as $\Lambda : a :: R : r$. Therefore then, since the said six Quantities are just so related as the Altitudes Λ, a , the Rectangles R, r , and the Bases B, b ; and also, since (by 48 *Prop.*) the Proportion of Λ to a is compounded of the direct Proportion of R to r and the reciprocal Proportion of b to B ; it is evident, that the Proportion of the first of the foresaid Quantities to the second, is compounded of the direct Proportion of the third and fourth,

fourth, and the reciprocal Proportion of the fifth and sixth, w. w. d.

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SUPPOSE now one was to prove, in Sir Is. Newton's short Way, the 6th Proposition, viz. that in uniform Motions, the Proportion of the Spaces run through is compounded of the direct Proportions of the Times and Celerities; which in his concise Stile would be expressed thus, *The Space run through is as the Time and Celerity conjunctly.*

His Proof is expressed thus. The Time being given, the Space is directly as the Celerity; and the Celerity being given, the Space is directly as the Time: Wherefore, neither the Time nor the Celerity being given, the Space is as the Time and Celerity conjunctly. The true Meaning is; the Times being the same or equal, the Spaces are (by 1 Prop.) directly as the Celerities; and the Celerities being the same or equal, the Spaces are (by 2 Prop.) directly as the Times:

Wherefore, neither the Times being the same or equal, nor yet the Celerities, the Proportion of the Spaces is, compounded of the direct Proportions of the Times and Celerities. Now that this is a true and conclusive Argument and Way of Reasoning, is evident from 49 Prop. because the two Spaces, the two Times, and the two Celerities here, are six Quantities just so related to one another, as those six in that Prop.

Suppose again one was to demonstrate in Newton's concise Method the 1 Cor. of 6 Prop. viz. that in uniform Motions, the Proportion of the Times is compounded of the direct Proportion of the Spaces, and the reciprocal Proportion of the Celerities; which in his brief Stile would be expressed thus, *The Time is directly as the Space, and reciprocally as the Celerity.*

His short Proof would be expressed thus: The Space being given, the Time is reciprocally as the Celerity; and the Celerity being given, the Time is directly as the Space :: Wherefore, neither the Space nor the Celerity being given, the
Time

Time is directly as the Space, and reciprocally as the Celerity. The true Meaning is ; the Spaces being the same or equal, the Times are (by 4 Cor. 6 Prop. which has no Dependence on 1 Cor.) reciprocally as the Celerities; and the Celerities being equal, the Times are (by 2 Prop.) directly as the Spaces: Wherefore, neither the Spaces nor Celerities being equal, the Proportion of the Times is compounded of the direct Proportion of the Spaces and the reciprocal Proportion of the Celerities. Now that this is a true and conclusive Argument, is evident from 50 Prop. because the Times, the Spaces, and the Celerities here, are six Quantities just so related and qualified, as those six in that Prop.

We shall conclude this short Treatise, with the Great *Newton's* own Demonstration of his general Law of centripetal Forces: In order whereto we shall premise the two following Lemmas.

PROPOSITION LI.

Lem. Fig. 36.

THE infinitely little or nascent Lines *cf*, *br*, produc'd or describ'd by a centripetal Force, in equal Particles of Time, are as the centripetal Forces or Impulses in the Points *r*, *p*; at least infinitely near.

This, I think, is incontrovertible; because the centripetal Forces in *r* and *p* are the Causes that produce the nascent Lines *cf* and *br*. See Sch. 44 Prop.

PROPOSITION LII.

Lem.

THE Spaces that a Body, constantly urged by a regular Force, describes, are, in the Beginning of the Motion, in a duplicate Proportion of the Times in which they are described.

This is Newton's 10 Lem. 1 Lib. Princip. and it is a plain Consequence of Schol.

22 *Prop. preced.* because a Force that constantly acts regularly, does, in the Beginning of the Motion, act uniformly or equally, at least infinitely near so.

We shall now give *Newton's* own Demonstration of his general Law of centripetal Forces, which, though for Clearness sake we shall enlarge it, I'm confident every Body will grant to be according to his Mind.

PROPOSITION LIII.

[Theor. Fig. 36.]

EVERY Thing being supposed as in 36 *Prop.* I say that the centripetal Force in any Point *P* of the Curve will be reciprocally as the nascent or evanescent Solid $\overline{SPq \times B D q}$.

The centripetal Forces and Times being denoted as in 36 *Prop.* if, in the infinitely small or nascent Figures $P R B D$, $pr b d$, the nascent Lines BR , br be described in equal Times, they are (by 51. *Prop*) as the centripetal Forces in the Points.

Points P, p , or as v, v ; and, if the centripetal Forces in P, p be equal, the nascent Lines BR, br are (by 52 Prop.) as the Squares of the Times in which they are described, or as τ^2, t^2 . Therefore, if neither the Times nor the centripetal Forces be equal, the Proportion of BR to br , is (by 49 Prop.) compounded of the Proportions of τ^2 to t^2 and of v to v , that is, $\frac{BR}{br}$ is $= \frac{\tau^2}{t^2} \times \frac{v}{v}$. Whence $\frac{v}{v}$ is $= \frac{BR}{br} \div \frac{\tau^2}{t^2} = \frac{BR}{br} \times \frac{t^2}{\tau^2}$.

But (by 32 Prop.) $\frac{t^2}{\tau^2}$ is $= \frac{Sp b}{SPB} =$ (by

41. Y Eucl.) $\frac{Sp \times bd}{SP \times BD}$, and consequently

$\frac{v}{v} = \frac{Spq \times bdq}{SPq \times BDq}$. Therefore is $\frac{v}{v} = \frac{BR}{br} \times \frac{Spq \times bdq}{SPq \times BDq}$. Whence (as in 36 Prop.) v

is to v as $\frac{Spq \times bdq}{br}$ is to $\frac{SPq \times BDq}{BR}$; or the centripetal Force in P is reciprocally as the nascent or evanescent Solid $\frac{SPq \times BDq}{BR}$.

W. W. D.

A Scholy.

THIS Demonstration, though perhaps obscurer to some than that of Dr. Gregory's deliver'd in 36 *Prop.* is yet preferable, being more general, because agreeing to all Sorts of regular Curves that can be described by a projectile and a centripetal Force: Whereas Gregory's Demonstration agrees only to those Curves to which equicurve Circles may be described, as is evident, since it depends upon Cor. 35 *Prop.*

F I N I S.

APPENDIX.

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APPENDIX.

HAVING finished the former Treatise, we shall here for the Diversion and Exercise of the Minds of Youth, controvert some material Things that have been demonstrated there; and advance some Arguments that seem to overthrow them. And since Mr. Gordon is so good at contriving Fallacies of his own, he may, if he pleases, try his Faculty at detecting those of others. But what need I speak so, since 'tis to be supposed that he would be glad, that the Knot I am going to tie could not again be loosed. In order to my present Design, I shall premise the two following *Lemma's*, and afterwards compare them together.

Q

Lemma

Lem. 1. Fig. 36.

THE nascent or evanescent Subtenses RB , fc of the Angle of Contact in a Curve $APBP$, are in the simple Proportion of their conterminal Arches PB , PC .

For RB being an infinitely small or nascent Line, the Point B must be infinitely near the Point P , and consequently PB must be an infinitely small or nascent Arch: Whence the said Arch is infinitely little different from a right Line: Therefore the Triangles RPB , fcP (the Point c being between B and P) are to be considered as rectilinear Triangles; but they are also equiangular, because fc and RB are parallel. Therefore (by *4. 6. Eucl.*) as is $RB : fc :: PB : PC$.
W. W. D.

This Demonstration seems very plain; natural, and convincing.

Lem.

Lem. 2. Fig. 36.

THE nascent or evanescent Subtenses RB , fc of the Angle of Contact in a Curve, are in the duplicate Proportion of their conterminal Arches PB , Pc .

This *Lemma* which indeed contradicts the former, was before deduced as an Inference or *Corollary* from 35 *Prop. preced.* Now we shall enquire whether has the Advantage of the other.

The Demonstration of the first *Lemma* supposes only, that the infinitely little Arch PB is a right Line, which, though it be not accurately so, is infinitely near so; and this Supposition seems very natural and allowable. Whence it will (by 4.6. *Euch.*) immediately, necessarily, and rigorously follow, that as $RB:fc::PB:Pc$. The Proof then of the said *Lemma* only supposes one infinitely small Inaccuracy.

The Demonstration of the second *Lemma* supposes several small Inaccuracies. First, The Demonstration of the first Case

of 34 *Prop.* whereon the said *Lemma* depends, supposes that the Arches ΔD , Δd (see *Fig. 33*) coincide with their Chords ΔD , Δd ; which is not rigorously true, and is the first Inaccuracy of the same Nature with that in the first *Lemma*. From whence, *Secondly*, It will follow that the Arch ΔD is a right Line, the same with its Chord ΔD ; and that the Arch Δd , a Part of the Arch ΔD , is a Part of the Chord ΔD : And consequently d must be consider'd as a Point of the Chord ΔD . But this being supposed, though the Angle ΔDC (by 31. 3. *Euc.*) be rigorously a right one, the Angle Δdc will not rigorously be a right one; and then it will not rigorously follow that Chord Δd is $= b.d \times \frac{A C}{A C}$, or $\frac{\text{Chord } \Delta d}{A C}$ is $= b.d$, which is a necessary Step in the Demonstration of the first Case of 35 *Prop.* and consequently the said Case it self will not rigorously follow. This is a second Inaccuracy. Therefore, since there are two Inaccuracies in the Demonstration of the said first Case, there must also be two in the second *Lemma* that depends upon it.

But,

But, *Thirdly*, There is yet another Inaccuracy, perhaps as great as, if not greater than either of the other two, in the Demonstration of *Cor. 35 Prop.* which *Corollary* is the second *Lemma* we are just now speaking of, in accommodating the Curve to the Circle: This, I believe, will be very obvious to any Body that attentively considers the said *Corollary*.

From all which it appears very plain, that the first *Lemma* has so far the Advantage of the second, that the first is to be admitted for Truth, and the second rejected. But I have yet something farther to say in behalf of the first *Lemma* and against the second, which is this. Mr. Gordon's *Theorem* about the Fall of a Body, describing a Curve, from the Tangents, which I have given at the Beginning of *Scholy 44 Prop.* looks really so like a Paradox as to be utterly incredible: Yet if the second *Lemma* be admitted for Truth, that *Theorem* is really and truly demonstrable from it; see the just now mentioned *Scholy*. But if the first *Lemma*:

be granted to be true, the said Theorem will easily be overthrown, and that which appears to be the Truth, proved, viz. That, if (in Fig. 43) ad be an infinitely small Arch of a Curve, and Tangents be drawn at all the Points of Division of the said Arch divided into an Infinity of Parts; a Body describing the said Arch will fall from the Tangents certain Spaces, and the Sum of all these Spaces will be (infinitely near) equal to the versed Sine kd of that Arch. Which is thus demonstrated.

Suppose the infinitely small or nascent Arch ad bisected in c ; then (by 1 Lem.) as is $ec : kd :: ac : ad :: 1 : 2$, therefore is $ec = \frac{1}{2} kd$. When the Body has come from a to c , the centripetal Force tending to s , has made it fall, from the Tangent of the Point a , the Space ec ; and when it is come to d , the said Force has made it fall from the Tangent of c , another Space equal to ec , at least infinitely near so; the Sum of which two Spaces, viz. $2 ec$ is $= kd$, because (as before) ec is $= \frac{1}{2} kd$. If again we suppose

pose the Arch ad divided into three equal Parts ab , bc , cd ; the Body, by the Force tending to s , will fall from the Tangent of a the Space fb , from the Tangent of b as much, and from the Tangent of c also as much, *that is*, it will fall from the three Tangents of a , b , c , three Spaces, every one of which is equal to fb , the Sum whereof, *viz.* $3fb$ is $= kd$, because (by 1. *Lem.*) as is $fb:kd::ab:ad::1:3$. In like Manner, if we suppose the Arch ad divided into four equal Parts, the Sum of the Spaces fallen from the Tangents of the Point a and the next three Points of Division, will be equal to kd : And so forth; be the Points of Division ever so many. Therefore the Sum of the Spaces fallen from all the Tangents, while the Body is describing the infinitely little Arch ad , is (at least infinitely near) equal to the versed Sine kd of that Arch. *w. w. d.*

The first *Lemma* being now admitted for Truth, the general Law of centripetal Forces will be found very different from that of the Great *Newton* formerly de-