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ELECTRICAL

ENGINEERING LEAFLETS

BY

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AND

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ADVANCED GRADE

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PREFACE.

THE Electrical Engineering Leaflets have been prepared for the purpose of presenting, concisely but accurately, some of the fundamental principles of electrical science, as employed in engineering practice. They have been arranged under three grades; namely, the Elementary, the Intermediate, and the Advanced.

The Elementary Grade is intended for those electrical artisans, linemen, motormen, central station workmen, or electrical mechanics generally, who may not have advanced sufficiently far in their studies to warrant their undertaking the other grades. Here the mathematical treatment is limited to arithmetic, and the principles are illustrated by examples taken from actual practice.

The Intermediate Grade is intended for students of electricity in high schools and colleges. In this grade a certain knowledge of the subjects of electricity and physics generally is assumed, and a fuller mathematical treatment is adopted. These leaflets, moreover, contain such information concerning the science of electricity, as should be acquired by those desiring general mental culture.

The Advanced Grade is designed for students taking special courses in electrical engineering in colleges or universities. Here the treatment is more condensed and mathematical than in the other grades.

Although the three grades have been especially prepared for the particular classes of students referred to, yet it is believed that they will all prove of value to the general reading public, as offering a ready means for acquiring that knowledge, which the present extended use and rapidly increasing commercial employment of electricity necessitates.

Laboratory of Houston & Kennelly,
Philadelphia, March, 1895.

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Electrical Engineering Leaflets,

—BY—

Prof. E. J. Houston, Ph. D.
AND
A. E. Kennelly, F. R. A. S.

ADVANCED GRADE.

ELECTRICAL EFFECTS.

1. The development of electrical excitation by friction, as is well known, is due to the contact of dissimilar material surfaces. The discovery of the existence of an electric force is ascribed to Thales, B. C., 600. Not only is the exact mechanism whereby electrical excitation is evoked by friction unknown, but even the nature of the excitement itself yet remains to be discovered. The electric force is, however, associated with a stress in an all-pervading medium called the ether. When two dissimilar substances are brought into contact, a stress in the ether is produced at the contact surfaces, and, on separating the bodies, a condition of deformation, or strain, pervades the ether in the surrounding space.

Whatever the nature of the strain may be, it is certainly polarized as regards direction, as is evident from the fact, that the condition of excitement, which appears to exist at the surface of one of the bodies, is different

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from, but supplementary to, the condition of excitement at the surface of the other body, and this difference or *polarity* is arbitrarily referred to as a positive and negative charge respectively.

An electric charge is generally supposed to reside on the surface only of the charged body, and, so far as manifestations of force are concerned, one might readily believe this to be the case, but it has been clearly proved that the active disturbance exists in the medium between the two excited bodies, and that the so-called charge is merely an effect of the discontinuity of this strain at their surfaces.

2. Contact between dissimilar materials produces an *electromotive force* in the ether between them, and it is this electromotive force or stress, which establishes the strain in the ether. The establishment of such a strain is called an *electric displacement and can only be maintained in non-conductors or dielectrics.* Electric displacement is of the nature of a *flux*, and follows, in its distribution, either the motion of displacement in an incompressible fluid or the strain in a compressible isotropic solid; namely, that as much flux must issue from any portion of space as enters it, provided no electric charge exists within that space. This is only another way of stating the fact that discontinuity of the flux exists at the surfaces of the excited bodies or the boundaries of the E. M. F.

The passage of a displacement flux constitutes an electric current; a momentary electric current, therefore, accompanies the charge and discharge of a dielectric, and such current is oppositely directed on charge to what it is on discharge. An electric current in a dielectric is accurately defined as the time-rate of change of the displacement, as will be afterwards more fully explained.

3. The effects produced by an electric discharge or current are extremely varied. Among the most important are the following:

(*1.*) Radiant effects.
(*2.*) Thermal effects.
(*3.*) Magnetic effects.
(*4.*) Electrolytic effects.
(*5.*) Physiological effects.

All these effects are believed to be different kinds of motions in the ether or in matter. To the motion of the ether belong the effects of magnetism and of radiant energy; *i. e.*, heat and light; while in the motion of the molecules of matter we have the purely thermal phenomena connected with temperature, and in the motions of the atoms and radicals, we have the phenomena of electrolysis.

4. It is necessary to distinguish between the terms force, work, and energy.

Force is that which sets a body in motion, arrests its motion, or changes the direction or velocity of its motion; *i. e.*, briefly that which alters the motion of matter. Force manifests itself in a great variety of forms, viz: muscular force, gravitational force, magnetic force, electric force, mechanical force and the forces of elasticity.

Work is done when force moves matter through a distance, and no work can be done unless the force or forces applied do produce such motion.

Energy is the capability of doing work and is of two kinds, namely, *kinetic* and *potential*. Kinetic energy is either the energy in moving bodies, or energy actually doing work; *i.e.*, the energy of motion. Potential energy is the energy of bodies at rest, or energy not actually displayed in motion, but connected with the potentiality of doing work.

5. It is possible that potential energy is only an unrecognized form of kinetic energy, just as the heat energy in a body, which measures its temperature, was at one time considered to be potential energy, but is now admitted to be a form of molecular kinetic energy. As knowledge advances it is probable that other forms of energy now regarded as potential may prove in reality to be due to motion, either in matter or in the ether; *i.e.*, kinetic energy.

6. The doctrine of the conservation of energy underlies all the phenomena in the physical sciences, and the appreciation of this doctrine underlies all engineering. This doctrine may be stated as follows:

The sum of the energy in the whole universe, as known to us, is constant.

When energy disappears in one form, an equal amount invariably appears in some other form; and force is manifested whenever energy changes form or changes magnitude.

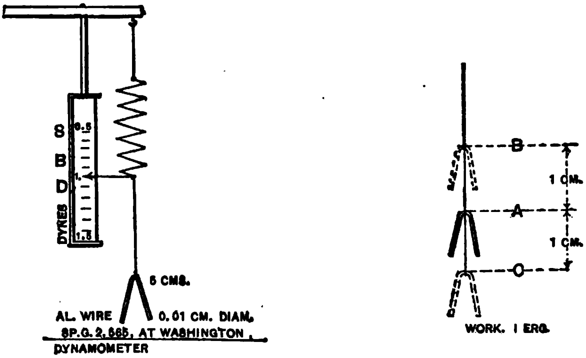
All natural phenomena are evidences of energy, and are brought about by forces acting on matter.

7. For ready expression and calculation it is convenient to refer the magnitudes of force, work and energy to certain fundamental scientific units, based upon the centimetre as the unit of length, the gramme

as the unit of mass, and the second as the unit of time. These are, therefore, called the centimetre-gramme-second units, or the C. G. S. units.

8. The C. G. S. unit of force is the *dyne*, and is that force which, after acting for one second on a mass of one gramme, imparts to it a velocity of one centimetre-per-second. The force of gravitation is commonly expressed by the formula,

$$g = 980.6056 - 2.5028 \cos 2\,l - 0.000003\,h,$$



FIGS. 1 AND 2.

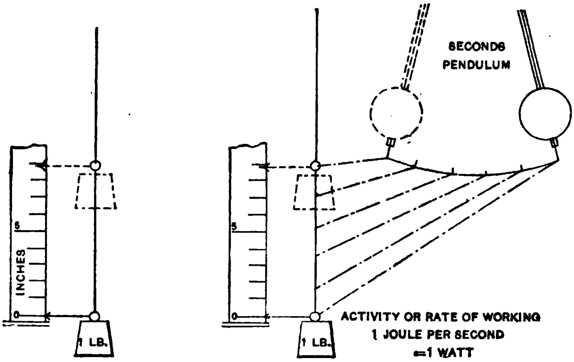
where *l*, is the latitude of the station, and *h*, its height in centimetres above the sea level, and, consequently, the weight of a body in *dynes*, *i.e.*, the force with which the earth attracts the body, is the product of the body's mass in grammes and this force, or

$$F = m\,g$$

The dyne is approximately equal to the weight of one milligramme (1.0203 mg.) at Washington. (See Fig. 1.)

Since the dyne is often an inconveniently small unit of force, the megadyne (one million dynes) is frequently employed in ordinary applications.

The megadyne is approximately equal to the earth's gravitational force on one kilogramme of matter, *i.e.*, to the weight of one kilogramme (1.0203 kgm. at Washington.) A column of mercury, 760 cms. high, at 0° C., which represents one atmosphere, presses upon its base with a force of approximately one megadyne per square centimetre. (1.0126 megadyne at Washington.)



ACTIVITY OR RATE OF WORKING
1 JOULE PER SECOND
=1 WATT

1 LB. LIFTED 0.738 FOOT—8.84″
PERFORMANCE OF 1 JOULE OF WORK AT WASHINGTON
=10 MEGALERGS

FIG. 3.

9. The erg, or the unit of work, is the work done when the force of one dyne acts through the distance of one centimetre. (See Fig. 2.) The erg is, therefore, the dyne-centimetre. Since the erg is too small a unit of work for practical purposes (so small a unit as one foot-pound, or the work done when a pound of matter is raised through the vertical distance of one foot, being about 13,550,000 ergs at Washington), the megerg, or one million ergs is more commonly employed.

One foot lb. = 13.55 megergs. } at Washington.
One megerg = 0.0738 foot lb. }

It may be remarked that the total amount of energy aggregated in the sun by the concentration from infinite diffusion to his apparent bulk under the influence of gravitation would be 2.6 × 10^{48} ergs, or 2.6 × 10^{41} joules.

In the measurement of electrical energy, the joule, which is 10 megergs (10,000,000 ergs), is the unit generally employed.

A joule is, therefore, = 0.738 foot pounds at Washington, or 1 foot pound = 1.355 joules at Washington.

10. When work is done, say, in lifting a weight against gravitational force, the same amount of energy is obviously expended (disregarding air resistance), whether the weight is lifted in one minute or in one second; but in the latter case it is evident that energy is expended sixty times more rapidly than in the former. This rate of expending energy, or of doing work, is called *activity*, as is diagrammatically shown in Fig. 3.

The C. G. S. unit of activity is the dyne-centimetre per second, *i.e.*, the erg per second; but the practical unit usually employed in engineering is the watt, which is one joule per second.

The average activity of a laborer, working with pick and shovel, is about 50 joules per second, or 50 watts. The average activity of the standard horse introduced by Watt into engineering, *i.e.*, 550 foot pounds per second is 746 watts. The activity in a nominal 2,000 candle power arc light is rated at 450 watts or 0.225 watts per candle, and in a 16 candle power incandescent lamp is about 50 watts or 3⅛ watts per candle.

For many engineering purposes the kilowatt (1,000 watts) is the unit of activity employed. Dynamos, for example, are commonly rated in kilowatts.

11. The amount of energy absorbed by a machine is called its *intake*. Since energy is never destroyed, the amount of work done by the machine must be rigorously equal to the intake. Since, however, some energy is always expended in the machine uselessly, the useful amount of work delivered by the machine, called the *output*, must always be less than the intake. The output divided by the intake is called the *efficiency* of the machine. Very large, well constructed dynamos of, say, 1,000 kilowatts capacity have an efficiency at full load of 0.96.

SYLLABUS.

Contact between dissimilar materials establishes a *stress* in the ether called electromotive force (E. M. F.)

The stress produces a strain flux in the ether called *electric displacement*. The strain accompanying an electric *charge* resides in the *dielectric* medium. The time rate of change of this flux is an electric current, flowing in the positive direction when increasing, in the negative direction when diminishing, the direction of the displacement being taken as positive.

All electrical effects are believed to be referable to different kinds of motion either in the ether or in matter.

All energy is either kinetic or potential. At any instant the sum of the kinetic and potential energies in the known universe is believed to be constant.

Electrical energy is conveniently measured in joules.

The rate of exchange or development of energy is termed *activity* and is measured in watts.

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ADVANCED GRADE.

ELECTROMOTIVE FORCE.

12. By the term electromotive force is meant the unknown cause or force which produces or tends to produce an electric current. Although but little is known concerning the exact nature of electromotive force, (abbreviated E. M. F.), it is believed to be associated with some variety of stress in the ether. This stress produces a strain called *displacement* which in the ether is permanent. In ordinary matter the displacement strain varies according to whether the matter is electrically conducting or non-conducting. If conducting, the strain cannot be maintained; if non-conducting, it is progressive, or advances with time, like elastic fatigue in materials under stress. The establishment of an electromotive force is, therefore, invariably attended, in the case of ether or non-conductors, with the establishment of a temporary current, and, in the case of conductors or a conducting circuit, of a continued current.

The entire series of phenomena accompanying the

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stress of E. M. F., and the establishment of its displacement strain, is called *electrification.*

13. Although the exact nature of a displacement current is unknown, yet, quantitatively, it suggests an analogy to the following phenomena. Suppose, for example, that an electromotive source has its poles connected respectively with two elastic conducting globular or other surfaces placed in a homogeneous deformable but incompressible jelly; and, that under the influence of the E. M. F., push and pull stresses are respectively exerted on the jelly at the surfaces connected with positive and negative poles, producing a positive pressure or outward thrust at the positive pole, and a negative pressure or inward thrust at the negative pole, measured in dynes per square centimetre of surface. Then the deformations, *i. e.* strains or displacements in the jelly produced by the elastic expansion of the positive pole, the elastic contraction of the negative pole, and the yielding of the jelly between them, correspond to the electrical displacements in the ether under the influence of E. M. F. The amount of jelly displaced through a bag surrounding either conducting surface would always be the same, viz., the increment of volume of the electro-positive pole, or the decrement of volume of the electro-negative pole. The rate of displacement in the jelly would evidently be a *flow*, just as the rate of electrical displacement in the ether constitutes an *electric current.*

The amount of displacement which will take place in the jelly for a given pair of polar surfaces and a given pressure in them, will vary with the distance between them; so too the electrical displacement that will take place between two conducting surfaces at a given E. M. F. varies with their distance and disposition.

It should be remembered, however, that the preceding is an analogy only, and that the actual nature of displacement may be quite different from what has just been suggested.

14. An electromotive force is a *vector quantity*, that is, a force which like a mechanical force has necessarily both direction and magnitude, and, like a mechanical force, can be resolved into components.

In the ether or any non-conductor, the E. M. F. producing displacement strain is a *localized vector*; that is, possesses a definite magnitude at every point in space. In a conducting circuit, however, the electromotive force which is acting to send a current is equal to the sum or line integral of all the E. M. F.'s residing in the circuit.

In the International C. G. S. system the practical unit of E. M. F. is the volt.

15. E. M. F. like all other forces does no work unless it is producing motion, *i. e.*, an electric current, and, just as the amount of work done by a force, is the product of that force into the distance through which it moves, or

$$W = Fd,$$

so the work done by an E. M. F. is equal to the product of the E. M. F. and the amount of electric motion or *quantity*. The practical unit of quantity in the international C. G. S. system is called the *coulomb*, and when one volt acting on a circuit causes one coulomb of electric quantity to pass through the circuit, one joule of work, derived from the electric source, is expended by the E. M. F. in the circuit.

Generally, therefore, if E, be the E. M. F. in a circuit (expressed in volts), and q, the quantity of electricity passing through the circuit (expressed in coulombs), then the *work done by* the E. M. F. is

$$W = E\,q \text{ joules;}$$

(W, is positive if E and q have the same sign; and negative if E and q have different signs. Thus in Fig. 4. the battery of E volts sends in a given time an electric quantity of q coulombs through the conducting circuit. The equation shows also that when W is negative, the work done by the E. M. F. is negative, or work is *done on* an E. M. F. when it opposes a current, this work appearing in the electric source supplying the E. M. F.

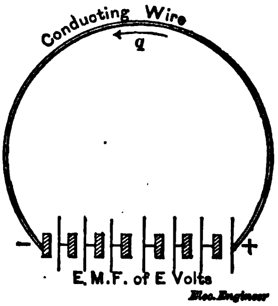


Fig. 4.
Expenditure of $E\,q$ joules.

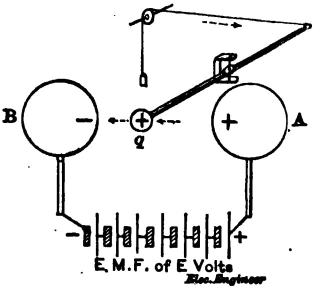


Fig. 5.
Expenditure of $E\,q$ joules.

16. This law is true both for conductors and non-conductors, though in non-conductors no current can be sustained. If, however, an E. M. F. E, be in communication with two conductors insulated from each other, such as two metal spheres, A and B, (Fig. 5.) separated by an air space, an electro-positive or + charge

of q coulombs on a third electric conductor C, say a small insulated metal globe, between the two conductors A and B, will, in effect, be simultaneously attracted by B and repelled by A. The total work which will be expended upon C, in the passage from A to B, under these forces will be

$$W = E\,q \text{ joules, as before.}$$

If, therefore, the quantity in the movable conductor C, be one coulomb, then the amount of work in joules expended in the transfer from A to B, gives in volts the E. M. F., E. But since work is expended continuously throughout the entire journey, the work done in joules from departure at A, to any intermediate point, measures in volts the *difference of potential* between A, and the intermediate point.

The difference of electric potential in volts between two points, is the energy expended in joules which one coulomb of electricity will perform in passing between the points.

The sum of all the intermediate potential differences (abbreviated P. D.'s.) in the path of C, between A and B, is obviously equal to the E. M. F. E; and, generally, the E. M. F. of any electric source at its poles is equal to the total P. D. between its poles in the external circuit. Any difference of potential is an E. M. F., but an E. M. F. is not necessarily accompanied by a difference of potential. An electric current tends to flow from a higher to a lower potential, just as a thermal current (heat) tends to flow from a higher to a lower temperature.

17. Devices for producing E. M. F.'s are called *electric sources.* They may be divided into classes according to the nature of the energy they absorb when in action, *i. e.*, when they supply a current.

(*1.*) Voltaic cell	Chemical Potential Energy.
(*2.*) Charged storage cell	
(*3.*) Selenium cell	Radiant Energy.
(*4.*) Thermo cell	
(*5.*) Frictional electric machine.	Mechanical Energy.
(*6.*) Influence electric machines.	
(*7.*) Magneto machines	
(*8.*) Dynamo machines	
(*9.*) Plants and animals	Vital Energy.

18. The following table gives the E. M. F. of a number of electric sources:

CELL.	PLATES.		ELECTROLYTE.	E. M. F. Volts.
	+	−		
Bichromate or Grenet	zinc	carbon	electropoin	1.9
Bichromate double fluid	zinc	carbon	dilute sulphuric acid	2.0
Bunsen	zinc	carbon	dilute sulphuric acid dilute nitric acid	1.96
Daniell	zinc	copper	zinc sulphate, copper sulphate	1.072
Edison-Lalande	zinc	copper	dilute caustic soda	0.667
Fuller	zinc	carbon	dilute sulphuric acid bichromate of potash	2.0
Grove	zinc	platinum	dilute sulphuric acid dilute nitric acid	1.93
Leclanché	zinc	carbon	sal ammoniac, manganese dioxide	1.47
Silver chloride	zinc	silver with chloride	sal ammoniac	1.03
Secondary Cell or Storage Battery			dilute sulphuric acid	2.00

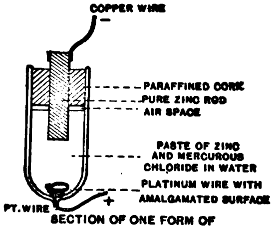
The following E. M. F's are not capable of precise limitation. Average values and limits are given.

Plating dynamos ....5 to 100 volts.
Continuous current incandescent dynamos, 50 to 150 volts.

Arc light dynamos	250 to 10,000 volts.
Street railway dynamos	300 to 700 volts.
Alternators for transmission of power	1,000 to 4,000 volts.

Frictional machines	500 kilovolts and over.
Influence machines	500 kilovolts and over.

Thermo-couple	a few millivolts, depending on metals used and temperatures of their junction.



SECTION OF ONE FORM OF CLARK STANDARD CELL

Elec. Engineer

FIG. 6.

19. The Clark cell is employed for accurate comparison and measurements of E. M. F., and, when properly prepared, is considered to have an E. M. F. of 1.434 International volts at 15° C. It is made up with pure mercury and pure zinc as the metallic elements, and pastes of mercurous sulphate and zinc sulphate as the electrolyte.

A common form of such cell is shown in FIG. 6, in which a platinum wire *Pt*, is sealed into the glass cell at its lower extremity. The part within the cell is first amalgamated with pure mercury and is then surrounded by a paste consisting of a mixture of mercurous and zinc sulphate. The temperature coefficient of this cell is usually taken as 0.077% per °C., so that

$E = 1.434\ [1 - 0.00077\ (t - 15)]$ International volts.

When the highest accuracy is required in the use of this cell, certain precautions are necessary in its preparation, which are accurately described in specifications issued by the British Board of Trade, a copy of which is published in vol. x, of the Transactions of the American Institute of Electrical Engineers, 1893, (page 19).

SYLLABUS.

Electromotive force is the name given to the unknown cause or force which produces or tends to produce an electric current.

During the establishment of an E. M. F. displacement currents are produced.

An E. M. F. is a vector quantity, *i. e.*, possesses both direction and magnitude.

An E. M. F. does no work unless it is producing motion, *i. e.*, an electric current.

E. M. F's. are measured in International volts of which a Clark cell is regarded as producing 1.434 at 15° C.

The practical unit of electric quantity is called the International coulomb.

The transference of one coulomb through a difference of potential of one volt is accompanied by an expenditure of energy of one joule.

Difference of electric potential constitutes E. M. F.

Laboratory of Houston & Kennelly,
Philadelphia.

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ELECTRIC RESISTANCE.

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20. The resistance of a conductor is that quality in virtue of which it limits the flow of electricity through it under a given electromotive force. The exact nature of resistance is unknown.

The resistance of a uniform, homogeneous conductor varies directly as its length and inversely as its cross sectional area. The resistance of a body of given shape, depends upon the nature of the body.

21. The practical unit of electrical resistance is called the *International ohm*, and, as nearly as can be determined, is equal to the resistance offered by a uniform column of mercury, 14.4521 grammes in weight and 106.3 centimetres in length, at the temperature of melting ice. Such a column would have a cross-section of one square millimetre.

22. Multiples and submultiples of the ohm are conveniently represented by certain prefixes as below.

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	MULTIPLES.	DERIVATIVES.
deka..	10	ten..............10
hecto..	100	one hundred.....10^2
kilo...	1,000	one thousand. ..10^3
mega..	1,000,000	one million .....10^6, megohm.
bega..	1,000,000,000	one billion .....10^9, begohm.
trega..	1,000,000,000,000	one trillion......10^{12}, tregohm.
quega	1,000,000,000,000.000	one quadrillion ..10^{15}, quegohm.

	SUB-MULTIPLES.	DERIVATIVES.
deci.....	0.1	one tenth...........10^{-1}
centi....	0.01	one hundredth......10^{-2}
milli....	0.001	one thousandth.....10^{-3}
micro...	0.000001	one millionth.......10^{-6}, microhm.
bicro....	0.000000001	one billionth . ....10^{-9}, bicrohm.
tricro..	0.000000000001	one trillionth ....10^{-12}, tricrohm.

The fundamental C. G. S. unit of resistance is a *bicrohm*, and its value is such that one C. G. S. unit of current passing through it, expends an amount of energy equal to one erg each second.

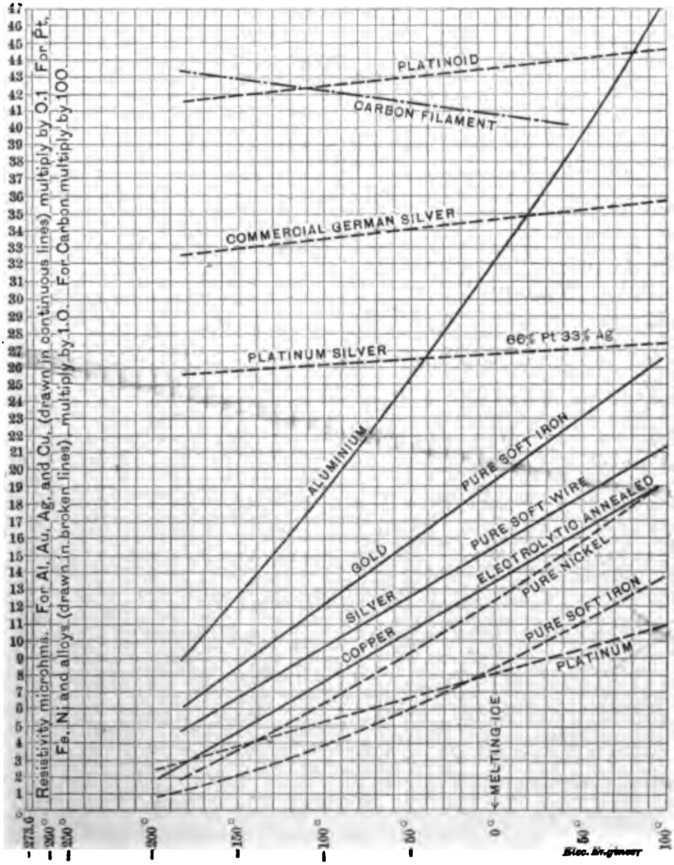
23. By the *resistivity* or *specific resistance* of a body, is understood the resistance of a cubic centimetre of the body, measured between opposite faces, at the temperature of zero centigrade. The following is a table of the resistivities of various common substances expressed in *International* ohms. Thus the resistivity of pure, soft or annealed copper, the metal most frequently used in electrical instruments, is 1.594 microhms at 0° C., and increases, as shown by the temperature coefficient in the third column, according to Matthiessen, 0.388 per cent. per degree centigrade for a *small* variation of temperature. An inspection of the table will show that while the resistivity of almost all the pure metals is markedly different, yet the temperature-coefficient for the small

limits, already mentioned, is practically the same with the exception of liquid mercury. The temperature-coefficients of the alloys, however, is much less than that of the ingredient metals. The resistivity of carbon (graphite) is about 0.07 ohm, and. its temperature-coefficient is negative, that is to say, the resistance diminishes with the temperature. For example, in the incandescent electric lamp, the resistance of the carbon filament when cold, is about twice as great as at the incandescent working temperature.

Substance.	Temperature.	Resistivity.	Temperature Coefficient.	Authority.
Silver, annealed ..	0° C.	1.500 microhms	0.377	Matthiessen.
Silver, hard drawn	“	1.53 “	“	“
Copper, annealed (Matthiesen's standard).......	“	1.594 “	0.388	“
Copper, hard dr'wn	“	1.629 “	“	“
Gold, annealed....	“	2.052 “	0.365	“
Gold, hard drawn..	“	2.089 “	“	“
Aluminum, annealed	“	2.903 “	....	“
Zinc, pressed......	“	5.598 “	0.365	“
Platinum, annealed	“	9.030 “	....	“
Iron, annealed....	“	9.687 “	....	“
Nickel, annealed..	“	12.420 “	....	“
Tin, pressed......	“	13.17 “	0.365	“
Lead, pressed.....	“	19.57 “	0 387	“
Antimony, pressed.	“	35.40 “	0.389	“
Bismuth, pressed ..	“	130.8 “	0.354	“
Mercury, liquid ..	“	94.84 “	0.072	“
Platinum-silver alloy, 2 parts Pt. to one Ag. hard or annealed	“	24 32 “	0.031	“
German silver	“	ab't 20.9 “	0.044	“
Platinoid	“	“ 32.7 “	0.021	Fleming.
Hadfield's manganese steel. .	“	“ 68.0 “	0.122	“

Substance.	Temperature.	Resistivity.	Temperature Coefficient.	Authority.
Selenium	0° C.	ab't 59000 ohms	1.0	Mattheissen.
Retort carbon	"	" 0.07 "	about —0.5	Everett.
Graphite	"	from 0.0024 to 0.042 ohms		
Ice	— 12.4°	2.24 begohms	....	Ayrton & Perry.
Ice	— 0.2	0.284 "	....	"
Sulphate of zinc saturated solution	10°	33.6 ohms	....	Ewing & Macgregor.
Sulphate of zinc	10°	28.22 " Solutions of minimum resistivity	....	"
Common salt	"	4.7 "	....	Kohlrausch & Nippoldt.
Sal ammoniac	"	2.5 "	....	"
Sulphate of soda	"	11.3 "	....	"
Sulphuric acid in water	"	1.376 "	....	"
Nitric acid in water	"	1.287 "	....	"
Hydrochloric acid in water	"	1.316	....	"
Pure water	....	about 3.75 megohms	....	Kohlrausch.
Sample of hydrant water	15° C.	about 200,000 ohms	....	Kennelly.
Mica	20°	84 tregohms..	....	Ayrton & Perry.
Gutta-percha	24°	449 " ..	....	Latimer Clark.
Shellac	28°	9 quegohms..	....	Ayrton & Perry.
Hard rubber	46°	28 " ..	....	"
Paraffin	46°	34 " ..	....	"
Glass, flint	0°	16700 " ..	...	Foussereau.
Porcelain	0°	540 " ..	....	"
Commercial stearic acid.	15°	440 tregohms	....	Kennelly.
" olive oil	"	1 " ..	....	"
" lard oil.	"	350 begohms ..	....	"
" creosote	"	5.4 megohms	....	"
" benzine.	"	14 tregohms	....	"
" benzole.	"	1.3 begohms.	....	"

The resistivity of pure water has been observed by Kohlrausch to be 3.75 megohms, and since an exceed-



Elec. Engineer

FIG. 7.—DIAGRAM OF RESISTIVITIES AT DIFFERENT TEMPERATURES, (DEWAR AND FLEMING.)

ingly small trace of impurities greatly decreases its resistance, it is probable that Kohlrausch's value is much below the resistivity of pure water, which in the opinion of some would be almost infinite. The temperature-coefficient of all liquids is negative. The temperature-coefficient of all insulators is also negative.

In order to point out more clearly the relation of resistance to temperature in different substances, the diagram Fig. 7, gives a series of curves of observed resistivities for different substances at various temperatures.

Various formulæ have been advanced at different times for reducing the resistance or resistivity of a metal at one temperature to its corresponding value at another, but this temperature, variation appears to differ appreciably with different samples of even the purest metals, or, at least, the experimental results are not yet in sufficiently close accordance to make any formula reliable. The most convenient supposition is, that the variations in resistance are proportional to the variations in temperature which produce them, or that the curves in Fig. 7 are straight lines. Some of them do in fact appear from the observations to be sensibly straight lines. On this supposition

$$\rho_t = \rho_0 \, (1 + a \, t)$$

where ρ_t is the resistivity at any temperature $t°$ C., ρ_0 the resistivity at zero centigrade, and a is an experimentally determined constant.

From the measurements of Dewar and Fleming as represented in Fig. 7, the mean value of the constant a, between 0° and 100° C. is, for

Platinum	0.00358
Gold	0.00376

Silver	0.0040
Copper	0.00422
Aluminum	0.00475
Nickel	0.00548
Iron	0.00655
Carbon	0.00391
Platinum Silver	0.000224
Platinoid	0.000253
German Silver	0.000317

According to Matthiessen's observations which have hitherto been generally accepted, the formula of correction for pure copper to any temperature between 0° and 100° C. is approximately.

$$\rho_t = 1 + 0.003870\, t + 5.968 \times 10^{-6}\, t^2 - 1.177 \times 10^{-8}\, t^3 - 9.93 \times 10^{-11} t^4.$$

The resistance of any homogeneous conductor may, therefore, be calculated by the following formula—

$$r = \frac{l}{a}\, \rho_t,$$

where l = the length of the conductor in centimetres.
a = its cross-section in square centimetres.
ρ_t = the resistivity at the observed temperature t° C.

Thus the resistivity of a mile of copper wire of Matthiessen's standard, 1 sq. mm. in cross-section at 0° C. is

$$= \frac{160{,}933}{0.01} \times 1.594 \text{ microhms} = 25{,}652{,}720 \text{ microhms}$$

or 25.65 ohms approximately.

Syllabus.

The resistance of a uniform homogeneous conductor varies directly with its length and inversely with its area of cross-section.

The International ohm is the practical unit of electric resistance.

In order to conveniently designate the decimals, multiples and submultiples of a quantity, suitable prefixes are employed.

The begohm is the fundamental c. g. s. unit of resistance.

The resistivity of a homogeneous isotropic body is the resistance of a column of that body, having unit length and cross-section.

The resistivity of all metals increases with temperature.

The resistivity of carbon, selenium, liquids and solutions, as well as insulators diminishes with temperature.

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ADVANCED GRADE.

ELECTRIC RESISTANCE

24. *Conductivity* is the reciprocal of resistivity, and *conductance* the reciprocal of resistance. Thus the minimum resistivity of an aqueous solution of nitric acid being 1.287 ohms, the maximum conductivity is $\frac{1}{1.287} =$ 0.777 *mho*, the mho (ohm spelled backwards) being the unit of electrical conductance. A wire which has two ohms resistance has 0.5 mho conductance.

The total resistance of a number of separate resistances, connected in series, is equal to their sum, and the total conductance of a number of separate conductances, connected in parallel, is equal to their sum.

Thus, if a number of resistances of a, b, c, . . . n, ohms, respectively, be connected in parallel, their respective conductances will be $\frac{1}{a}, \frac{1}{b}, \frac{1}{c} \cdots \cdots \frac{1}{n}$. Their total conductance will be $\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \dots + \frac{1}{n}\right)$ and their total resistance $\dfrac{1}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \dots + \frac{1}{n}}$.

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Thus, if three resistances be connected in parallel of 20, 50, and 80 ohms respectively, their respective conductances will be 0.05, 0.02, and 0.0125 mho. Their joint conductance will be 0.0825 mho, and their joint resistance 12.12 ohms.

25. The resistance of any metallic wire increases with its temperature. Where a constant standard resistance is required, this is an objectionable feature. Since the effect of temperature is less marked on alloys than on their ingredients, alloys are generally used for standard resistances.

The alloys most frequently used for this purpose are platinum-silver, german-silver, manganin, and platinoid, or (german-silver with one or two per cent. of tungsten.) The resistance of some alloys is, however, owing to a tendency to crystallize, apt to be subject to slight variations with time. Platinum-silver or platinoid appear to be the two alloys most nearly free from this objection, and are, therefore, most frequently employed in the construction of standard resistance coils.

26. All gases at ordinary temperatures and pressures have such high resistivities that these have never yet been measured. When, however, the pressure is greatly reduced, the resistivity becomes very low, and in certain recent experiments with air at a pressure of 0.01 mm. mercury, or about 13 dynes per sq. cm., the resistivity of the air was apparently about 1.4 ohms.

27. Resistances may be measured in various ways which may be conveniently divided into classes according to the magnitude of the resistances and the degree of accuracy required.

(*1.*) Very high resistances (upwards of one begohm) by electrometer methods, in which the rate of loss of charge by leakage is observed.

(*2.*) Resistances from one megohm to one begohm, by galvanometer deflection, where a deflection through a known resistance serves to valuate the unknown resistance.

(*3.*) From $\frac{1}{100}$ ohm to one megohm, by a differential galvanometer or a Wheatstone balance.

(*4.*) Below $\frac{1}{100}$ ohm, by *potentiometer* measurements.

28. The method most frequently employed for the usual range of resistances is the Wheatstone bridge or balance, a form of which is represented in Fig. 8.

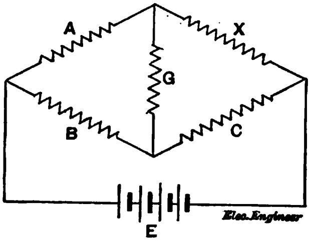

FIG. 8.—DIAGRAM OF WHEATSTONE BRIDGE.

In this method, as is well known, an unknown resistance is measured in terms of a known resistance by so proportioning the resistances A, B and C that no current flows through a galvanometer, represented by the resistance G, connected to the circuit as shown. When balance is obtained $X = C\frac{A}{B}$. When the arms A, and B, are equal, $X = C$ and the unknown resistance is equal to the resistance in the bridge. In practice A, and B, are some multiple of 10 ohms between 10 and 10,000 and the resistance C, can be adjusted by single ohms between 0 and 10,000, so that making the ratio $\frac{A}{B} = \frac{10}{10000}$

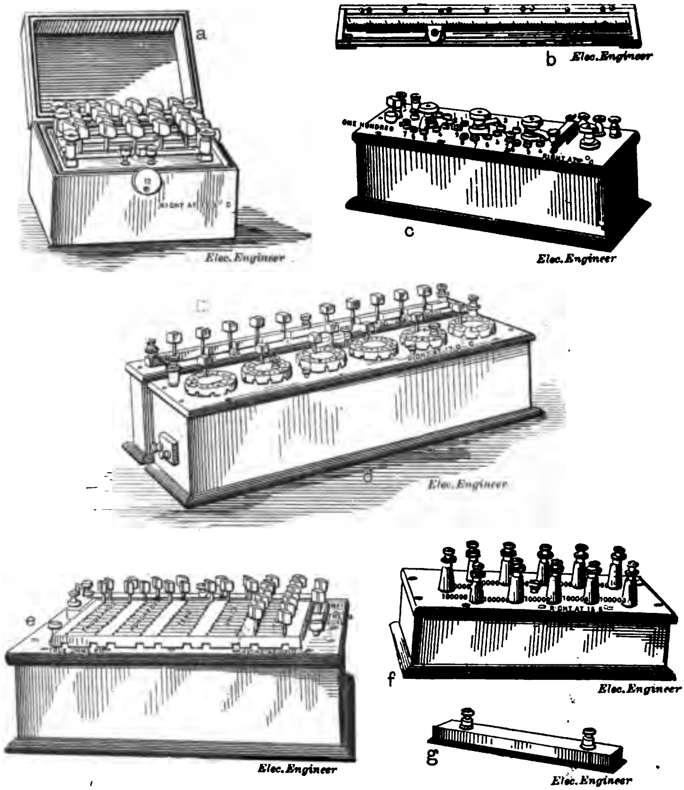

Fig. 9.—Various forms of Resistances.

or $\frac{10000}{10}$, the total range of the instrument is from $\frac{1}{1000}$th ohm to 10 megohms.

The best resistance to employ in A and B, depends upon the resistance of the galvanometer and that of the unknown resistance. Where these are small *A* and *B*, should be comparatively small; where they are large *A* and *B*, should be large.

Fig. 9 represents various forms of resistances. *a*, is a compact form of bridge with keys to close the battery and galvanometer circuits. *b*, is a slide form of meter bridge in which the arms *A* and *B*, (in Fig. 8) are altered simultaneously by the shifting of a contact along the wire. Such a form is only used for the measurement of low resistances, say under 20 ohms. *c*, is a form of dial bridge with keys attached. *d*, and *e*, are more elaborate forms of dial bridges. The dial pattern, though sometimes more expensive than the ordinary box type of Wheatstone bridge, has the advantage that there is less chance of additional resistance being introduced by bad contacts, only four or five plugs being used, so that the danger of additional resistance by loose contacts is largely avoided. At *f*, is shown a form of megohm, in wire, carefully insulated and divided into ten coils of 100,000 ohms each.

At *g*, is represented a form of megohm in carbon (pencil mark) on ground glass. This form of resistance is not reliable for accurate measurements and is only used for approximate work.

30. It is evident that the accuracy of measurement, by whatever method that may be employed, depends upon the accuracy of the standard resistances of comparison. It has been found in practice most convenient to make this fundamental standard of the value of an ohm. Three forms of standard ohms are shown in Fig. 10. At x is shown the form in common use. More

modern forms are shown at Y and Z. In all cases the difficulty of employing the instrument is in determining the true temperature of the coil of wire, the temperature being inferred from that of a mass of water or oil surrounding the coil. The improvements in Y and Z, Fig. 10, consist in exposing a large surface of the liquid and providing means for stirring the same in order to rapidly equalize the temperature within and without.

The standard megohms are employed for the purpose of calibrating galvanometers.

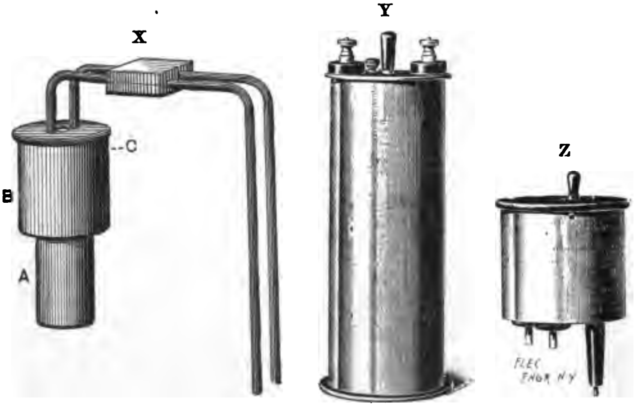

FIG. 10.—FORMS OF STANDARD RESISTANCES.

31. The materials of which the earth's crust is formed have an exceedingly high resistivity when wholly dry and may almost be classed as insulators, so that the resistance of the earth's crust as a whole, if perfectly dry, may be very great. In nearly all regions, however, the surface strata are permanently moist, and, since such moisture contains various saline matters in solution, the resistivity of the ground is usually less than 100 ohms.

Therefore, the ground may be used in place of a second conductor to complete a circuit between two stations.

32. It may be supposed that like all conductors the resistance of the ground as measured between two ground-plates would increase in proportion to the distance between them. In the case, however, of a large conducting mass such as that of the earth it can be shown that such is not the case, for the following reason. If two metallic hemispheres A and B (Fig. 11) be buried, as shown, in the surface of a conducting medium, infinitely extended below the infinite horizontal plane C A B D, the resistance as measured between A and B, will be

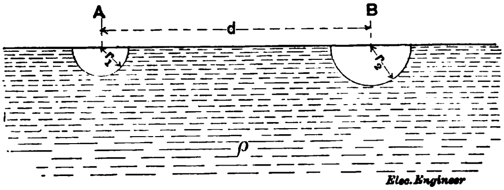

FIG. 11.

$$R = \frac{\rho}{2\,\pi}\left(\frac{1}{r_1} + \frac{1}{r_2} - \frac{2}{d}\right) \text{ ohms};$$

where ρ, is the resistivity of the medium in ohms; $\pi = 3.1416$, r, is the radius of each hemisphere in centimetres; and d, is the distance between the centres of the hemispheres in centimetres.

From which it is evident that the distance between the hemispheres does not appreciably affect the value of the resistance R, which may be taken as

$$\frac{\rho}{2\,\pi}\left(\frac{1}{r_1} + \frac{1}{r_2}\right);$$

or, when the hemispheres have equal radius,

$$R = \frac{\rho}{\pi\, r}.$$

Thus if ρ, the resistivity of the medium = 100 ohms, and r, the radius of the hemispheres is 1 metre, the resistance of the ground = 0.318 ohm.

The distance d, fails to appreciably affect the resistance R, owing to the fact that the current is by no means confined to the portions of the medium lying directly between A and B, but diffuses or spreads through the entire mass of the infinitely extended medium.

SYLLABUS.

Very large resistances are usually measured by electrometers or galvanometer deflections.

Very small resistances are usually measured by potentiometers.

Intermediate resistances are usually measured by means of the Wheatstone bridge by properly proportioning the resistance in the bridge arms. The Wheatstone bridge, as ordinarily constructed, may measure resistances from $\frac{1}{100}$th ohm to one megohm and is frequently constructed for $\frac{1}{1000}$th ohm to 10 megohms.

In order accurately to determine the resistance of a standard coil its true temperature requires to be known.

The resistivity of nearly all the materials forming the earth's crust is high. The presence of water, however, renders the resistivity of the mass much lower. The resistance of a ground return in a circuit may, therefore, be only a fraction of an ohm.

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ADVANCED GRADE.

ELECTRIC RESISTANCE.

33. If the resistivity of the insulating materials alone had to be considered their specifications would offer but little choice. In actual practice the resistance of an insulator is determined not so much by its dimensions and resistivity, as by the leakage afforded through the film of dust and moisture which collects on its surface. For this reason the best form of insulation is that which affords the longest and narrowest path for leakage. In cases where very high insulation is desired, some form of oil insulator is employed, the principle being to insert in the circuit of the leakage path a film of oil whose resistivity is not only great, but which is automatically kept clean by the tendency of dust to settle and fall to the bottom. (See Fig. 12.)

34. The insulation resistance of a line or conductor is measured in megohms, and this total insulation multiplied by the length of the line in miles, gives the average apparent insulation per mile in megohm-miles.

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On the other hand, the metallic resistance of a conductor divided by its length gives its apparent conductor resistance per mile. On long lines the effect of the escape of the measuring current by leakage will be to make the apparent insulation per mile, $R_{app.}$ too high, and the apparent conductor resistance per mile, $r_{app.}$ too low. These values, corrected for leakage through uniform insulation, are:

$$r = \sqrt{R_{app.}\, r_{app.}}\ \tanh^{-1} \sqrt{\frac{r_{app.}}{R_{app.}}}$$

$$R = \sqrt{R_{app.}\, r_{app.}}\ \coth^{-1} \sqrt{\frac{r_{app.}}{R_{app.}}}$$

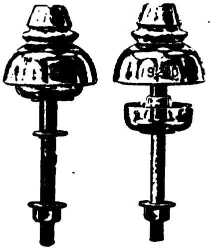

FIG. 12.—OIL INSULATOR.

For example: If the apparent insulation $R_{app.}$ of a uniform line 200 miles long, was 6024 ohms, and when grounded at the distant end its apparent conductor resistance was 2656 ohms, then the corrected conductor resistance $= \sqrt{6024 \times 2656}\ \tanh^{-1} \sqrt{\frac{2656}{6024}} = 4000 \times 0.8 = 3200$; that is, 16 ohms per mile; and its corrected insulation $R = \sqrt{6024 \times 2656}\ \coth^{-1} \sqrt{\frac{2656}{6024}} = 4000$

× 1.25 = 5000 ohms; that is, 1 megohm mile. The apparent values would be 13.28 ohms per mile and 1.2048 megohm miles. But 6024 × 2656 = 5000 × 3200 = 16,000,000, and in all cases where leakage through uniform insulation resistance has the effect of diminishing the conductor resistance by any ratio, the insulation resistance apparently increases in the same ratio, so that the product of insulation resistance and conductor resistance remains constant.

35. The best resistance to give to the coils of any electromagnetic relay or other receptive device operated at the receiving end in a ground return circuit, is the resistance which the circuit offers from the receiving end. This resistance, owing to the influence of leakage, may be considerably less than the conductor resistance of the line. This rule applies, strictly speaking, to a very slow rate of signalling in telegraphy, and for rapid signalling the resistance of the relay should be much lower.

35A. We have seen that the resistance of a conductor depends upon its resistivity and its geometrical form. When the form is simple, as in the case of wires, the computation of the resistance offers no difficulty. When, however, the form is more complex a difficulty arises, for example, in determining the insulation resistance of a uniform cable consisting of a conducting wire and concentric external sheath, the insulator having known resistivity and known dimensions. In this case the resistance per centimetre would be expressed by the formula,

$$R = \frac{\rho}{2\pi} \log_e \frac{D}{d} = 0.3665\, \rho \log \frac{D}{d}$$

where ρ, equals the resistivity (ohms.)
D, equals the external diameter.
d, equals the internal diameter.
ε, equals the Naperian base.

That is, if $\rho = 300$ begohms, $D = 1$ inch, $d = 0.5$ inch, then; $\dfrac{D}{d} = 2$, $\log \dfrac{D}{d} = 0.30103$;

$R = 0.3665 \times 300 \times 10^9 \times 0.30103$; $= 33.1$ begohms per centimetre; this divided by 160,933, the number of centimetres in a mile, gives 0.2057 megohms to the mile, *i. e.*, 0.2057 megohm-mile.

36. A common error in less recent text-books is found in the belief that electric resistance partakes of the nature of a velocity. This, however, while true for the existing system of electrical dimensions in the electromagnetic system, where resistance appears as a length divided by a time, is only a misconception derived from incomplete knowledge. The real nature of resistance is yet unknown.

37. When a galvanometer in a circuit gives too high a deflection, it is usual to reduce this deflection by the introduction of a bypath or shunt. For example, when the galvanometer of resistance G, has its terminals connected by the shunt S, its deflection will be reduced by the factor $\dfrac{S}{G+S}$, whose reciprocal, $\dfrac{G+S}{S}$ is called the multiplying power of a shunt. To obtain, therefore, a shunt with a multiplying power of 1000 for a galvanometer of 5000 ohms resistance, we have $\dfrac{5000+S}{S} = 1000$, so that $S = \frac{5000}{999}$ or $\frac{1}{999}$th part of the galva-

nometer's resistance, and generally the resistance of a shunt must be $(n - 1)$ times less than the resistance of the galvanometer in order to have the multiplying power of n.

38. In the use of any high resistance apparatus it is absolutely necessary that the insulation of the apparatus be as high as possible, for the effect of leakage may be to considerably reduce the resistance of the apparatus as computed. For example, a box containing one megohm in resistance might easily have a leakage resistance over the surface of the box between the terminals of one begohm. The effect of this very small leakage would be to shunt the megohm by a resistance one thousand times as great, and the effect would be to reduce the resistance of the box by about 1000 ohms, and leave the apparent total of 999,000 ohms approximately.

In practice the problem frequently presents itself of determining the size of wire required to fill a spool or bobbin of certain dimensions. To do this we first calculate the volume of space required to be filled by the wire, and then employ the following formula:

$$d = -t + \sqrt{t^2 + 0.0009432\sqrt{\frac{v}{r}}};$$

where d = diameter of the wire in inches
t = thickness of insulation (inches). If D be the covered diameter, $2t = D - d$.
v = volume of winding space in cubic inches
r = the resistance required in the winding (ohms)

The resistivity of the wire is here assumed to be 1.775×10^{-6} (copper of 0.97 Matthiessen's conductivity at 20°C.)

In practice t has the following values:

Silk, single covering, 0.0005 to 0.001 inch.
Silk, double covering, 0.0015 to 0.002 inch.
Cotton, single covering, 0.0035 to 0.004 inch.
Cotton, double covering, 0.005 to 0.007 inch.

The precise thickness of the coat depends upon the size of the wire. Large wires usually take heavier thicknesses of insulator.

Thus a spool of two inches interflange, *i. e.*, length between flanges, has an internal or core diameter of 0.5 inch, and an external diameter when fully wound of 1.0 inch. The resistance of the winding is to be 20 ohms, with a double silk covered copper wire, in which the insulation increases the diameter of the wire by three mils. Find the required diameter.

Here $t = 0.0015$; $v = 2 \times 0.5891 = 1.1782$ cubic inches;

$$r = 20, \; v/r = 0.05891 \quad \sqrt{v/r} = 0.2427.$$

$$d = -0.0015 + \sqrt{0.0000023 + 0.0002289}$$

$$= -0.0015 + 0.0152 = 0.0137 \text{ inch.}$$

The nearest size to this is No. 27. B. & S., 0.0142 inch, which, when covered with the required thickness of silk has a diameter of 0.0172 inch. There would be 15 layers of this wire, each layer having 116 turns, supposing the winding perfectly regular and complete. The total number of turns would, therefore, be 1740; and the mean turn length being 2.356 inch the total length of wire = 341.6 feet. = 18 ohms. The resistance is two ohms less than that required, owing to the difference between the diameter of the wire that has to be selected and the calculated diameter.

When the thickness of insulation is very small, the formula becomes approximately

$$d = -t + 0.03071\sqrt[4]{\frac{v}{r}}$$

Thus, taking the above case,

$$\sqrt[4]{\frac{v}{r}} = 0.4926,$$

and
$$d = -0.0015 + 0.0151$$
$$= 0.0136 \text{ inch.}$$

39. When plates of pure amalgamated zinc are immersed in an aqueous solution of pure zinc sulphate it has been observed that no appreciable resistance exists in the surface of contact between the two. If, therefore, the resistivity of zinc and the resistivity of the solution were known, the resistance of the combination could be determined. Generally, however, such contact surfaces between metals and liquids appear to possess a small definite resistance, called surface-contact resistance, in addition to the electromotive force which is usually established there.

SYLLABUS.

The insulation resistance of a line or conductor is usually measured in megohms and its apparent insulation per mile in megohm-miles.

The apparent conductor resistance of a line divided by its length in miles gives the apparent conductor resistance per mile.

An electromagnetic relay or other receptive device at the receiving end of a ground return circuit should with a slow rate of signalling, preferably have a resistance equal to the resistance which the circuit offers from the receiving end.

The resistance of a conductor depends upon its resistivity, on its geometrical form, but in all except very simple forms the computation becomes complex.

It is a mistake to believe that the nature of electric resistance is of the nature of velocity, its real nature being unknown.

A by-path or shunt is often employed with a galvanometer or other device which may carry too much current.

By the multiplying power of a shunt is meant the ratio in which the shunt reduces the current through the device shunted and is represented by unity plus the ratio of the resistance of the device to the resistance of the shunt.

A very small leakage in a high resistance apparatus may materially reduce the resistance proper to that apparatus.

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ELECTRIC CURRENT.

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40. It is a popular belief that an electric current consists actually of the passage of electricity through a conductor. Like most popular beliefs this is erroneous. According to modern views, when a telegraph circuit, say, between New York to Philadelphia, has an E. M. F. impressed upon its terminals at New York, it is no longer believed that electricity leaves the electromotive source or dynamo and flows through the conductor to Philadelphia like water through a pipe. When an electromotive source, such as a battery, is placed on open circuit, it produces in the surrounding ether a certain small electric stress. When, however, the conducting line or circuit, which may be 100 miles long, is connected to its terminals, this electric stress breaks down in the ether around the battery, and a flow of energy in the form of an electro-magnetic wave moves outward from the battery around and along the surface of the wire, with the velocity of light. According to this view, the real

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province of the conductor is to direct the flow of energy from the battery, the conductor acting as a sink or line of absorption in the dielectric medium, and determines the direction of movement of the energy. A conductor, therefore, directs the passage of energy through the medium, at the same time expending some of that energy in progress, converting it into heat within the substance of the wire. The electric current cannot, consequently, be conveniently regarded as passing through the wire, but rather through the ether surrounding the wire, the energy, as it moves forward, converging and being rained down upon the wire.

However true this theory may be, and it is in general acceptance by the advanced thinkers of to-day, it is much more convenient for practical purposes to retain the old notions concerning the passage of a current through the wire like water through a pipe, since the effects of electric currents can be more readily dealt with by the simpler theory than by one which is probably more nearly correct.

41. When an electromotive force is impressed on a condenser, energy enters the dielectric and the condenser receives an electric charge, a certain quantity of electricity entering the condenser. The unit of electric quantity is called the *coulomb*. While the electric charge is entering the condenser, its time-rate-of-change is called the *electric flow* or *current*. A time-rate-of-change of one coulomb per second is called an ampere. If q, be the quantity of electricity or charge, in coulombs, which is entering the conductor or passed through a conducting circuit, and I, be the strength of current in

amperes, flowing through the conductor, then at any instant,

$$I = \frac{d\,q}{d\,t};$$

that is, a current is the time-rate-of-change of the quantity, or the instantaneous rate of flow.[1]

When a quantity of electricity moves through a circuit under the action of an electromotive force, work is

---

[1] For the sake of readers who have not yet mastered the elements of differential calculus, the following explanation of the symbols here employed may be acceptable. If S be the space passed through by a moving body, reckoned from a given instant of time, then if the spaces moved over in equal intervals of time be equal, the body will be moving with uniform velocity, and the length of any portion of its path, divided by the time required to describe it, will always give the same value of the velocity. Thus, if the body be moving with the velocity of 20 feet a second, the quotient of any distance it has moved over, divided by the time occupied in travelling though it, will always be equal to 20; but if the velocity of the body be varying, the quotient of the space traversed by the time occupied in traversing will generally be different. Thus, a body falling to the ground from rest, is continually accelerating its velocity, and the quotient of space by time will vary, not only with the position of space selected, but also with the length of the space traversed. The velocity at any point of its descent is, therefore, only to be defined by the quotient of space traversed by time occupied in traversing, for an extremely short, and in theory, for an infinitely short, distance. For this conception a special symbol is introduced; thus, if $\frac{S}{t} = v$, be the velocity of a body, and different values of S, divided by their appropriate intervals of t, give different values of v; then, proceeding to the ideal infinitely small spaces and the infinitely short intervals of time in traversing it, which may be represented, respectively, by the symbols $d\,S$, and $d\,t$, the true velocity at any moment is expressed as before, by the quotient $v = \frac{d\,S}{d\,t}$.

A similar method is adopted for dealing with all variable quantities whose rates of variation are not constant. As for example, in the case before used in the context, where amperes $= \frac{d\,q}{d\,t}$. Here the quantity of electricity varying perhaps irregularly with time, shows that the current may not have the same value at different times, but the symbol $\frac{d\,q}{d\,t}$ gives us a means of expressing the actual instantaneous value at any moment.

done, just as when a mechanical force moves through a distance. If the quantity of electricity moved is known in coulombs, and the force with which it is moved in volts, their product will represent the volt-coulombs or the joules of work expended in the process, so that

$$W = E\,q \text{ joules.}$$

If now we know the rate per second at which the quantity is flowing or $\frac{d\,q}{d\,t}$, the current strength in amperes, and multiply this by the E. M. F., we obtain the volt-amperes or the activity of the circuit in watts,

$$\frac{d\,W}{d\,t} = P = E\,I \text{ watts.}$$

In mechanics the activity of a force is measured by the product of the force and the distance through which it acts or

$$P = F\frac{d\,S}{d\,t} \cos a \text{ ergs per second};^{1}$$

where P, is the activity; F, the force in dynes; $\frac{d\,S}{d\,t}$ the time rate of displacement in cms. per second; and a the angle between the directions of displacement and the direction of the force. So in electricity,

$$P = E\,\frac{d\,q}{d\,t} \cos a \text{ (watts)};$$

where P, is the activity, E the E. M. F. in volts, $\frac{d\,q}{d\,t}$ the current strength, and a the angle between the direction

---

[1] The symbol cos a, represents the cosine of the angle a, and is always some number between minus one and plus one, which can be found for any given angle from trigonometrical tables. For example, if $a = 60°$, cos $60° = 0.5$; so that in this case the number 0.5 might be substituted for the symbol cos a

of the E. M. F. and the current. In almost all cases the current and E. M. F. are in the same line, so that cos a becomes unity, and $P = E \frac{d\,q}{d\,t} = E\,I.$

42. Although the coulomb, or ampere-second, is the unit of electric quantity, yet the ordinary commercial unit of electric quantity is the ampere-hour. This is because the second is a too small a unit of time for practical use. The ampere-hour, is 3,600 coulombs; *i.e.*, the number of seconds in an hour.

43. When a uniform conductor in the form of a wire has a cross-section of, say, 0.6 sq. cm. and carries a steady current of 15 amperes, the current density would be $\frac{0.6}{15} = 0.04$ ampere per square centimetre, and would be uniform for the entire cross-section of the conductor. Current density is, therefore, the intensity of current per normal unit area, and is expressed in amperes per square centimetre.

44. Attempts have at different times been made to formulate rules for the carrying capacity of conductors and of copper wires by specifying a definite current density, for example, 1,000 amperes per square inch of cross-section. Such a rule, however, cannot prescribe uniform temperature elevations in conductors of different sizes, for the reason that the surface of the conductor only increases as the square root of the cross-sectional area, and the surface area is the principal factor determining the rate of escape of heat from the wire.

45. When the strength of a current is rapidly altering, as in pulsatory or alternating currents, it becomes necessary to define how that varying current strength shall be estimated.

For example, the mean magnetic strength of that current or the mean electrolytic effect of that current might be taken as determining its value. In these cases, the strength would be the arithmetical mean or average of the current strength in amperes during the time under consideration. In practice, however, rapidly varying currents are measured by their mean heating effects, and since the heating effect of a current depends upon the square of its strength, currents are determined in their effective values by the average of their squares taken during the period under consideration. If half an ampere of continuous current just suffices to heat the filament of an incandescent lamp up to a certain degree of incandescence, then any rapidly pulsatory or alternating current which will bring the lamp to the same degree of incandescence is just half an ampere in strength.

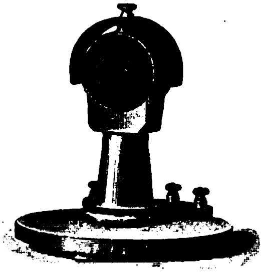

Fig. 13.—Thomson Mirror Galvanoscope for Signalling on Submarine Cables.

Fig. 14.—Thomson Marine Galvanometer for use on Rolling Vessel at Sea.

46. Various methods may be employed for measuring the strength of an electric current, but practically the magnetic method alone is employed.

When an electric current passes through a conductor, it is accompanied by the distribution of magnetic flux or magnetism in its vicinity. This flux is attended by stresses in the space so occupied, which stresses acting either on iron or active conductors, produce movements in the same, which movements are opposed by springs or gravitational forces. The amount of motion produced is usually read off by a pointer or index upon a graduated scale.

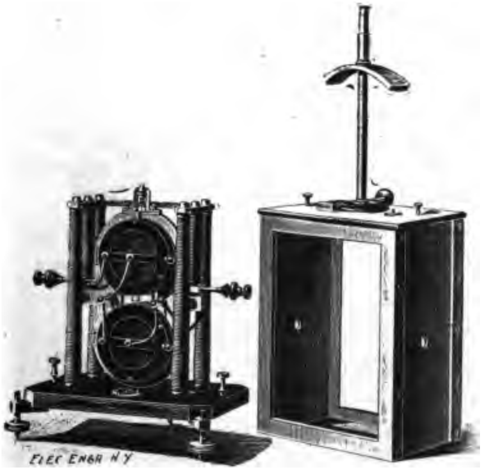

FIG. 15.—HIGH-GRADE THOMSON MIRROR GALVANOMETER FOR MEASURING HIGH INSULATION RESISTANCES.

Fig. 13 shows a common form of Thomson mirror galvanoscope employed for the reception of telegraph signals on long submarine cables. A circular coil of wire in the upper part of the instrument carries at its centre a small magnetic needle attached to the back of a small glass mirror, and suspended on a fibre.

Fig. 14 shows a form of this instrument intended for use on board ship. The coil and mirror are enclosed in a soft iron case one inch thick, in order to reduce as far as possible the disturbing effect of the earth's magnetic field on the suspended magnet when the ship turns about.

Fig. 15 shows a form of double coil Thomson galvanometer prepared for careful insulation tests. The coils are supported on long corrugated hard rubber pillars.

SYLLABUS.

It is no longer believed that electricity flows through a conductor but rather through the dielectric surrounding the conductor.

The presence of a conductor directs the energy, and at the same time absorbs some of it in progress.

The unit of electric quantity is called the coulomb.

The time rate of change in quantity that has passed through a circuit is the current in that circuit, and is expressed in units called amperes.

The unit of electric quantity, the ampere-second or coulomb is not used in commercial practice, being replaced by the ampere-hour.

The density of electric current is expressed in amperes per square centimetre. It is uniform only in the case of steady currents, and that only, where the conductors are very long and are uniform in nature and cross-section.

Galvanoscopes are used to indicate the presence of a current, and galvanometers to measure its strength.

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—BY—

Prof. E. J. Houston, Ph. D.
AND
A. E. Kennelly, F. R. A. S.

ADVANCED GRADE.

OHM'S LAW.

47. Ohm's law as discovered and annunciated by Dr. Ohm of Berlin in 1825, is generally expressed as follows: The current strength in any continuous current circuit is directly proportional to the total E. M. F., and inversely proportional to the total resistance, or

$C = \frac{E}{R}$; or, as written in foreign countries, $I = \frac{E}{R}$. (1.)

Ohm's law, as expressed above. assumes that the full current strength in the circuit has been reached. Strictly speaking, a continuous current requires an indefinitely long time to attain full strength, although practically, within the limits of measurement, the maximum strength is usually reached in a small fraction of a second.

At the International Electrical Congress at Chicago in 1893, it was recommended that a uniform system of notation should be internationally adopted, and since the symbol I was selected for current strength in this nota-

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tion, and C, for capacity, we propose to follow the international notation. From equation (1) we obtain,

$$E = IR \qquad (2.)$$

and
$$R = \frac{E}{I} \qquad (3.)$$

so that in any continuous current circuit, any two of the essential quantities, (electromotive force, resistance and current) being known, the third can be determined.

48. These formulæ apply not only to a complete circuit, but also to any portion of the same. Thus the electromotive force in a circuit distributes itself in such a manner that the current strength in the circuit is equal throughout. The electromotive force required to drive a current through any portion of a circuit against the resistance in that portion, is usually called the "*drop*" in that circuit. Thus with a dynamo supplying a continuous current at a pressure, across the brushes, of 125 volts, to incandescent lamps in parallel, it may be required to limit the drop in the supply mains to eight per cent., meaning that eight per cent. of the 125 volts, or ten volts, would be the limit of pressure required to drive the supply current through the mains, leaving 115.0 volts at the lamps.

If the total number of lamps was 500, each of 50 watts, and since the product of the current consumed and E. M. F. delivered is the activity in the lamp, the current supplied to each lamp would be $\frac{50}{115} = 0.4348$ amperes, so that the total current is $500 \times 0.4348 = 217.4$ amperes, representing a total activity of $115 \times 217.4 = 25{,}000$ watts, or 25 K. W. The drop of ten volts allowed in the two conducting wires, or five volts in each wire, requires that the resistance of each wire should in conformity with the formula, $r = \frac{e}{i}$, be $\frac{5}{217.4} = 0.023$ ohm. The resistance of each lamp must in conformity with the same law be,

$$r = \frac{e}{i} = \frac{115}{0.4348} = 264.4 \text{ ohms.}$$

The joint resistance of all the lamps is, $= \frac{r}{500} = 0.5288$ ohm.

If the resistance of the dynamo be 0.015 ohm, the drop in the dynamo must also be 0.015 × 217.4 = 3.26 volts, so that the E. M. F. in the circuit must be 125 + 3.26 = 128.26.

The total resistance in the circuit would be,

Dynamo armature......	0.015 ohm.
Leads..............2 × 0.023 =	0.046 "
Lamps	0.5288 "
	0.5898 "

So that the total current in the circuit would be

$$I = \frac{128.26}{0.5898} = 217.4 \text{ amperes.}$$

The activity in the circuit, $IE = 217.4 \times 128.26 = 27.884$ K. W.
Of this the activity in the dynamo is $Ie_1 = 217.4 \times 3.26 = 0.709$ K. W.
The activity in the leads is, $I(e_2 + e_3) = 217.4 \times 10 = 2.175$ "
The activity in the lamps is, $Ie_4 = 217.4 \times 115 = 25.000$ "
27.884 "

The electrical efficiency of distribution is the ratio of the energy in the lamps to the energy in the circuit or $\frac{25}{27.884} = 0.897$.

49. Applying the same law to branch, derived or shunt circuits, the current in any particular branch is equal to the electromotive force at the terminals of that branch, divided by its resistance. Thus in the preceding figure, the current in any one lamp, is 115 volts divided by 264.4 ohms = 0.4348 ampere. This is true no matter how complex the network of conductors may be.

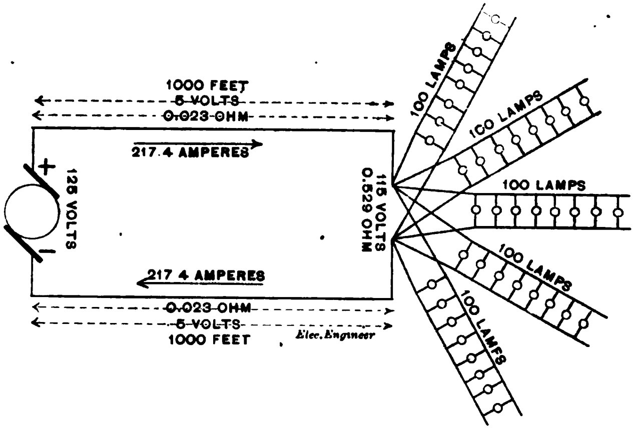

Fig. 16.—Application of Ohm's Law to a Circuit.

50. In complete networks of circuits, there are certain corollaries of Ohm's law which enable the current strength to be deduced in any branch. These may be expressed as follows:

(*1.*) *No current can be absorbed at any branch point.* Thus in Fig. 17, $i_1 = i_2 + i_3$ because if this identity did not hold, the current arriving at the point *A*, would be greater or less than the current leaving it, so that generation or absorption of current would occur at the point *A*.

(*2.*) *The total* E. M. F. *in any closed loop must be equal to the sum of the potential differences in the loop due to IR.*

Thus in Fig. 17, $E - e = i_1 r_1 + i_3 r_3 - i_4 r_4 + i_5 r_5$, because if this identity did not hold, the total E. M. F. acting in the loop would be greater or less than the total counter E. M. F. of *IR* established by the current, whereas, in any continuous current loop or circuit, these two quantities must be equal.

(*3.*) *The* P. D. *at the extremities of any line must be equal to the sum of the* P. D.'S *due to IR, in that line together with the sum of* E. M. F.'S *contained in it.*

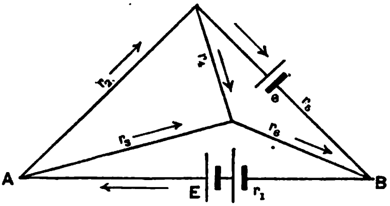

FIG. 17.—NETWORK OF CONDUCTORS.

Thus calling *U*, the potential difference between *A* and *B*

$$U = i_3 r_3 - i_4 r_4 + i_5 r_5 + e.$$

This follows by the same reasoning as in the preceding case, of which it is a direct consequence, for, $U = E - i_1 r_1$.

(*4.*) *The current in any branch is the sum of the currents that all the* E. M. F.'S *in the network would produce if each of the* E. M. F.'S *were successively permitted to act singly.*

Thus calling i_m the current which would be established in r_3 if the E. M. F., e, existed alone, and i_n, the current which would be established in r_3, if E existed alone, then, when both E. M. F.'s exist, as shown in the figure, the resulting current $i_3 = i_m + i_n$.

(*5.*) Taking any two branches such as r_3 and r_5, the current which would be set up in r_5 by inserting a given E. M. F. in r_3, is equal to the curreat strength which would be set up in r_3 by the insertion of the same E. M. F. in r_5.

Thus considering the figure as representing the connections of a Wheatstone bridge, the current set up in the galvanometer branch r_4 by the testing battery in r_1, is equal to the current which would be set up in r_1 by removing the testing E. M. F. to r_4.

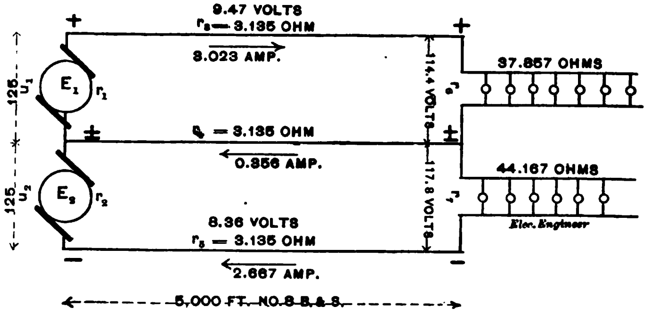

FIG. 18.—THREE-WIRE SYSTEM.

These rules can all be deduced directly from Ohm's law. Numbers (1) and (2) are frequently called Kirchoff's laws. In all cases care must be taken to observe the directions of the various E. M. F.'s and currents, since the geometrical and not the mere arithmetical sums of these quantities are under discussion. Also the E. M. F.'s must be considered independently of the resistance which practically accompany them. In (5) for example, when we consider the trausference of the E. M. F. in a battery, we have to regard the resistance of the battery as immovable, and the E. M. F. only to be transferred.

51. In order to determine the current strength in any or all the n branches of a conducting network in which all the E. M. F.'s and resistances are known, it is customary to write down n independent simultaneous equations with the aid of (1) and (2), and then solve these equations by the ordinary algebraic processes. Thus Fig. 18 represents the connections of a "three-wire system" in which two dynamos in series operate two groups of incandescent lamps with a "neutral" wire from the connection of the dynamos to the connection of the lamp groups. We may determine the current strength in the various conductors as follows:

$$u_1 = i_3 (r_3 + r_6) + i_4 r_4$$
$$u_2 = i_5 (r_5 + r_7) - i_4 r_4$$
$$i_3 = i_4 + i_5$$

Solving these equations for i_3, i_4 and i_5, we obtain

$$i_3 = \frac{U r_4 + u_1 R_2}{r_4 (R_1 + R_2) + R_1 R_2} \qquad (a)$$

$$i_4 = \frac{u_1 R_2 - u_2 R_1}{r_4 (R_1 + R_2) + R_1 R_2} \qquad (b)$$

$$i_5 = \frac{U r_4 + u_2 R_1}{r_4 (R_1 + R_2) + R_1 R_2} \qquad (c)$$

where $R_1 = (r_3 + r_6)$; $R_2 = (r_5 + r_7)$; and $U = u_1 + u_2$

It follows, therefore, from (b), that whatever the two resistances r_6 and r_7, may be, that is to say, whatever the two incandescent lamp loads may be, balance in the system will be obtained, and no current will flow through r_4 when $u_1 : u_2 :: R_1 : R_2$.

For example, suppose (Fig. 18) that the positive load consists of seven lamps of 265 ohms each, and the negative load of six lamps of 265 ohms each, while the pressure at dynamo terminals is 125 volts $= u_1 = u_2$, and the resistance of each conductor = 3.135 ohm.

Then $r_6 = 37.857$ ohms, and $r_7 = 44.167$.
$R_1 = 40.992$ " " $R_2 = 47.302$.

$$\begin{aligned} \text{Then by } (a)\ i_3 &= \frac{250 \times 3.135 + 125 \times 47.302}{3.135 \times 88.294 + 40.992 \times 47.302} \\ &= \frac{6696.5}{2215.8} = 3.023 \text{ amperes.} \\ i_5 &= 2.667 \text{ “} \\ i_4 &= 0.356 \text{ “} \end{aligned}$$

The drop in r_3 = 9.47 volts.
r_5 = 8.360 "
Pressure at A = 114.38 "
B = 117.79 "

250.00

Syllabus.

The current strength in any continuous current circuit after full strength has been attained, is directly proportional to the total E. M. F., and inversely proportional to the total resistance. This is called Ohm's Law.

Ohm's law applies not only to an entire circuit, but also to any part of a circuit.

The fall of pressure or "drop" in a conductor carrying a current is the pressure required, to drive the current through the conductor, and is equal to IR, the product of the current and the resistance.

Laboratory of Houston & Kennelly,
Philadelphia.

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—BY—

Prof. E. J. Houston, Ph. D.
AND
A. E. Kennelly, F. R. A. S.

ADVANCED GRADE.

ELECTRIC CIRCUITS.

52. The term circuit, as generally understood, embraces a complete conducting path established between the electric source and the electro-receptive devices that in practice are connected therewith. Ordinarily, the circuit consists of a true conducting path, principally or wholly of metal; but circuit paths may, however, lie through a non-conducting dielectric, as in the case of the dielectric circuit. In the electric circuit, the current strength may be unvarying; in the dielectric circuit, it can never be uniform. All conducting circuits belong to the former type; all electrostatic circuits, to the latter type.

53. Conducting circuits may be conveniently grouped into four general classes; namely,

(*1.*) Series circuits.
(*2.*) Multiple circuits.
(*3.*) Multiple-series circuits.
(*4.*) Series-multiple circuits.

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54. In the series connection of electro-receptive devices, the separate devices are placed so as to be successively traversed by the current. The current strength is maintained constant no matter how many

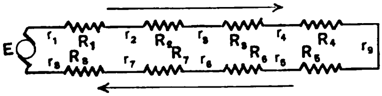

FIG. 19.—SERIES CIRCUIT.

separate devices are included in the circuit; consequently, in all such cases, the electromotive force of the source must be varied in accordance with the number of devices in circuit. This variation is generally obtained automatically from constant-current dynamos.

Fig. 19 shows a dynamo of E. M. F., E, connected with a number of electro-receptive devices connected by the conducting circuit r_1, etc. Here the total resistance of the circuit is $(R_1 + R_2 + \ldots + R_8) + (r_1 + r_2 + \ldots + r_8)$ and the current in the circuit is equal to the E. M. F., E, divided by this resistance. Calling this current I, the activity yielded by the source is $E\,I$ watts, and the activity in any separate device is $I^2\,R$, developed entirely as heat, while the activity in any section of conducting wire is $I^2\,r$; so that the activity in the circuit is entirely ex-

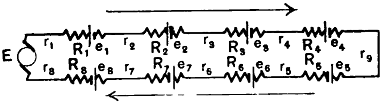

FIG. 20.—SERIES CIRCUIT CONTAINING COUNTER E. M. F.

pended in producing heat either in the receptive devices, or in the conducting wires connecting the same. Such activity is commonly called $I^2\,R$, *activity*, and the loss of energy accompanying such currents is the $I^2\,R$, *loss*.

Fig. 20 shows a dynamo of E. M. F., E, connected in series with a number of electro-receptive devices each of which supplies not only the resistance R, but also a counter electromotive force e. The resistance in this circuit is obtained as in the preceding case, but the current is now,

$$I = \frac{E - (e_1 + e_2 + e_3 + \ldots + e_8)}{(R_1 + R_2 + \ldots + R_8) + (r_1 + r_2 + \ldots + r_8)} = \frac{E - \Sigma e}{\Sigma R + \Sigma r}$$

the activity yielded by the dynamo to the external circuit is $E\,I$, watts as before, but the energy liberated in each device is now $(I^2 R + e\,I)$ watts. The first term is entirely thermal, but the energy corresponding to the second term may be mechanical, chemical, or thermal, depending upon the nature of the receptive device. In the case of the arc lamp, $e\;I$, is thermal; in the case of the magnetic device, $e\,I$, is generally mechanical; and in the case of electrolysis, $e\;I$, may be partly mechanical, but is principally chemical.

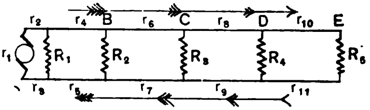

Fig. 21.—Multiple Circuit.

55. In the multiple connection of electro-receptive devices, all the positive terminals of the devices are connected to a single positive lead, and all the negative terminals similarly connected to a single negative lead. Thus in Fig. 21 a number of receptive devices represented by the resistances R_1, R_2, etc. are connected as shown with the positive and negative leads respectively of the dynamo of electromotive force E. In this

case the resistance of the circuit is the complex expression obtained in the following manner:—

The resistance at $E, R_E = R_5$.

The resistance at $D, R_D = \dfrac{1}{\dfrac{1}{R_4} + \dfrac{1}{r_{10} + r_{11} + R_5}}$.

Resistance at $C = R_C = \dfrac{1}{R_C + \dfrac{1}{r_8 + r_9 + R_D}}$.

56. In the multiple-series connection of electro-receptive devices, the electro-receptive devices are connected in series groups, and these groups subsequently connected in parallel. Thus in Fig. 22, a number of receptive devices r_1, r_2, etc., are connected, as shown, in three separate groups in series, and these groups subsequently connected in multiple. If R_1, R_2, R_3, and R_4, are the resistances of each group, then the total joint resistance of the circuit

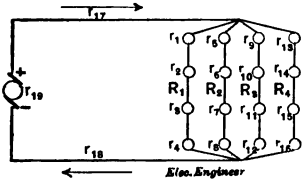

FIG. 22.—MULTIPLE SERIES CIRCUIT.

$$= \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} + \dfrac{1}{R_4}} + r_{17} + r_{18} + r_{19}.$$

57. In the series-multiple connection of receptive devices, the separate devices are connected in groups in multiple, and these separate groups subsequently connected in series.

Such an arrangement is indicated in Fig. 23, where four groups of four lamps each are operated in series. By this means the current in the supply mains, for a given number of lamps equally divided into groups, is reduced four times, and the operating pressure at main terminals is increased four times. Such an arrangement, with the addition of the equalizing wires C D, E F, and G H, is practically employed, and is called a *five-wire system.*

58. All E. M. F.'s developed in a circuit by a continuous current are counter E. M. F.'s. A current I, flowing through a resistance R, sets up a virtual counter E. M. F. of $I\ R$, volts. When flowing through an electrolyte it usually establishes a counter E. M. F. of polarization. When passing through a coil, loop, or electromagnet, it sets up, during the initial period of rise to full strength, a counter E. M. F. of induction, as will be subsequently explained. When it passes through a motor and causes its armature to rotate, it develops in the rotating armature a counter E. M. F. In all these cases the current does work upon the counter E. M. F. and expends that work as heat in the resistance, as chemical energy in the electrolyte, as magnetic energy sometimes taking the form of mechanical work in the coil or magnet, and as mechanical work in the rotating motor.

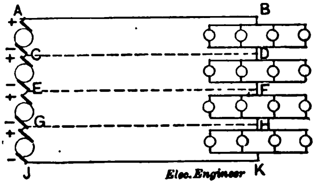

Fig. 23.—Series-Multiple Circuit.

If the E. M. F. developed in a circuit by a current were not a counter E. M. F. but aided the current, it would do work on the current. This work would have to be drawn from the source of the aiding E. M. F., so that it would be only necessary to expend some work in the circuit, to involve the expenditure of additional work from some other portion or portions, indicating a condition of unstable electrical equilibrium.

In all cases, therefore, where work is done in a circuit by the action of a current, that work is due to the development and action of a counter E. M. F. and the greater this counter E. M. F. for a given current strength, the greater the amount of energy that is absorbed by its source, and the greater the amount of work which may be expended. A counter E. M. F. is, therefore, not prejudicial to the action of an electric source. On the contrary, it is the means by which work can be delivered to the circuit.

59. The preceding facts may be tabularly arranged as follows:

VARIETIES OF COUNTER E. M. F. DEVELOPED BY ALTERNATING OR CONTINUOUS CURRENTS.

Varieties of E. M. F.	Types.	Varieties of Energy.
1. Virtual Counter E. M. F.	$(I R)$....	$(I R)\, I = I^2 R =$ Heat in Resistance.
2. Counter E. M. F. of Polarization.	(e).......	$(e\, I) =$ chemical energy in electrolyte.
3. Counter E. M. F. of Induction...	(e)......	$(e\, I) =$ magnetic energy and mechanical work in coil in magnet.
4. Motor E. M. F	(e)......	$(e\, I) =$ mechanical work in rotation of motor.

60. Ohm's law is essentially the law applying to any circuit or to any conductor forming portion of a circuit. The fundamental law which underlies the expression $I = \frac{E}{R}$ is, however, as follows,

$$i = \frac{e}{\rho}, \text{ or } e\,g;$$

where i is the current density at any point of a circuit, e, the E. M. F. at that point, being considered as a vector or directed quantity, and being also equal to the drop of pressure per centimetre of length, ρ, the resistivity of the medium at that point, and g, its reciprocal, the conductivity. This law, therefore, expresses the fact that wherever the electromotive force is acting, the current density is in direction of that E. M. F., and equal to the value of that E. M. F. multiplied by the local value of g, the conductivity. This is Ohm's law expressed for localized action and not for generalized action.

The formula $I = \frac{E}{R}$

is generally employed with the practical units of the ampere, ohm and volt. It is evident, however, that it is also applicable to the fundamental C. G. S. units. So that when E is expressed in units of 10 bicrovolts, or 10^{-8} volts, and R in bicrohms; *i. e.*, 10^{-9} ohms, the current strength becomes evaluated in C. G. S. units of 10 amperes each.

SYLLABUS.

In an electric circuit the current strength may be uniform; in a dielectric circuit it can never be uniform.

Conducting circuits may be series, multiple, multiple-series, or series-multiple.

A series circuit is frequently called a constant current circuit. In such a circuit the current is usually maintained constant even though the resistance be variable.

A multiple circuit is frequently called a constant potential circuit. The current in the leads is variable, but the E. M. F. is constant.

The E. M. F.'s developed in a circuit by a current are all counter E. M. F.'s. These counter E. M. F.'s may be divided into four classes; viz., the virtual counter E. M. F. of resistance, the counter E. M. F. of polarization, the counter E. M. F. of induction, and the motor electromotive force of a motor.

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Prof. E. J. Houston, Ph. D.
AND
A. E. Kennelly, F. R. A. S.

ADVANCED GRADE.

THE VOLTAIC CELL.

61. When a plate of chemically pure zinc and a plate of chemically pure copper are plunged into a dilute solution of sulphuric acid, no visible action takes place as long as the plates are electrically disconnected outside the acid liquid. When, however, the two plates are connected outside the liquid by a conducting wire, the completion of the electric circuit is immediately attended by the establishment of an electric current through the circuit, and a more or less visible action on the zinc, as evidenced by the evolution of hydrogen and the gradual solution of the zinc plate. Such an arrangement forms what is called a *voltaic cell.*

62. The electromotive force which produces the electric current, exists at the contact surfaces of the two plates with the liquid. As we have already seen, whenever an E. M. F. acts in the direction of an electric current, energy is absorbed from the source of the E. M. F.; that is, work is done on the current and

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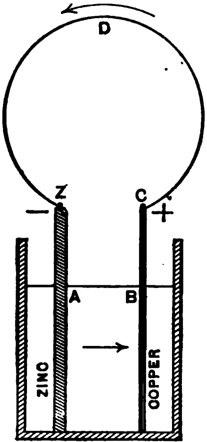
appears in the circuit. So here, when an E. M. F. sets up a current in the voltaic circuit, energy is absorbed at the source of the E. M. F.; namely, at the junction surfaces between the plates and the liquid in which they are immersed. Such a combination or *couple* of two plates or *elements*, with the conducting solution or *electrolyte* is called a *voltaic cell.* It follows, therefore, that the particular combination of elements and electrolyte in a voltaic cell, which will insure the most powerful E. M. F., will be that combination which will ensure the maximum resultant amount of work being absorbed at the immersed surfaces when the circuit is closed. In an active voltaic cell, one of the elements or plates is dissolved by the electrolyte while the other plate remains unattacked. The plate which is dissolved is called the *positive plate* of the couple and the other the *negative plate.*

The current flows through the liquid from the positive to the negative plates, as shown by the arrows, when the external circuit C D E is completed, and the terminal C, at which the current leaves the cell is, according to convention, called the *positive pole* or terminal, and the terminal Z, where it returns to the cell, the *negative pole* or terminal. It will be seen, therefore, that the negative terminal is connected with the positive plate, and the positive terminal is connected with the negative plate. This classification, although generally used, is apt to mislead. In reality, the plate A, (allowing for any existing drop of pressure) must have the same potential throughout its substance; and, similarly, the plate B, must be positive throughout. Since the electromotive force is resident at the surface of contact between the plates and the liquid, it follows that the entire plate B, is positive to the liquid and the entire plate A, negative to the liquid. The terms positive plate and negative plate, however, are in universal use, and if properly understood introduce no error. We shall, therefore, hereafter employ them.

63. It has frequently been stated in text-books of less recent date that the seat of the E. M. F., for example, the zinc-copper couple shown in Fig. 24 is at the metallic contact of the zinc plate and the copper wire outside the cell. This is, however, erroneous. The only E. M. F. which can exist at this metallic junction is what is called a thermo-electric E. M. F., and as such is altogether two small to account for the E. M. F. of the cell. As has already been pointed out, wherever an E. M. F. aids or opposes a current, work is done by or on the E. M. F. If, therefore, the E. M. F. of the voltaic cell resided at the zinc-copper contact outside the cell, it would follow



Elec. Engineer

FIG. 24.—SIMPLE FORM OF VOLTAIC CELL.

that the energy of the cell would be supplied at this contact, *i. e.* outside the cell, and would therefore be independent of such energy relations as existed within the cell. In point of fact, however, the energy is absorbed, not at this contact, but at the contact surfaces of the plates with the electrolyte, and it is to these surfaces, therefore, that we have to look for the E. M. F. of the cell.

64. In every voltaic cell there are two distinct sources of E. M. F., viz:

(*1.*) An E. M. F. at the contact surface of the positive plate with the liquid.

(*2.*) An E. M. F. at the contact surface of the negative plate with the liquid.

When a cell, such as shown in Fig. 24, has a positive plate of zinc, a negative plate of copper, and an electrolyte of dilute sulphuric acid, on the closing of the circuit hydrogen sulphate, $H_2\,SO_4$, in the electrolyte is decomposed. The negative radical, (SO_4) enters into combination with the zinc, to form zinc sulphate, $ZnSO_4$, and the hydrogen radical H_2, is liberated at the negative plate in the form of bubbles of gas. Before closing the voltaic circuit, we have a condition of affairs in the cell represented by the chemical expression

$$Zn + H_2\,SO_4 + Cu,$$

and after closing, $ZnSO_4 + H_2 + Cu.$

65. The most economical voltaic cell that has been yet developed, is incapable of producing, on a large scale, electric energy as cheaply as a dynamo electric machine. A careful consideration will show how hopeless it is to expect any existing voltaic cell economically to compete with an efficient dynamo. As we have

already pointed out, the source of energy in the cell is the chemical potential energy in the positive plate and electrolyte. For example, taking for the positive element the metal which experience has shown to be the most economical and suitable, namely zinc, one pound of zinc, of the requisite degree of purity, costs, when made up in large quantities into plates, say $0.07. This pound of zinc dissolved in a voltaic cell without loss by local action, produces a delivery of about 1,347,500 coulombs, and if the E. M. F, of the cell be two volts, 2,695,000 volt-coulombs, *i. e.*, 2,695,000 joules of electrical energy, equal to 0.7486 kilowatt- hours, so that, leaving out of consideration the cost of the electrolytes employed in the battery, as well as labor, interest and depreciation, the cost of a kilowatt-hour in zinc consumed, is 9.352 cents per kilowatt-hour in the battery circuit. On the other hand, it is known that 1.8 lbs. of coal in the best large steam plants will furnish one *average* indicated horse-power-hour; or 2.4 lbs. of coal, one average indicated kilowatt-hour, which with an efficiency of conversion in dynamo machines of 0.9, represents an expenditure of 2⅔ lbs. coal per kilowatt-hour of electrical energy in the dynamo circuit. With coal costing $3.00 per ton of 2,240 lbs., the cost of a kilowatt-hour is thus approximately 0.357 cent for coal consumed, leaving out of consideration water, oil, waste, labor, interest and depreciation. In a large steam dynamo central station, the total cost of producing and delivering a kilowatt-hour over a long line to consumers is sometimes seven cents.

66. If the working E. M. F. of a cell be denoted by e (volts) and its internal resistance by r (ohms) then $\frac{e^2}{r}$ may be called the *electrical capability* of the

cell, and is equal to the activity of the cell, when short circuited, expressed in watts. If now an activity of P watts is required from the battery in its external circuit, the minimum number of cells which will supply this activity is $4\,P \div \left(\frac{e^2}{r}\right)$. This number of cells will, therefore, represent the most economical installation or first cost.

Thus if a given type of cell has an E. M. F. of 2 volts, and a resistance of 0.1 ohm, its electrical capability is 40 watts. If a battery of these cells has to yield 160 watts in its external circuit, the minimum number of cells required is $4 \times \frac{160}{40} = 16$. Each cell will yield to the circuit 20 watts, or one-half of its capability, and will yield to the external circuit 10 watts or one-quarter of its capability. In other words, minimum installation cost requires an efficiency of 0.5 from the battery, and half the capability of each cell.

67. When the requisite number of cells has been determined by the preceding rule, the grouping of the cells does not alter their activity. In the case considered, if 16 cells be operated in series the terminal E. M. F. would be 16 volts and the current 10 amperes. If the battery was arranged in 2 rows of 8 cells, the terminal E. M. F. would be 8 volts, and 20 amperes, similarly for 4 rows of 4 cells, 8 rows of 2, or 16 rows of 1, the output would be 160 watts. The grouping adopted would in practice depend upon the nature of the receptive device; *i. e.* the motor or lamp operated.

68. It sometimes happens that the activity required from a cell for minimum installation cost, namely one-half of its capability, is greater than the cell can

sustain. In such cases the rule must be modified. If i, be the maximum current strength in amperes which the cell can sustain, then $e\,i$, is its maximum yield to the circuit, and $e\,i - i^2\,r$, its maximum delivery to the external circuit. Hence if P, be the external activity required, the minimum number of cells which will yield it is $\frac{P}{e\,i - i^2\,r}$.

For example, in the cells already considered, the theoretical current obtainable from them on short-circuit would be $\frac{2}{0.1} = 20$ amperes and the most economical working current in regard to first cost of battery would be ten amperes. But if the maximum current practically obtainable from these cells without undue polarization was four amperes, then the minimum number required to yield 160 watts in the external circuit would be $\frac{160}{8 - 1.6} = 25$, and this might be arranged in one, or five rows, according to requirements. The efficiency of the battery would no longer be 0.5. In this instance it would be 0.8.

69. It should be observed that in order to obtain the best economy in operating and maintaining a battery for a given activity, the cost of materials and superintendence have to be considered as well as interest, depreciation, and first cost, so that the number of cells for best working economy, may be very different from the number of cells for lowest first cost of installation.

70. The value of any type of cell, in regard to the minimum number that must be installed for the supply of a given power is proportional to its capability,

and if p, be the price in dollars of any given type of cell, its economic value for cost of installation is proportional to $\frac{e^2}{p\,r}$. Thus, two cells would have equal economic value for the delivery of a small quantity of power, reckoned on the basis of first cost in installation, if one had two volts 0.1 ohm, and cost $1.00; while the other had 0.6 volt, 0.012 ohm and cost $0.75.

71. Whenever in order to secure the maximum first cost of battery by obtaining an efficiency of 0.5, the cells of a battery have to be joined up in multiple series, it is preferable to employ larger cells in a single series if possible. Such a proceeding is not only more economical, since large cells cost relatively less than smaller ones, but will also be a safegard against defective action by the failure of cells in any series, thereby allowing the neighboring series to discharge through it thus seriously interfering with the effectiveness of the battery.

SYLLABUS.

The seat of E. M. F. in a voltaic cell is at the contact surfaces of the plates or elements of the voltaic couple with the electrolyte or electrolytes.

It is erroneous to ascribe the seat of E. M. F. in a voltaic cell to the metallic junction outside the cell.

In any cell, the ratio of the square of the E. M. F. to the resistance, may be called the capability of the cell.

The economic value of a cell, so far as regards first cost, is its capability divided by its price.

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ADVANCED GRADE.

THE VOLTAIC CELL.

72. The conduction of an electric current by an electrolyte is of a different nature from the conduction afforded by metals. In the latter case, apart from an elevation of temperature, the passage of the current is attended by no change in the metal. In the case of an electrolyte, however, the passage of an electric current is invariably attended by a decomposition or dissociation of some of the constituent molecules. This is accounted for on the supposition, that in liquids, only the *ions*, *i.e.*, the dissociated molecules, are capable of carrying an electric current, and, if no dissociated molecules existed in the solution, that solution would act as an insulator. The action of an E. M. F. on an electrolyte is, therefore, to direct the movement of the ions.

Since atoms possess definite electric capacity, differing for different kinds of atoms, but always the same for the same kind of atom, it follows that the passage of a definite quantity of electricity, say one coulomb, must necessitate

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the transfer of a definite number of atoms or radicals, different in the case of hydrogen than in that of oxygen; consequently a definite quantity or mass of any given radical is invariably dissociated and transferred by the passage of one coulomb of electricity. This mass expressed in grammes is called the *electro-chemical equiv-*

TABLE OF ELECTRO-CHEMICAL EQUIVALENTS.

Element.	Atomic Weight.	Valency.	Electro-chemical Equivalents. Milli-grammes per Coulomb.	Coulombs per Gramme.	Ampere-hours per pound.
Hydrogen	1.	1	0.01038	96340.	12140.
Potassium	39.03	1	0.4051	2469.	311.1
Sodium	23.00	1	0.2387	4189.	527.8
Silver	107.7	1	1.118	894.5	112.7
Copper, in cuprous combinations	63.18	1	0.6558	1525.	192.1
Mercury, in mercurous combinations	199.8	1	2.074	482.2	60.75
Chlorine	35.37	1	0.3671	2724.	343.2
Iodine	126.54	1	1.3134	761.4	95.93
Bromine	79.76	1	0.8279	1208.	152.2
Copper, in cupric combinations	63.18	2	0.3279	3050.	384.3
Mercury, in mercuric combinations	199.8	2	1.037	964.3	121.5
Tin, in stannous combinations	117.4	2	0.6093	1641.	206.8
Iron, in ferrous combinations	55.88	2	0.2900	3448.	434.4
Nickel	58.6	2	0.3042	3287.	414.1
Zinc	64.88	2	0.3367	2970.	374.2
Lead	206.4	2	1.071	933.7	117.6
Oxygen	15.96	2	0.08283	12070.	1521.
Gold	196.2	3	0.6789	1473.	185.6
Iron, in ferric combinations	55.88	3	0.1934	5171.	651.5
Aluminum	27.04	3	0.0935	10700.	1348.
Nitrogen	14.01	3	0.04847	20630.	2599.
Tin, in stannic combinations	117.4	4	0.3046	3283.	413.6

alent of the radical. For example, one coulomb of electricity passing through an electrolyte will transfer 0.00001038 gramme of hydrogen.

A table of electro-chemical equivalents is given on page 74.

73. It will be seen from the table that the electro-chemical equivalent of silver is 1.118 milligrammes; therefore, each coulomb of electricity will liberate 1.118 milligrammes of silver, and each ampere-hour will liberate $3600 \times \frac{1.118}{1000} = 4.0248$ grammes of silver.

It will also be observed that the electro-chemical equivalent of any monad element, such as hydrogen, chlorine, nitrogen, iodine, etc., is directly proportional to its atomic weight. Thus the electro-chemical equivalent of chlorine is 35.37 times the electro-chemical equivalent of hydrogen. This is tantamount to the statement that all monad atoms or radicals carry the same electric charge; that all dyad atoms or radicals carry twice the charge of a monad atom; that all triad atoms or radicals carry three times the charge of a monad atom; that all tetrad atoms or radicals carry four times the charge of a monad atom. Consequently, when an electric current passes in series through solutions of various chemical substances, there will be liberated one-fourth as many tetrad, one-third as many triad, and one-half as many dyad, as monad atoms or radicals.

Thus, one coulomb of electricity will liberate $0.00001038 \times \frac{63.18}{2}$ grammes of copper from cupric solutions, for the reason that the atomic weight of copper is 63.18, and copper, in such salts, is a dyad radical.

Generally, in the case of any element or radical, the mass in grammes liberated by one coulomb is,

$$0.00001038 \times \frac{\text{atomic weight}}{\text{valency}}$$

74. Whenever a definite chemical combination occurs, a certain amount of energy is either absorbed or liberated. In the case of the mere chemical formation of zinc sulphate from metallic zinc and sulphuric acid, energy is liberated as heat. If, however, the same combination is effected in a voltaic cell, when the circuit is closed, this energy is no longer liberated as heat, but as electric energy, and the same number of joules appear in the circuit as before. It is evident, therefore, that the E. M. F. is capable of being calculated when the *thermo-chemical equivalents* of its formation products are known.

By the thermo-chemical equivalent of a substance is meant the amount of energy liberated by the chemical combination of its molecular weight with any other substance. This energy is usually expressed in gramme-calories, *i. e.*, the amount of heat necessary to raise the temperature of one gramme of water 1° C., but may be expressed in joules.

Suppose that exactly one coulomb of electricity flows through a circuit from a Daniell cell. There will be $0.00001038 \times \frac{64.88}{2} = 0.0003367$ grammes of zinc dissolved and converted into zinc sulphate. There will also be $0.00001038 \times \frac{63.18}{2} = 0.0003279$ grammes of copper deposited on the negative plate.

Heat is evolved by the normal formation of zinc-sulphate from zinc, and, one gramme of zinc, converted into $ZnSO_4$ and dissolved in water, is known by experiment to yield 16,090 joules of energy in the form of heat, so that 0.0003367 gramme of zinc dissolved by ordinary chemical processes would yield 5.417 joules.

But, on the other hand, work requires to be expended in order to reduce copper from a solution of copper sulphate, and it has also been found by measurement that the energy necessary to resolve one gramme of copper in this manner is 13,190 joules, or to resolve 0.0003279 gramme, 4.324 joules. Thus for every coulomb of electricity generated by the cell, chemical changes are effected within it which would represent 5.417 joules produced at the positive plate, and 4.324 joules expended at the negative plate, leaving a balance of 1.093 joules developed in the cell.

If the chemical changes occurred under chemical action alone, the 1.093 joules would be expended in heating the contents of the cell, just as water is heated by the admixture of sulphuric acid. As, however, the changes occur under electrical action, the 1.093 joules do not appear as heat in the cell, but in the entire circuit as electrical energy, of the type $E\,q$, (see Paragraph 15), and since $E\,q = 1.093$ joules, and q is here chosen as unity, we have $E = 1.093$ volts.

75. According to the principles of the conservation of energy we have thus determined that the E. M. F. of the Daniell cell must be 1.093 volts, in order that the energy accompanying the chemical changes in the cell should be wholly developed in the circuit, and assuming that the experimentally determined energy valuations of these chemical changes, *i.e.*, their thermochemical equivalents, have been accurately measured.

In fact, 1.09 volts is very nearly the E. M. F. of a

Daniell cell freshly set up, with pure metal plates, and pure saturated solutions. In practice, owing to a variety of causes, the E. M. F. is usually lower than this, and, on closed circuit work, may even be below one volt.

We have remarked that the E. M. F. was calculated on the assumption that the chemical energy developed in the cell was not liberated there as heat. Practically, however, some heat is generated in the cell during its electric activity. This is only a secondary consequence of the electric resistance of the cell; and its share of the total resistance in the circuit, determines the proportion of heat that will be developed within the cell. If the cell be so constructed as to offer a negligibly small resistance, then the amount of heat electrically developed by the current would be negligibly small, and all the chemical energy developed by chemical changes in the cell would be liberated outside the cell, *i.e.*, in the external circuit, by the electric agencies.

It should be remarked, however, that owing to the incompleteness of our knowledge of thermo-chemical equivalents, and of the exact nature of the electro-chemical actions in the cell, the E. M. F. of a cell can in only a few instances be practically predetermined.

A voltaic cell is, therefore, a device for liberating outside the cell the energy developed by its chemical activities, and if the amount of those chemical activities is completely known, the E. M. F. of the cell is determinate by purely thermo-chemical measurements.

Irrespective of the cost of materials, the best type of cell would be one in which the thermo-chemical energy developed in the changes effected at the positive plate would be a maximum, and in which the expenditure of thermo-chemical energy developed in the changes effected at the negative plate would be a minimum. Such a cell would possess the maximum E. M. F.

76. From the hundreds of combinations that have been tried, the zinc-carbon chromic acid type of cell appears to possess the greatest E. M. F. practically available; viz., about two volts.

The disadvantage of this cell, however, lies in the fact, that since the chromic acid around the negative plate freely attacks zinc, it is necessary to employ a porous jar to contain it, and even this resource only retards diffusion and causes waste of zinc on open circuit, besides adding considerably to the resistance of the cell.

SYLLABUS.

In metallic conduction no visible change in the conducting circuit is produced beyond a change in its temperature.

In electrolytic conduction the passage of the current is invariably attended by the dissociation of some of the constituent molecules of the liquid conductor, *i.e.*, of its electrolyte.

The electrical capacity of the ultimate atoms of matter differs for different kinds of atom, but is invariably the same for the same kind of atoms.

In electrolytic conduction the passage of one coulomb of electricity necessitates the transfer of a number of atoms or radicals dependent on their electro-chemical equivalents.

The electric carrying capacity, or charge, of all monad atoms is the same. The charge of all dyad atoms is twice, of all triad atoms three times, and of all tetrad atoms four times that of a monad atom; consequently, the electro-chemical equivalent of an atom of any element will be proportional to its atomic weight divided by its valency.

The E. M. F. produced by any voltaic cell may be calculated when the thermo-chemical equivalents of its formation products are known, and is equal to the total resulting joules developed in the cell by its chemical action, per coulomb of electricity passing through it.

In every voltaic cell thermo-chemical energy is liberated at the positive plate and absorbed at the negative plate. The maximum E. M. F. will, therefore, be produced when the former action is greater than the latter.

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THE VOLTAIC CELL.

77. The hydrogen evolved at the surface of the negative plate adheres to the plate, and produces by its contact an electromotive force, counter or opposed to that of the cell. This is called the *counter E. M. F. of polarization.* Various methods are employed to prevent polarization. These consist, practically, of methods by which the hydrogen is either prevented from being evolved at the negative plate; or, if evolved, prevented from forming there by entering into combination with some suitable substance surrounding the negative plate. This substance is called a *depolarizer.*

78. Voltaic cells may be divided into the following classes according to the presence or absence of a depolarizer and its character, viz.:

(*1.*) Single-fluid cells or those which possess an exciting fluid but no depolarizer.

(*2.*) Single-fluid cells with solid depolarizers surrounding, or in contact with, the negative plate.

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(*3.*) Double-fluid cells, or those with an exciting fluid and a fluid depolarizer.

79. In single-fluid cells no steps are taken to avoid polarization. It is evident, therefore, that the cells cannot be successfully employed for furnishing a continued electric current. Indeed, even for temporary currents, they are inferior to most forms of efficient cells with depolarizers.

One of the earliest forms of single-fluid cell was the Smee cell, a zinc-silver couple, immersed in an electrolyte of dilute sulphuric acid in water. This cell was at one time largely used in telegraphy and in electroplating. It has now almost entirely disappeared, being replaced by more efficient types of cell, or by dynamo electric machines.

80. A form of single-fluid cell, which is still employed for producing powerful currents for brief intervals of time, is the zinc-carbon couple immersed in a solution of sal-ammoniac in water.

Like all cells of the single-fluid type without depolarizers, this cell requires long intervals of rest in order to regain its full E. M. F.

81. The Grenêt, the bichromate, or the Poggendorf cell, as it is indifferently called, consists of a zinc-carbon couple immersed in an electrolyte of bichromate of potash and sulphuric acid. The reactions occuring in this cell, when at work, are probably represented as follows:

Before action,

$$3\,Zn + 7\,H_2\,SO_4 + K_2\,Cr_2\,O_7 + C.$$

After action,

$$3\,ZnSO_4 + 7\,H_2\,O + K_2\,SO_4 + Cr_2\,(SO_4)_3 + C.$$

In the Grenêt cell, the zinc plate should be removed from the electrolyte when not in use, since otherwise deleterious *local action*, or irregular consumption of the zinc will occur. For this purpose some arrangement is generally made to lift either the zinc only, or both the zinc and carbon from the liquid, as shown in Fig. 25, where a form of cell suitable for use in driving the motor of a phonograph is illustrated. Although the Grenêt cell may be regarded as a single-fluid cell without any separate depolarizer, yet, in point of fact, the exciting solution acts both as an exciting liquid and as a depolarizer, since no free hydrogen makes its appearance at the negative plates. Such a cell as is represented in Fig. 25 will supply three amperes steadily.

FIG. 25.—FORM OF GRENET CELL.

FIG. 26.—FORM OF GRAVITY DANIELL CELL.

82. In double-fluid cells, in order to prevent the depolarizing liquid from mixing with the exciting liquid, the depolarizing liquid is generally placed in a porous jar. The presence of the porous jar greatly in-

creases the internal resistance of the voltaic cell on account of the high resistivity of the unglazed earthenware of which it is formed.

The Daniell cell possesses the great advantage of giving a continuous, steady current, provided the current density in the cell is not excessive.

The reaction which occurs in this cell is probably expressed by the following equation:

Before action,

$$Zn + H_2\,SO_4 + Cu\,SO_4 + Cu.$$

After action,

$$ZnSO_4 + H_2\,SO_4 + Cu + Cu.$$

83. In practice, the inconvenience arising from the use of a porous jar, led Callaud to modify the Daniell cell for closed-circuit work. In the Callaud cell the porous partition is entirely dispensed with, and the zinc sulphate and copper sulphate solutions are separated entirely by reason of their differences of density. Fig. 26 shows the Callaud or gravity cell. The copper element consists of a sheet of copper, bent as shown, placed at the bottom of the cell and provided with an insulated wire passing out at the top. The zinc plate has generally the form of a star or crowfoot, and is suspended near the top of the jar. After the battery has been in use for some time, the zinc sulphate formed by its action, will be seen as a clear transparent liquid layer separated from the dense blue copper sulphate solution, in the lower part of the jar, by a sharply marked boundary. After the cell has been in use some time, some of the zinc sulphate solution requires to be drawn off, a handful of copper sulphate crystals thrown in, and fresh water added. A film of

oil is frequently poured on the liquid so as to avoid the effects of creeping, and to prevent evaporation.

The ordinary form of Callaud cell should not be called upon to deliver more than ¼ ampere steadily. Special forms of gravity cell are sometimes employed, called Tray cells, which will supply five amperes steadily.

84. Fig. 27 shows the Leclanché cell, which has a zinc-carbon couple. The zinc is in the form of a rod. The carbon is placed inside a porous cell and closely packed with powdered carbon and black oxide of manganese, the latter acting as a solid depolarizer. The exciting liquid is a solution of sal-ammoniac in water. This cell is made in a variety of forms. In one form, called the agglomerate form, the porous cell is dispensed with, and the crushed carbon and manganese dioxide are moulded around the carbon plate under great pressure.

FIG. 27.—FORM OF LECLANCHE CELL.

FIG. 28.—PARTZ GRAVITY CELL.

The action of the Leclanché cell is probably represented as follows:

Before action,

$$Zn + 2\,NH_4\,Cl + 2\,MnO_2 + C.$$

After action,

$$ZnCl_2 + 2\,NH_3 + MN_2\,O_3 + H_2\,O + C.$$

When the cell is overworked, the reactions that take place are very obscure.

85. Fig. 28 shows a Partz acid gravity cell. It is composed of a zinc-carbon couple; the carbon plate rests on the bottom of the jar, while the zinc is suspended near the top.

On charging, the jar is partly filled with an aqueous solution of common salt or of sulphate of magnesia. A specially prepared salt, consisting of a mixture of chromic and sulphuric acids is now added through the funnel tube shown at the side. This salt, on reaching the bottom of the cell, dissolves and spreads over the surface of the carbon plate and acts as a depolarizer. Its greater density keeps it at the bottom of the vessel.

Fig. 29.—Fuller Cell.

Fig. 30.—Edison-Lalande Cell.

86. Fig. 29 shows a Fuller mercury bichromate cell, which consists of a zinc-carbon couple with the carbon immersed in an electrolyte, consisting of a solution of bichromate of potash, sulphuric acid and water, and the zinc, which is usually in the form of a truncated cone, placed inside a porous cell filled with dilute sulphuric acid in water. A small quantity of mercury is

poured into the porous cell to thoroughly amalgamate the zinc. This cell gives a high electromotive force and a fairly steady current, but possesses the disadvantage attending all cells with porous partitions of a comparatively high internal resistance, and waste of chemicals on open circuit.

87. Fig. 30 shows an Edison-Lalande cell, which consists of a zinc-copper couple in a solution of caustic soda in water. A solid depolarizer is used in this cell consisting of a block of black oxide of copper supported in a frame or grid of copper. The action consists in the formation of a zincate of soda, and the reduction of the copper oxide to metallic copper on the external surface of the block. Owing to the fact that the zinc and copper plates have large surfaces placed in close proximity to each other, the internal resistance of this cell is remarkably low, and forms the nearest approach, on the part of a primary battery, to the internal resistance of a storage cell. For this reason it is capable of supplying strong currents, although its E. M. F. is comparatively low ($\frac{2}{3}$ volt). Its local action is usually negligible.

88. The chloride of silver cell consists of a zinc-silver couple immersed in a solution of sal-ammoniac in water. The depolarizer is a mass of chloride of silver fused as a rod around a silver wire. The electromotive force of this cell is very uniform, but owing to the expense of silver plates these can not be given a large surface, and hence the cells possess a comparatively high internal resistance and are unsuited to the delivery of strong currents. For testing purposes, however, the

portability and constancy of the cell renders its use admirable, and it is frequently made up into portable batteries, one of which is shown in Fig. 31.

89. Fig. 32 shows two of Clark's standard cells enclosed in a brass box with a hard rubber cover and a thermometer suitably placed to indicate the temperature of the interior.

FIG. 31.—BATTERY OF SILVER CHLORIDE CELLS IN SERIES.

FIG. 32.—PAIR OF CLARK STANDARD CELLS ENCLOSED IN A CASE WITH A THERMOMETER.

SYLLABUS.

Voltaic cells are of three general classes, viz.: single-fluid cells with an exciting liquid and no depolarizer; single-fluid cells with an exciting liquid and a solid depolarizer, and double-fluid cells with both an exciting liquid and a liquid depolarizer.

Voltaic cells without depolarizers are generally unserviceable.

Voltaic cells with solid depolarizers have come into very extended use.

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Magnetomotive Force

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90. The space surrounding a magnet, known technically as a magnetic field, is a region traversed or permeated by magnetic flux. Although the physical nature of magnetic flux is unknown, yet it possesses both magnitude and direction, and its presence is accompanied by a condition of stress in the ether. It is assumed that the direction of magnetic flux is such that it issues from the positive or north-seeking pole of a magnet, commonly called the north pole, and, after passing through the region outside the magnet, re-enters it at its negative or south-seeking pole, completing the circuit through the substance of the magnet. The direction of the flux, at any point of a path, is that which would be assumed, at that point, by the magnetic axis of a freely suspended small magnetic needle. The strength of this magnetic flux rapidly diminishes with the distance from the magnet.

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91. Outside the magnet the magnetic flux is strongest in the neighborhood of the poles, where its density is said to be a maximum. The unit of magnetic flux is called the *weber*. *Flux-density*, or *flux intensity*, is defined as being the quantity of flux passing through a normal or perpendicular square centimetre of area, so that unit flux-density is a flux of one weber through a normal square centimetre, and is called a *gauss*. Flux-density is usually represented by the symbol $\mathcal{B}$. In the open country, in the neighborhood of New York, the flux-density of the earth's magnetic field is about 0.6 gauss, directed downwards at an inclination of about 72°. Although the flux-density, in the space outside a magnet, is greatest in the neighborhood of its poles, yet, in the case of a homogeneous magnet, the flux-density in the magnetic circuit is usually greatest at some point within the magnet.

92. If the distribution of magnetic flux outside a long cylindrical bar magnet be mapped out by the aid of a small magnetic needle, it will be found that the flux-paths are similar in direction and magnitude to the stream-lines which would exist in an incompressible fluid surrounding the magnet, if this were a hollow tube containing a force pump so operated that the fluid was continually being forced out at one end and in at the other.

93. A magnetic flux differs from any material flux with which we are acquainted, in that while it is associated with energy, since in all cases it requires an expenditure of energy to establish it, yet, it requires no expenditure of energy to maintain its existence after it has been once established. The amount of energy

stored up in magnetic flux is represented by $\frac{\mathcal{B}^2}{8\pi}$ ergs per cubic centimetre of air or other non-magnetic material, so that the amount of energy existing in a cubic centimetre of air, in the neighborhood of New York, owing to the existence of the earth's magnetic flux, is about $\frac{0.6 \times 0.6}{25.133} = .01432$ erg.

A marked difference exists in this respect between the electric and magnetic circuit, since, in the electric circuit, a constant expenditure of energy to the amount of $i^2 \rho$ ergs per cubic centimetre—where i, is the current density, and ρ, the resistivity of the material in the circuit—has to be maintained as long as the current is flowing; whereas, in the magnetic circuit, no energy is required to maintain the flux when once established, as, for example, in the permanent magnet.

94. Just as the presence of electric flux or current necessitates the existence of an E. M. F. to produce it, so the presence of a magnetic flux necessitates the presence of a magnetomotive force (abbreviated M. M. F.) to produce it.

Magnetic flux, unlike electric flux, cannot be insulated. Therefore, magnetic flux cannot be confined to a particular conductor.

95. There are two varieties of M. M. F. The *permanent* and the *transient*. A permanent magnetomotive force is found in the case of a permanent magnet. Here an expenditure of energy has been initially necessary to magnetize the bar, but since the bar maintains the flux indefinitely after the withdrawal of the magnetizing force, there must exist in the magnetized

bar a true M. M. F. which sustains the flux. It is assumed, that in the case of the magnetizable metals, the ultimate atoms or the molecules naturally possess true M. M. F.'s, which, however, distribute their magnetic circuits in all directions and thus neutralize each other's influences. On the application of the magnetizing force, however, these separate molecular M. M. F.'s are brought more nearly into a common line or direction, thus mutually assisting each other, and exerting a definite external influence. When all the molecules in a bar are thus brought into line, its M. M. F. is at a maximum and the magnet is said to be *magnetically saturated.*

96. Most practical magnetic circuits extend almost throughout their entire length through iron. In order to force magnetic flux through a circuit, it is necessary to wind the circuit with turns of insulated wire and to send a current through these turns of wire. The M. M. F. depends upon the number of turns so linked with the circuit, and upon the strength of the current. Their product, usually expressed in ampere-turns, measures the magnetomotive force produced.

97. The unit of M. M. F. is the gilbert, and is such a M. M. F. as would be produced by $\frac{10}{4\pi}$ or 0.7958 (roughly 0.8) ampere-turn.

The gilbert and the ampere-turn, as units of M. M. F., are related in a similar manner to the kilowatt and the horse-power as units of activity in engineering; *i. e.*, by a numerical ratio. It is commonly convenient to express activities in horse-power, in order to conform to usage, and the classification of existing machinery; but for computations and simplicity of reasoning and description, it is usually advantageous to employ the more fundamental and scientific unit, the kilowatt. Similarly, in dealing with M. M. F.'s it is commonly convenient to express their values in ampere-turns, but for purposes of computation and simplicity of reasoning, it is usually advantageous to employ the more fundamental and scientific unit, the gilbert.

98. In order to follow the effects of iron on the magnetic flux produced by a current, as already pointed out, let us take the simple case of an air-core solenoid, or hollow anchor ring, of the form shown in Fig. 33. If such a ring were wound with 100 turns of insulated wire carrying a current of five amperes, the M. M. F. exerted would be 500 ampere-turns = 628.5 gilberts. We may suppose that in a ring of the dimensions shown, the flux through the core produced by this M. M. F. would be 62.85 webers. If the ring were composed of copper or wood, or any material except the magnetic metals, this total flux would be practically the same. If, however, Fig. 33 represents an iron solenoid of the same size and wound with the same wire, then the magnetic flux set up in this iron core, on the passage of the same magnetizing current, would be, perhaps, 500 times greater. Here the additional flux is due to a M. M. F., previously existing in the iron in a freely distributed state, but now aligned and brought into action by the current. On the cessation of the current in the solenoid, this structural M. M. F. in the iron may remain largely intact, as shown in Fig. 34, producing a flux of say 20,000 webers, which is called residual magnetism. Although the above are the conditions as they appear to actually exist in a ferric magnetic circuit, *i.e.*, a magnetic circuit of iron, still it is practi-

cally much more convenient to assume that the iron is destitute of M. M. F., but that it conducts magnetic flux much more readily than air. That is to say, it is practically more convenient to suppose that the iron does not act as a source but as a good conductor of flux.

The form of magnetic circuit shown in the above figure is the only form known in which the magnetic flux-paths are definitely limited, being confined, at least for a theoretically wound solenoid of this type, entirely to the interior of the coil. The flux-paths are, therefore, all circles, and the density is uniform around any circle.

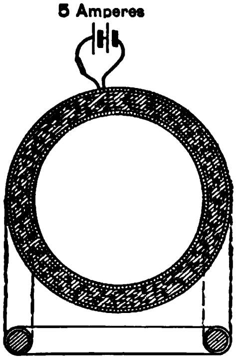

FIG. 33.—M. M. F. OF 500 AMPERE-TURNS, OR 628.5 GILBERTS APPLIED TO A CLOSED CIRCULAR COIL WOUND ON AN IRON CORE, ESTABLISHING A M. M. F. OF 250,000 AMPERE-TURNS OR 312,250 GILBERTS. FLUX, 31,250 WEBERS.

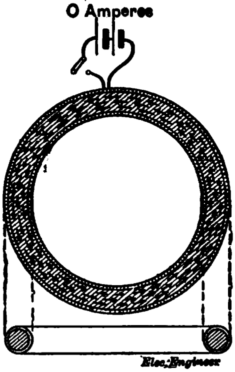

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FIG. 34.—SAME COIL AND IRON CORE WITH PRIME M. M. F. REMOVED, LEAVING A RESIDUAL M. M. F. IN THE IRON OF ABOUT 200,000 GILBERTS. RESIDUAL FLUX, 20,000 WEBERS.

99. When a bar of iron is brought into a magnetic flux and the flux passes lengthwise through it, the bar thereby becomes magnetized. The end where the

magnetic flux enters, becomes of south-seeking polarity, and the end where it leaves, of north-seeking polarity. It is, for convenience, generally assumed that such a bar concentrates the flux of the field in which it is placed owing to the greater magnetic permeability or conductivity of the iron for flux. Although this is a convenient way of treating the matter for practical purposes, yet it is inconsistent with the facts. A new or local magnetic circuit is, in reality, called into existence by the prime flux, having its local M. M. F. in the iron bar, and its flux similarly directed in the mass of the bar and oppositely directed in the air outside it, to the prime flux.

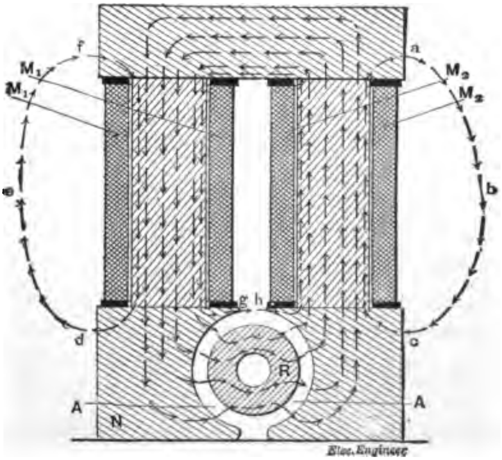

FIG. 35.—SECTION OF A COMMON TYPE OF DYNAMO WITH MAGNETIC CIRCUIT INDICATED.

100. Fig. 35 represents diagrammatically the distribution of magnetic flux in the magnetic circuit of a particular type of bipolar dynamo. Here the magnetizing coils M_1 M_1 and M_2 M_2, when excited by the passage of a continuous current, become the source of a M. M. F. which drives magnetic flux through the circuit. Most of this flux passes through the cores, yoke, and pole pieces of the magnets, through the air gaps A A and the armature core R. Some of the flux, however, completes its circuit by *leakage paths* such as *a b c* and *d e f*, through the surrounding air. The M. M. F. of the coils M_1 M_2, evidently depends upon the number of turns of wire, and upon the strength of the circulating current, *i.e.*, upon the number of ampere-turns or gilberts.

SYLLABUS.

A magnetic field is a region traversed by magnetic flux, and is attended by a stress in the surrounding ether.

The unit of magnetic flux is called the weber.

The unit of magnetic flux-density is called the gauss, or one weber per normal square centimetre.

The unit of M. M. F. is the gilbert, and is the M. M. F. produced by 0.7958 ampere-turn, approximately.

Magnetic flux-density is usually denoted by $\mathcal{B}$, and has direction as well as magnitude.

The energy attending a magnetic flux amounts, in non-magnetic media, to $\frac{\mathcal{B}^2}{8\pi}$ ergs per cubic centimetre.

A closed circular coil or solenoid, with or without an iron core, has no external magnetic influence; *i.e.*, all its magnetic circuit is confined to the interior of its coil.

Magnetic flux is due to the existence of M. M. F.

There are two varieties of M. M. F., the permanent and the transient.

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Magnetic Reluctance.

101. The *reluctance* of a magnetic circuit corresponds to resistance in the electric circuit, and is that quantity which limits the flux of magnetism under a given M. M. F. The flux in any magnetic circuit can only be increased by either increasing the M. M. F., or by diminishing the reluctance.

The unit of reluctance is named the *oersted*, after Hans Christian Oersted, who, in 1820, discovered the magnetic action of an electric current. The oersted is the reluctance offered by a centimetre cube, of air-pump vacuum, between opposed surfaces.

The specific reluctance of a body is called its *reluctivity*, and is the reluctance offered by a centimetre cube of the body between opposed parallel faces, just as the specific electric resistance of a body is called its resistivity. The reluctivity of nearly all substances, other than the magnetic metals, is sensibly that of vacuum, is equal to unity, and is independent of the flux density.

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102. If an anchor ring of copper, wood, glass or other non-magnetic material be uniformly wrapped with a magnetizing coil, the reluctance of the circuit will depend upon the length of the circuit, and on its area of cross-section. In this particular case all the magnetic flux will be confined to the interior of the winding, a compass needle held outside the winding indicating no deflection. Thus, if the mean diameter of the ring in Fig. 36, be 20 cms., and its cross-section five square centimetres, the mean length of the magnetic circuit will be 62.83 centimetres, and the reluctance of the circuit *approximately*, $\frac{62.83}{5}$ = 12.566 oersteds.

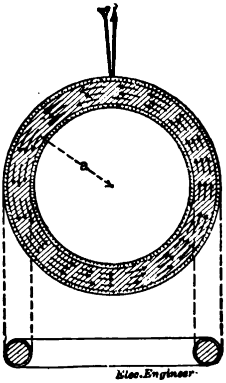

FIG. 36,

A non-ferric magnetic circuit in which the magnetizing force or flux density is uniform along any circle of radius, such as *a*, and equal to the M. M. F. divided by the circumference, $2\pi a$.

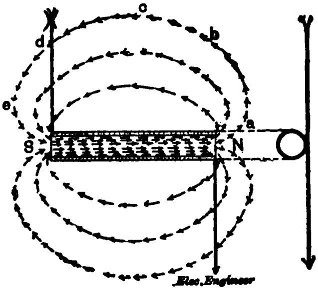

FIG. 37.

A non-ferric magnet circuit in which the magnetizing force or flux density (in the absence of iron) is not the same at different parts of the circuit, and where the quotient of M. M. F. by the flux path length only gives the average intensity.

103. If now the same coil be wound on a core of iron of the same dimensions, the magnetic flux within the iron will be far greater than within the previous copper, wood and glass core.

104. In the voltaic circuit, as we have seen, there exists in nearly all cases that practically occur, a distribution of electric potential, and the total difference of potential expressed in volts is the E. M. F. in the circuit.

Similarly, in the magnetic circuit, in nearly all cases that practically occur, there exists a distribution of magnetic potential and the total difference of potential expressed in gilberts is the M. M. F. in the circuit.

The distribution of potential is diagrammatically indicated in any magnetic circuit by broken lines, which are the sections, in the plane of the paper, of surfaces connecting all points in the magnetic circuit having the same potential, that is, *equipotential surfaces.* These imaginary equipotential surfaces may be made as numerous as desired. If correctly drawn, they would everywhere intersect the magnetic flux-paths or stream-lines at right angles. In other words, the flux at a point is always normal to the equipotential surface through the point.

105. The magnetizing force, or, as it is sometimes termed, the *magnetic force* in the circuit, is the space rate of change or gradient of the potential. That is to say, the magnetizing force at a point is numerically equal to the number of gilberts variation of potential per centimetre of flux-path. Its direction is along the lines of flux, normal to the equipotential surfaces. Where the rate of change of potential along the flux-path is one gilbert per centimetre, as guaged by an indefinitely small excursion, the magnetic force is unity, or one gauss. If the rate of change in potential per centimetre of flux-path were 500 gilberts, the magnetizing force there ex-

isting would be 500 gausses. It is customary to express magnetizing force by $\mathcal{H}$, and by the foregoing definition, $\mathcal{H} = -\frac{d\,\Phi}{d\,n}$; where Φ, is the magnetic potential and n, the normal to the equipotential surface at the point. It will be seen that if, gilbert by gilbert, all the equipotential surfaces in a magnetic circuit are drawn; where they lie close together, the number of gilberts per centimetre will be great, and $\mathcal{H}$ is great; and where they lie far apart, the gradient of potential is small, and $\mathcal{H}$, is small. When the equipotential surfaces lines are straight and parallel, they are also equidistant, so that $\mathcal{H}$, is uniform, and has everywhere the same strength and direction. Where they are concave or convex in the direction of the flux, there $\mathcal{H}$ is either convergent and increasing, or divergent and decreasing.

When the circuit is non-ferric, the magnetic force $\mathcal{H}$, is identical with the flux density, which we have hitherto denoted by $\mathcal{B}$. When, however, the circuit contains iron, the distribution of $\mathcal{H}$, or the prime flux, sets up a structural M. M. F. in the iron, whose flux, merged with $\mathcal{H}$, gives a resultant distribution, represented by $\mathcal{B}$.

It is evident that since $\mathcal{H}$, is the gradient of the potential, the product of the gradient and a small length of flux-path gives the fall of magnetic potential in that length. Summing up in this way, along any flux-path, the product of gradients and small distances, in succession, the sums will be the total difference of magnetic potential in the circuit, or the M. M. F. In other words, the M. M. F. is the line integral of the magnetic force.

Thus, referring to Fig. 36, if a, be the radius in centimetres of any flux-path, then the length of that path is $2 \pi a$, and since the intensity $\mathcal{H}$, in gausses, has the same value all round this circle, the line integral once round the circuit on this flux path is

$$2 \pi a \mathcal{H} = \mathcal{F} \text{ gilberts; or, } \mathcal{H} = \frac{\mathcal{F}}{2 \pi a} = \frac{\mathcal{F}}{L} \text{ gausses;}$$

where L, is the length of the flux-path considered.

106. In general, however, this rule can not be applied.

For if, in the non-ferric circuit shown in Fig. 37, formed by a helix of, say, twenty turns carrying one ampere, if we divide the M. M. F. of 20 ampere turns, *i. e.*, 25.14 gilberts, along any path of flux, such as $a\ b\ c\ d\ e\ f$, by the length of the path, we only obtain the average magnetic force. The magnetic force will be greater than this mean value within the helix, and less than this mean value outside the helix where the paths diverge.

The magnetic force receives its name from the fact that if a unit magnetic pole could be isolated (a physical impossibility) and introduced into the magnetic circuit at any point, such as c, the mechanical force which would be exerted upon this unit magnetic pole would be equal in dynes to the value of $\mathcal{H}$, in gausses, at that point.

It has been found that a comparatively simple relation holds between the magnetizing force exerted through iron or steel, and the apparent magnetic reluctance it offers. We have already pointed out that it is not the reluctance, but a structural M. M. F. which varies under the action of an impressed prime magnetizing force. Practically, however, it is convenient to regard the effect as one of change of reluctance. Fig. 38 represents the variation of the apparent reluctance of various samples of iron and steel under a continuously increased magnetic force. It will be seen that the reluctance commences in

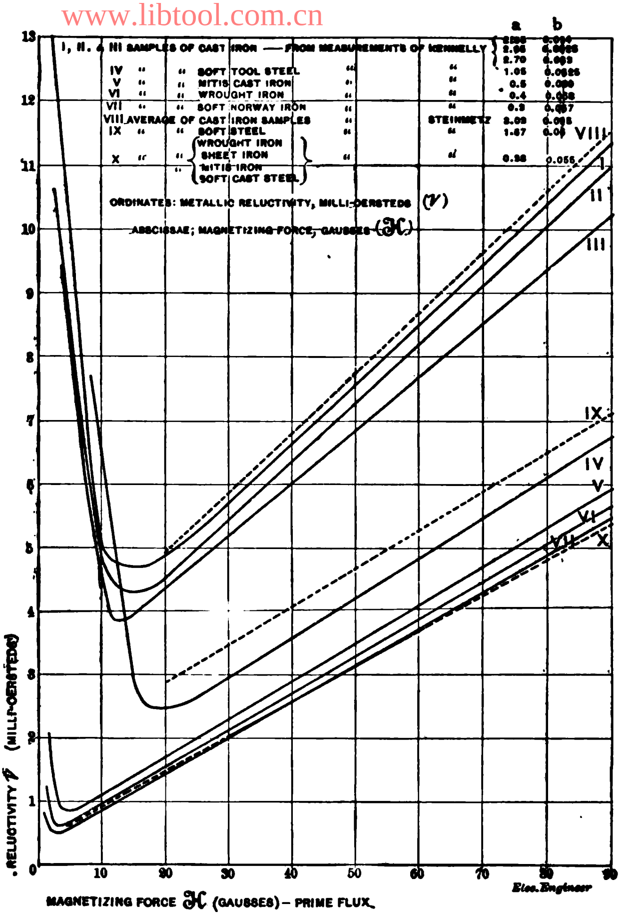

FIG. 38.

Curves of reluctivity in iron and steel in relation to magnetizing force.

all cases at a certain value, and diminishes as the magnetic force is increased, to a critical value, where the reluctance turns and commences rising steadily in a straight line.

Thus the lowest full line curve, No. VII, represents the reluctivity of soft annealed Norway iron. For magnetizing forces above 3 gausses, the reluctivity follows an ascending straight line, and at 90 gausses reaches 5.45 millioersteds. If reluctivity be denoted by the Greek symbol ν, we have, therefore, beyond the value of $\mathcal{H} = 3$,

$$\nu = (0.3 + 0.057\,\mathcal{H}) \div 1000,$$

and similarly for other samples of iron or steel.

Reluctivity is, strictly speaking, expressed in the C. G. S. system, as a numeric. In Fig. 38, it is, for convenience, expressed in millioersteds, and the curves may be therefore directly interpreted as representing the reluctance of a centimetre cube.

107. Since the ether pervades even the densest matter, the reluctivity of any medium may be regarded as the reluctivity of that ether and of the medium taken in parallel. For low values of the magnetizing force, the reluctivity of the ether is so much greater than that of iron (say, 1,000 times greater), that it may be neglected. This linear relation $\nu = a + b\,\mathcal{H}$, which appears to hold from experimental evidence, refers only to the *metallic reluctivity* of the iron or steel, independently of the ether which prevades the metal. When, however, very high magnetizing forces are reached, the reluctivity of the iron increases greatly and becomes much greater than that of the ether, whose reluctivity therefore controls. The reluctivity of iron and the

ether together can, therefore, never be greater than unity. For practical purposes, however, iron is always worked at such magnetizing forces, that its metallic reluctivity is always much lower than unity, and consequently the metallic reluctivity may be taken within the limits of the diagram to be sensibly equal to the real reluctivity of the ether and iron together.

SYLLABUS.

Reluctance is that quantity in a magnetic circuit which limits the flux under a given M. M. F.

The reluctivity of any medium is its specific reluctance, and, in the C. G. S. system, is the reluctance offered by a cubic centimetre of the body between opposed faces.

The unit of reluctance is called the oersted, and is the reluctance of a cubic centimetre of air-pump vacuum.

The reluctivity of all media with the exception of the non-magnetic metals is practically the same, *i. e.*, unity.

The reciprocal of magnetic reluctivity is called magnetic permeability, and both quantities are mere numerics in the existing C. G. S. system of units, but are probably not simple numerics in the, as yet, undiscovered true relations of this system.

It is erroneous to suppose that magnetic permeability or reluctivity varies in iron under magnetic force, except, perhaps, within small limits. The apparent variation is due to the existence of a structural M. M. F. induced in the iron under the influence of a prime magnetic force.

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MAGNETIC FLUX.

108. The fundamental equation of the magnetic circuit is

$$\Phi = \frac{\mathfrak{F}}{\mathfrak{R}}; \text{ or, the webers} = \frac{\text{gilberts}}{\text{oersteds}}. \tag{1}$$

From this we obtain

$$\mathfrak{F} = \Phi\,\mathfrak{R}, \tag{2}$$

and

$$\mathfrak{R} = \frac{\mathfrak{F}}{\Phi}. \tag{3}$$

There are, therefore, two ways of varying the magnetic flux in any circuit; namely, by increasing the M. M. F., and by decreasing the reluctance.

As we have already seen, a linear relation exists between the reluctivity of a magnetic metal and the magnetizing force, but in many practical magnetic problems it is the flux density, rather than the magnetizing force, which is known, and from which the reluctivity has to be determined. It becomes necessary, therefore, to know

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the value of the reluctance of the circuit from point to point at the existing flux density, in order that the total reluctance of the circuit may be determined.

From the relation

$$\nu = a + b\,\mathcal{H} \tag{4}$$

and

$$\mathcal{B} = \frac{\mathcal{H}}{\nu}, \tag{5}$$

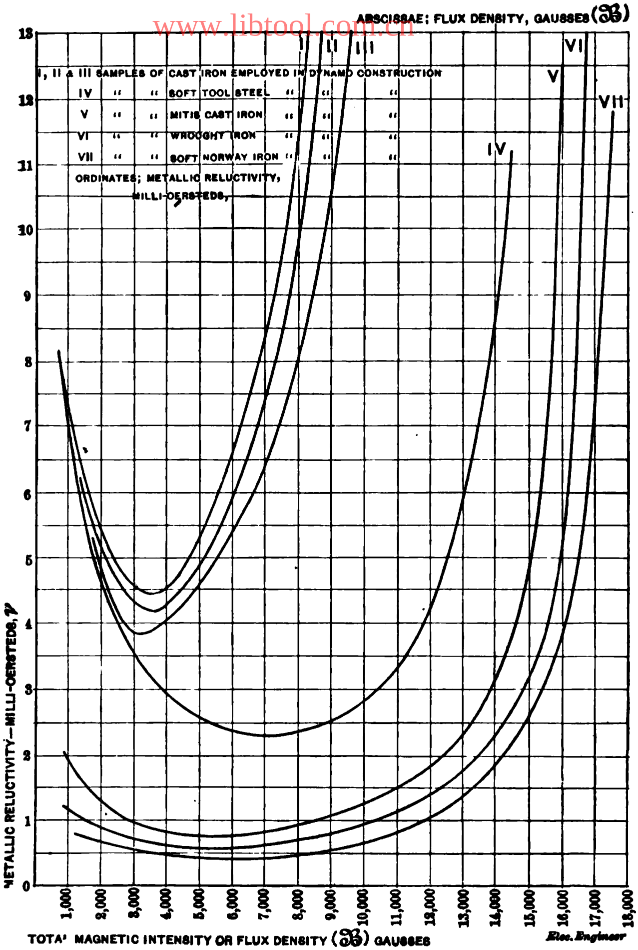
corresponding to $i = \frac{e}{\rho}$ for the electric circuit, we obtain

$$\nu = \frac{a}{1 - b\,\mathcal{B}}. \tag{6}$$

109. Fig. 39 represents a series of curves showing the reluctivity of various samples of iron and steel at different flux densities up to 19 kilogausses, taken from actual observations with materials employed in dynamo construction. These curves conform to equation (6), through the ascending branch, the corresponding flux densities of which are those practically employed in designing dynamo machinery. The descending branches are not expressed by equation (6). They belong to the reluctivity at early stages of the magnetizing force or flux density.

In order to show the application of the preceding formulæ, we will consider some cases similar to those which may arise in practice. We will first take the simple case of the ferric circuit, in anchor ring form, shown in Fig. 40, uniformly wound so as to have no leakage.

110. The corresponding case of electric flux is shown in Fig. 41, where a number of voltaic cells are connected in a circle in series. Here, neglecting the influence of temperature, the resistance of the circuit becomes independent of the current density. The case is



Elec. Engineer

FIG. 39.

Curves of reluctivity in iron and steel in relation to flux density, from measurements by Kennelly.

not an exact analogue unless the reluctivity of the electric conductor be modified to suit the magnetic intensity. Suppose this ring composed of Norway iron, to be of the dimensions shown, and wound with 300 turns of insulated wire which carries a current of 4 amperes. The M. M. F. for this winding will be 1,200 ampere-turns = 1508.4 gilberts. The magnetizing force

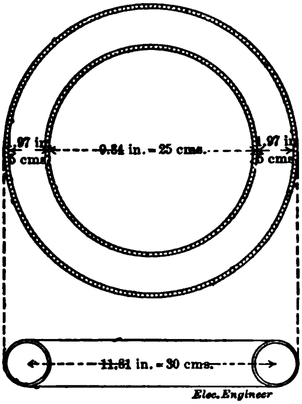


FIG. 40.

Sections of a Norway iron ring. Ferric magnetic circuit. Mean circumference 94.25 cms. Cross-section 19.635 sq. cms.

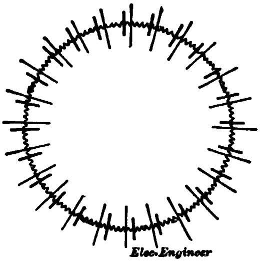


FIG. 41.

Diagrammatic representation of electric circuit. The analogue of magnetic circuit in Fig. 40.

will be this M. M. F. divided by the length of the magnetic circuit, which will vary between the limits of the outer and inner circumferences. Taking the mean circumference, the mean prime intensity will be $\frac{1508.4}{94.25} = 16.01$ gausses, and this would be the flux density if the ring were made of wood instead of iron. By reference to Fig. 38, it will be seen that, at this magnetizing force, the reluctivity of Norway iron is 0.00121; and, since the cross-section of the core is 19.635 sq. cms. the reluctance of the circuit is $\frac{94.25}{19.635} \times 1.21 = 5.807$ millioersteds = 0.005807 oersted, and the flux produced in the circuit will, therefore, be $\frac{1508.4}{0.005807}$ = 272,200 webers or 272.2 kilowebers, with a mean flux density of $\frac{272.200}{19.635}$ = 13,860 gausses, or webers per square centimetre of cross-section, *i.e.*, 13.86 kilogausses.

111. Taking now the case of an electro-magnet of the dimensions shown in Fig. 42, the magnetic circuit having two air-gaps in series, we will first assume

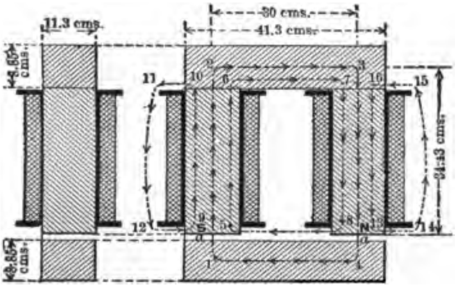


FIG. 42.

Electromagnet of wrought iron, æro-ferric circuit. Air gaps ½ in. = 1.27 cm. Mean length of magnetic circuit 140.24 cms. Cross-section of magnetic circuit 25 sq. cms.

that the leakage is negligibly small. The magnetic circuit is, therefore, considered as entirely confined to the path of the arrows, 1, 2, 3, 4, leaving the other paths for later consideration. Let us suppose that it is required to find the M. M. F. which will be necessary to produce a flux of 250 kilowebers, through the keeper. Assuming that this magnet is entirely constructed of wrought iron, and that the air-gap is fixed, as, for example, by means of wooden wedges, so that the keeper is unable to approach the poles, then the reluctance in each gap is $\frac{1.27}{25}$ = 0.0508 oersted. The density of the flux in the iron of the circuit is 10 kilogausses, and by referring to

Curve VI., Fig. 39, it will be seen that at this density the reluctivity of ordinary wrought iron is 0.00095. Since the mean length of the circuit in the iron is 137.7 cms., and its cross-section is 25 sq. cms., the reluctance of the iron will, therefore, be $\frac{137.7}{25} \times 0.95 = 5.23$ milli-oersteds = 0.00523 oersted, and the total reluctance of the circuit 0.1068 oersted. The M. M. F. required to send

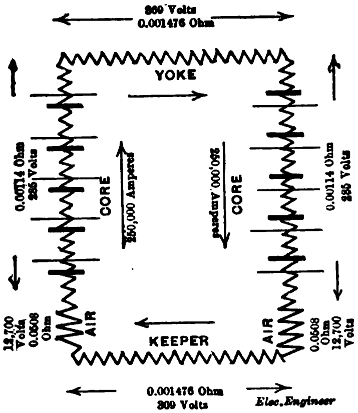


FIG. 43.—ELECTRIC CIRCUIT ANALOGUE. WITHOUT LEAKAGE.

250,000 webers through this circuit, will be 250,000 × 0.1068 = 26,700 gilberts = 21,256 ampere-turns. If the spools have the same winding there must be 10,628 ampere-turns on each spool.

The corresponding electric case is shown in Fig. 43.

112. Let us now assume that the electro-magnet possesses an appreciable leakage, and let us assume that this leakage takes place, as shown diagrammatically in Fig. 42, along the paths 5, 6, 7, 8,—9, 10, 11, 12— and 13, 14, 15, 16. Let it be ascertained that this leakage amounts to 33⅓ per cent. of the total flux, so that for

every 100 webers of flux in the interior of the field cores only $66\frac{2}{3}$ pass through the keeper. It is required to find the M. M. F., which will enable 250 kilowebers, as before, to pass through the keeper under these circumstances.

The effect of leakage is not only to reduce the effective cross-section, which may carry the main circuit flux, but also, owing to the increase in density, to increase the reluctivity of that reduced cross-section.

Similarly, if it be known that the leakage flux through the yoke, in the path 6, 7, is 50 kilowebers, the total flux through the yoke will be 300 kilowebers, and the flux density there $\frac{300}{25} = 12$ kilogausses. At this density, the reluctivity of wrought iron by Curve VI., Fig. 39, is seen to be .001316.

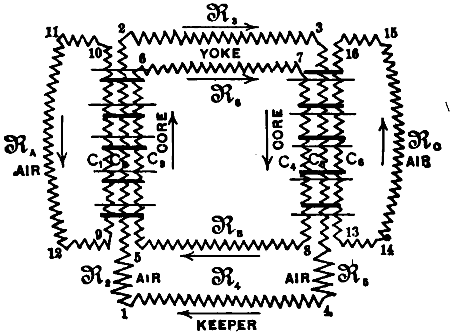


FIG. 44.—ELECTRIC CIRCUIT ANALOGUE. WITH LEAKAGE.

113. Referring to Fig. 44, which represents the electrical analogue of this case, observe that each core may be regarded as the seat of an E. M. F. impressed on three independent circuits, numbered to correspond with Fig. 42. $\mathcal{R}_A$, $\mathcal{R}_B$, and $\mathcal{R}_C$, are fixed reluctances through air, depending upon the dimensions and arrangement of

the various parts of the magnet. They have perfectly definite values, but these values may be very tedious and difficult to compute. The reluctances C_1, C_2, C_3, C_4, C, C_6, R_3 and R_6 depend upon the share of iron allotted to each branch circuit, and also to the flux density. We proceed to determine the various reluctances C'_2, R_3, C_5, R_5, R_1, and R_2, in the main circuit, and calling their sum $\mathcal{R}$, we have the simple relation

$$\Phi_{1,2,3,4} = \frac{\mathcal{F}}{\mathcal{R}}.$$

Portion.	Length, Cms.	Total Section, Sq. Cms.	Share of Section Carrying Main Flux Sq. Cms.	Total Flux, Kilo-webers.	Density, Kilo-gaus. es.	Reluctivity, Milli-oersteds	Reluctance Oersteds.
Core	30	25	16.667	375	15	3.077	$\frac{30}{16.667} \times \frac{3.077}{1000} = 0.005539$
Yoke....	38.85	25	20.833	300	12	1.316	$\frac{38.85}{20.833} \times \frac{1.316}{1000} = 0.002454$
Core	30	25	16.667	375	15	3.077	$\frac{30}{16.667} \times \frac{3.077}{1000} = 0.005539$
Air-gap..	1.27	25	25	250	10	1000	$\frac{1.27}{25} \times 1 = 0.050800$
Keeper ..	38.85	25	25	250	10	0.95	$\frac{38.85}{25} \times \frac{0.95}{1000} = 0.001476$
Air-gap.	1.27	25	25	250	10	1000	$\frac{1.27}{25} \times 1 = \frac{0.050800}{0.116608}$

To force 250 kilowebers through the main circuit through this reluctance, a M. M. F. will be needed of 250,000 × 0.116608 = 29,152 gilberts, or 23,200 ampere-turns—11,600 to each spool.

SYLLABUS.

The fundamental equation of the magnetic circuit is

$$\Phi = \frac{\mathcal{F}}{\mathcal{R}} \text{ or the webers} = \frac{\text{gilberts}}{\text{oersteds}}.$$

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ELECTROMAGNETS.

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114. The polarity of an electromagnet may be determined by noticing the character of the magnetizing coils, whether right-handed or left-handed, and the direction in which the current traverses them. A right-handed helix carries the flux in the same direction as that in which the current advances along the helix, while a left-handed helix carries the flux in the opposite direction of the advance of the current.. As already pointed out, the flux in an electromagnet is of two distinct characters; namely, the prime or magnetizing flux, and the induced or structural magnetic flux, which is called into play by the action of the prime flux.

115. Electro-magnets may be divided into two classes; namely, the tractive and the portative; the former are designed to exert a pull on their armature at some distance from the poles, and the latter are designed to support a pull upon the armature when the armature is placed in contact with the poles.

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In all practical cases an electromagnet exerts a pull upon the iron or steel of the armature whether the armature be separated from the poles by an air-gap, or whether it be in actual contact with the poles, and the same process of calculation has to be employed in each case in order to compute the attractive force. This process is substantially as follows: The reluctances of the different parts of the circuit have to be determined and summed, the M. M. F. acting in the circuit has then to be ascertained, and from these the flux in the circuit is deduced. From the flux and its distribution, the flux density in the air-gap between the keeper and poles has to be found, and from this flux density, the intensity of the attractive force is determined from point to point. The total attractive force on the armature will be the surface integral of this attractive force over the area of the attracting surfaces.

116. The fundamental law of attractive force is as follows: At any element of surface on iron or steel at which flux enters or emerges perpendicularly, the attractive force in dynes, exerted upon the element, will be the product of the elementary surface area into the square of the flux density (expressed in gausses), divided by 8π; that is,

$$d F = d S \frac{\mathfrak{B}^2}{8\pi} \text{ dynes,}$$

where $\mathfrak{B}$, is the normal flux density; $d F$, the element of attractive force, and $d S$, the element of surface in square centimetres; and this force will be exerted along the flux paths, or perpendicular to the surface. If, however, the entering or emerging flux makes an angle θ, with the normal to the surface, then the above rule requires slight modification. The equivalent normal surface, on which the attractive effort is exerted, is $d\,S \cos\theta$, so that the attractive force becomes,

$$d\,F = \frac{d\,S \cos\theta\, \mathcal{B}^2}{8\,\pi} \text{ dynes,}$$

exerted in the direction of the flux-paths of which the component perpendicular to the surface is,

$$d\,F = \frac{d\,S \cos^2\theta\, \mathcal{B}^2}{8\,\pi} \text{ dynes.}$$

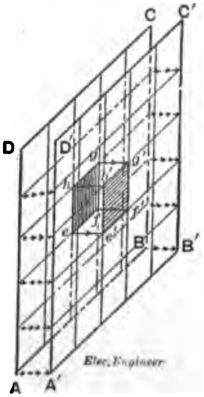


Fig. 45.
Flux normal to opposed plane parallel polar surfaces.

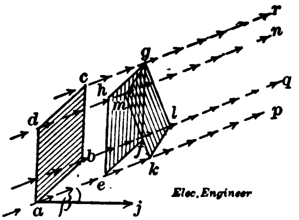


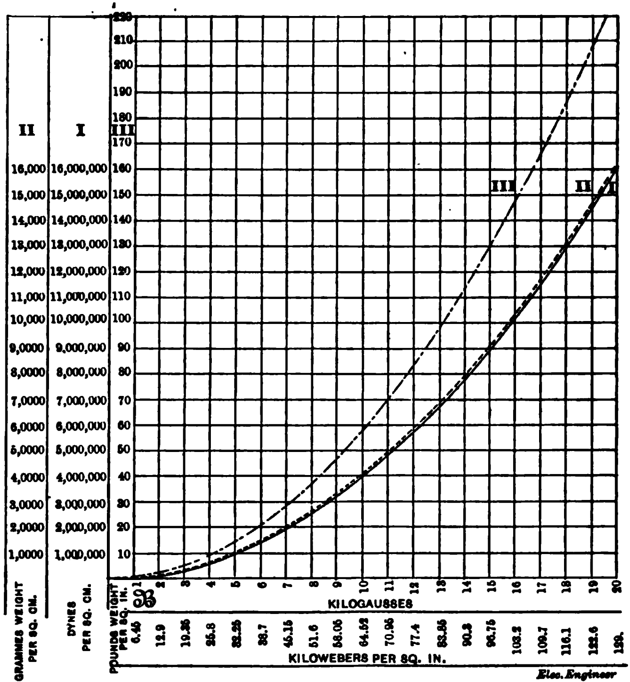
Fig. 46.
Flux oblique to polar surfaces.

117. Let A B C D, and A′ B′ C′ D′, Fig. 45, be portions of two parallel plane polar faces of iron between which the magnetic flux passes perpendicularly across the intervening space A A′, or C C′. If the flux intensity is uniform over these surfaces and equal to five kilogausses, then the mechanical force exerted between any pair of opposed unit areas, such as the shaded portions $e\,f\,g\,h$, and $e'\,f'\,g'\,h'$, each one square centimetre, will

be $\frac{5000 \times 5000}{8\,\pi}$ or approximately 994,800 dynes, or 1,015 grammes weight (2.238 pounds), at Washington. Since the total area A B C D, or A′ B′ C′ D′ is 25 square cms., the total mechanical force exerted between these surfaces will be 25.375 kilogrammes (55.95 pounds). The magnitude of the attractive force does not depend upon the distance A A′, separating the polar faces, nor does it depend upon the direction of the flux between them. All that is essential is that the flux should be perpendicular to the faces. Increasing the air-gap will, in practice, usually diminish the total flux, and, therefore, the flux intensity over the surfaces, also causing the flux paths to deviate from the perpendicular by lateral diffusion; but if these secondary effects could be compensated and removed, the attraction between the surfaces would not vary with the length of air-gap.

118. Fig. 46 represents a case where the flux passes between the parallel polar faces at an angle β, with their normal. If $a\ b\ c\ d$, and $e\ f\ g\ h$, are areas limited by the flux-paths $a\ e\ p$, $b\ l\ q$, $c\ g\ r$ and $d\ h\ n$, then the attractive force between these areas will be such as would be experienced by two surfaces each of the area $k\ l\ g\ m$, standing perpendicularly across the flux. If $a\ b\ c\ d$, and $e\ f\ g\ h$, have each an area of one square cm., the surface $k\ l\ g\ m$, will have an area of $\cos\beta$ square cms. With 10 kilowebers passing through each of the shaded areas, the flux density will be $\mathcal{B} = \frac{10000}{\cos\beta}$. The attractive force between two opposed parallel surfaces of area $k\ l\ g\ m$, will be $\frac{\mathcal{B}^2}{8\,\pi}\cos\beta$ ex-

erted along the flux paths. The component of this tension exerted across the actual surfaces *a b c d*, and *e f g h*, will be $\frac{\mathfrak{B}^2}{8\pi}\cos^2\beta$, and the component tending to make



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Curves representing the intensity of magnetic stress, for all values of $\mathfrak{B}$ from O to 20 Kilogausses. O to 129 Kilowebers per sq. in.
I in dynes per square centimetre. II in grammes weight per square centimetre
III in pounds weight per square inch.

FIG. 47.

these surfaces shear across one another would be $\frac{\mathfrak{B}^2}{8\pi}\sin\beta\cos\beta$.

In practice, the useful flux exerting tractive force between the polar surfaces of electromagnets may be considered as crossing those surfaces at right angles, so that the simpler formula may generally be applied.

119. As a consequence of the preceding formulæ it is evident that the active polar surfaces of a portative electromagnet should have as great an area as possible, provided that the flux density over them be made as great as possible. In other words, the polar surfaces should have maximum areas consistent with their magnetic saturation. The curves in Fig. 47 show that at a density of 18 kilogausses, which is readily obtainable in ferric magnetic circuits employing soft Norway iron, the attraction becomes 13.14 kilogrammes per square cm., or 186.6 pounds per square inch of opposed polar surfaces.

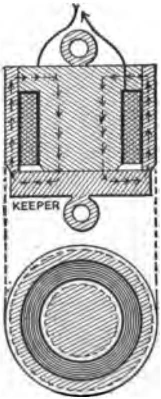

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FIG. 48.

Section of Portative Electromagnet through Axis, and Polar Surfaces.

120. A convenient practical form of electromagnet for sustaining heavy weights is shown in Fig. 48. The magnetic circuit is indicated by the lines of arrows.

The external surface of the magnet when the keeper is in place, being entirely of iron, the magnet is usually described as belonging to the *ironclad* type. The contact polar surfaces should be carefully planed and kept clean if the best attractive results are to be obtained. The space allowed for the exciting coil is made as small as is consistent with saturation of the polar surfaces. The limiting M. M. F. that can be employed for a given winding space depends upon the heating of the coil by the current, and not upon the size of the wire. Practically, however, a large wire with few turns can be better protected against damage from a high temperature, than a small wire with many turns. It is essential that the flux density should be a maximum in the circuit at the polar surfaces, and for this reason the surface area of the inner core and outer ring should be kept equal while a slight constriction in the iron should be made at the poles.

If such a magnet have a cross-sectional area of 20 square cms. at the inner or core polar surfaces, and also 10 square cms. at the outer or annular polar surfaces, the portative power of the magnet may readily be 40 × 15 = 600 kilogrammes weight.

121. When an electromagnet has to exert a tractive force upon its armature at a distance, through one or more air-gaps of given length, the best area of polar surface to employ with a fixed M. M. F., and the size of magnet are those which make the reluctance of the air, equal to the reluctance of the iron in the circuit. If, for example, an electromagnet has two poles each four centimetres in diameter, and the air-gap or distance between poles and armature be 0.25 cm., then the area of

each pole face will be 12.57 square cms., and the reluctance of the air 0.0398 oersted. If the reluctance in the iron be, say, 0.050 oersted, under these conditions with the M. M. F. employed, it will be advantageous to increase the air reluctance by constricting the polar surfaces until the air and iron reluctances equate. This assumes, however, as negligible, leakage and diffusion of flux at the polar surfaces. In consequence of leakage and diffusion, it is preferable to make the air reluctance somewhat less than the iron reluctance.

SYLLABUS.

The direction of magnetic flux within a coil is in the direction along which the current traverses the coil if the coil be right-handed, and opposite to the direction of the current if the helix be left-handed.

The fundamental law of tractive force upon a magnetized surface, at which flux enters or issues perpendicularly, is $d\,F = d\,S\frac{\mathfrak{B}^2}{8\,\pi}$ dynes.

Portative electromagnets are designed to have as large an area of saturated polar surfaces as possible.

Powerful tractive magnets are designed to have their reluctance about equally divided between air and iron.

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122. Whenever relative motion exists between magnetic flux and an electric conductor, so that one moves across the other, an E. M. F. will be set up in the conductor. This relative motion between flux and conductor may occur in two ways; namely,

(*1.*) When the conductor moves across the flux.
(*2.*) When the flux moves across the conductor.

Both cases may occur together, but in (1) we suppose that the flux may be considered as at rest, and in (2), that the conductor may be considered at rest.

123. We will now consider case (1), in which the conductor moves across a magnetic flux. Although the mechanism by which E. M. F. is induced is unknown, yet the E. M. F. produced is directly proportional to the total amount of flux per second cut by the conductor, and this clearly depends on two quantities; namely, upon the velocity of the conductor across the flux, and, upon the intensity of the flux.

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124. Consider, first, a uniform magnetic flux, whose intensity at a given point is equal to $\mathcal{B}$, and is directed as shown by the arrows, Fig. 49. A rectilinear conductor A B, normal to the flux, is moved in a direction normal to the flux with a velocity v, which would carry it in one second to A′ B′. Then the total amount of flux cut per second would be the amount passing through the rectangle A B B′ A′, and this is clearly equal to

$$e = v\, l\, \mathcal{B} \qquad \text{C. G. S. units of E. M. F.}$$

Suppose, however, that a conductor lying in a position oblique to the flux, as shown in Fig. 50, is moved in a direction normal to the uniform flux with a uniform velocity v; then the total amount of flux cut per second will be that enclosed by the rectangle A b b' A′, where A b, is the virtual length of the conductor equal to A B cos β, that is, the projected length normal to the flux, and the E. M. F. is equal to

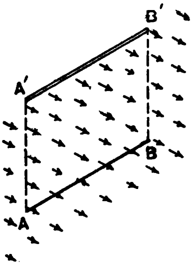

Fig. 49.

Conductor normal to flux, moving in direction normal to flux.

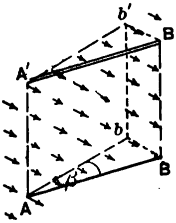

Fig. 50.

Conductor oblique to flux, moving of conductor in direction normal to flux.

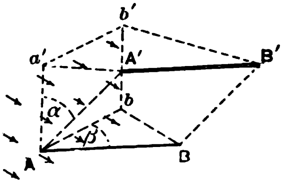

Fig. 51.

Conductor oblique to flux, moving in direction oblique to flux.

$$e = v\, l \cos \beta\, \mathcal{B} \qquad \text{C. G. S. units.}$$

If both the position of the conductor, and its motion are oblique to the direction of the flux, as in Fig. 51, then the total flux cut per second will be that enclosed by the rectangle A b b' A$'$, where A B, is the virtual length of the conductor, as before, and A a', is the virtual velocity normal to the flux, or A A$'$ cos α. So that

$$e = v \cos \alpha \, l \cos \beta \, \mathfrak{B} \qquad \text{C. G. S. units.}$$

Since one volt equals 10^8 C. G. S. units of E. M. F., the E. M. F. as above obtained, must be divided by 10^8 in order to obtain its value in volts.

FIG. 52.

125. The direction of the induced E. M. F. varies both with the direction of the flux and with the direction of the motion. The simplest rule for memorizing this direction is, probably, *Fleming's hand rule.*

If the right hand be held, as shown in Fig. 52, with the extended *f*ore-finger pointing in the direction of the *f*lux, and the thu*m*b in the direction of the *m*otion, then the E. M. F. induced will be directed along the direction in which the middle finger points.

126. In practice when a conductor is moved through a magnetic flux, it generally happens that neither the intensity of the flux nor the velocity in the direction of motion is uniform. Nevertheless, the above law is true for any small element of the conductor at any moment, when its direction and the intensity of the flux in which it moves are taken into account.

127. Turning now to case (2) where the flux moves across a conductor. The fundamental rule remains the same as in the preceding case; if v, be the velocity of the field at any point, where the intensity is $\mathcal{B}$, and l be the virtual length of conductor at right angles to the flux and the motion, then the E. M. F. in C. G. S. units is $v \mathcal{B} l$, as before.

128. In order that the induced E. M. F. in a conductor may produce a current, the circuit of that conductor must be closed, that is, a conducting loop must be formed, although only a portion of this loop may be active in cutting through flux and generating E. M. F. It is obviously the same whether we speak of the rate at which the portions of the loop are cutting through flux, or of the rate at which a loop is enclosing flux, since the sum of all the lines cut through per second around a loop must be equal to the amount of flux enclosed by the loop in that time. Similarly, when flux is withdrawn from a loop, the E. M. F. will be introduced around the loop in the opposite direction. All these results may be included in the following equation:

$$e = \frac{d\,\Phi}{d\,t},$$

where Φ, is the flux enclosed by the loop (webers) in the positive direction, and $\frac{d\,\Phi}{d\,t}$ the instantaneous rate of change of that flux.

129. A consideration of Figs. 49, 50 and 51 will render it evident that E. M. F. is never produced by the relative motion of magnetic flux and a conductor, unless a change exists in the amount of flux enclosed by the conducting circuit. It is evident, therefore, that if the conducting ring, shown in Fig. 53, though normal to the uniform magnetic flux, and moving at right angles to such flux, so as to cut the flux, has, nevertheless, no resultant E. M. F. generated in it, since at any moment of time the flux it encloses is constant; or, if regarded from the standpoint of Fig. 49, the E. M. F. generated by the cutting in the upper half of the loop, is exactly equal and opposite to the cutting in the lower half.

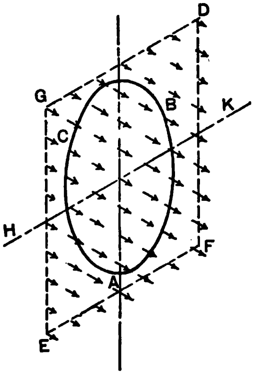

FIG. 53.

Conducting ring normal to flux, moving in plane normal to uniform flux.

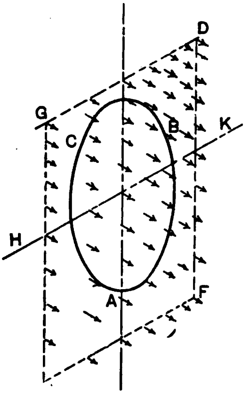

FIG. 54.

Conducting ring normal to flux, moving in a plane normal to a non-uniform flux.

The conducting ring, shown in Fig. 54, placed normal to the non-uniform flux, if moved in its own plane in any direction except in the directions F G, or G F, will have a resultant E. M. F. generated in it, since the amount of flux enclosed by the loop will otherwise increase or diminish.

130. If a conducting loop, placed in a uniform magnetic flux, be rotated about any axis, as, for example, about the axis H K, in Fig. 55, a resultant E. M. F. will be induced in it, since the amount of flux enclosed by the loop will vary. Thus, in rotating the loop from A to A′, the flux enclosed will be reduced from the total area A B C, to the virtual area *d e f*. Since the value of the E. M. F., at any instant, is the time rate of change of the flux enclosed by the circuit, it is evident that the maximum E. M. F. is produced when the plane of the loop is parallel to the direction of the flux.

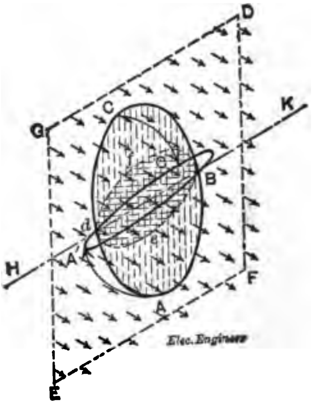

FIG. 55.
Rotation of a Loop in a Magnetic Flux.

131. When a current through a conductor is changing its strength, there will, as we have already seen, be a change in the amount and intensity of the flux surrounding the conductor. Since the entire conducting circuit, in which the current flows, may be regarded as a loop, or combination of loops, these variations in the flux linked with the conducting loop will induce in the conductor an E. M. F. of exactly the same strength as though the current remained unchanged, but the same flux variations passed through the conducting loop. This E. M. F. induced in a circuit by variations in its current strength is known as a *self-induced* E. M. F., or an E. M. F. of *self-induction*.

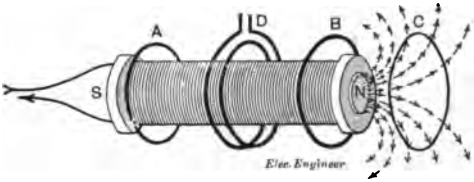

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FIG. 56.
Projection of Magnetic Flux through Conducting Loops.

132. If a current be sent through the electromagnetic helix, shown in Fig. 56, in such a direction as to produce the poles S, N, then the flux established is shown in part by the curved arrows, in the neighborhood of the north pole. As this flux emerges, it will pass through the loops A, D, B, and C; but whereas the same amount of flux passes through each of the two loops A, or B, is greater than that which flows through C, the E. M. F. generated in A, or B, during the change will be greater than that in C, while the double loop D, will have double the E. M. F. in it. The direction of the E. M. F. in these loops,

on making the magnet circuit, is opposite to that on breaking: for in one case the flux passes through the loop to the right, and in the other case, to the left. Even when the existence of the flux that passes through the loop cannot be determined by the aid of a compass needle, as, for example, in Fig. 57, where the closed circular coil is linked with three separate conducting loops *j k l*, *h e f g*, and *m*, yet on varying the current in the coil the same E. M. F. will be induced in all three loops. This case is apparently an independent demonstration of the fact that the velocity of the propagation of magnetic disturbances is finite.

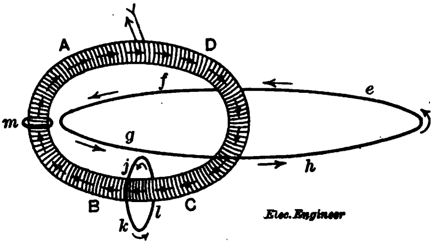

FIG. 57.
Closed Circular Coil linked with three Loops of Conductor.

SYLLABUS.

If Φ, be the total flux in webers linked with a conducting circuit in the positive direction, then $\frac{d\,\Phi}{d\,t} \times 10^{-8}$ is the E. M. F. induced in that circuit in International volts.

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A. E. Kennelly, F. R. A. S.

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THE DYNAMO.

133. A continuous current dynamo, depending as it does for the production of its E. M. F. on the movement of conductors through a magnetic flux, determines the magnitude of that E. M. F. in accordance with the relation,

$$E = \Phi\, n\, w \qquad \text{C. G. S. units,}$$
$$= \Phi\, n\, w \qquad 10^{-8} \text{ volts.}$$

where Φ, is the total useful magnetic flux through one pole passing into the armature (webers); n, the number of revolutions made by the armature per second; and w, the number of conductors on the surface of the armature counted in one complete revolution. That is to say, the E. M. F. is directly proportional to the product of the total useful magnetic flux through each pole, the rate of speed of the armature, and to the number of turns of wire. Thus, Fig. 58, represents a four-pole generator for 550 volts. The useful flux through each

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pole may be, say, 55 megawebers; there are, say, 200 conductors embedded in the surface of the armature, and the speed of rotation is assumed to be 300 revolutions per minute, or five revolutions per second. The E. M. F. generated by the machine is, therefore,

$$E = 5 \times 200 \times 55 \times 10^6 \times 10^{-8} = 550 \text{ volts.}$$

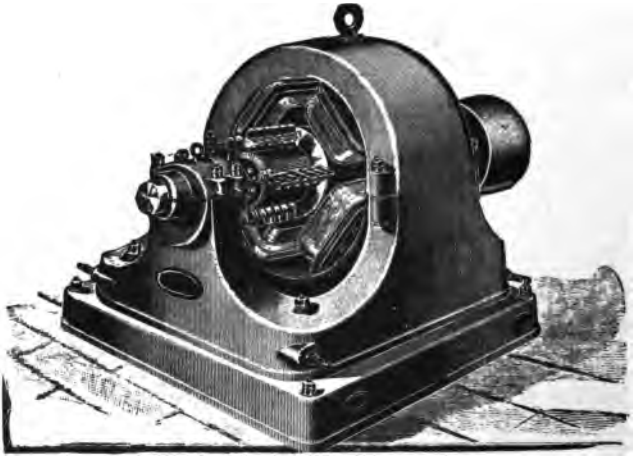

Fig. 58.
550 Volt Four-Pole Generator.

134. The output of a generator, or its capacity, is usually expressed in kilowatts, and is the electrical activity which the machine can maintain at its terminals. This, within wide limits, is independent of the electromotive force; that is to say, the 100 K. W. machine can be constructed for 1,000 volts and 100 amperes, or for 100 volts and 1000 amperes, or 50 volts and 2,000 amperes output. There is, however, a practical limit to the E. M. F. which can be obtained from a machine without

altering its rating; for, very small E. M. F.'s may require such massive construction in their commutators and adjacent conducting parts, to carry off the enormous corresponding currents generated, that the whole structure of the machine may require to be modified, while, when high electromotive forces are reached the great thickness of insulating material required, may so reduce the available winding space as to seriously reduce the output.

135. When a dynamo-electric generator is operated on open circuit, it is, of course, doing no work, since the resistance of its external circuit is infinite. As the resistance of the external circuit is decreased, the amount of work in its circuit increases, until, when the external circuit has no resistance, or the machine is short-circuited, the amount of work in the electric circuit of the machine would be a maximum, and, if it could be reached, would be expressed by the formula, $P = \frac{E^2}{r}$ watts; where E, is the E. M. F. generated in the armature, and r, is the internal resistance of the machine. This activity may be called the *electric capability* of the machine, and is similar in nature to the electric capability of a voltaic cell. Of course, no machine of any considerable size could be made to run on short-circuit.

136. Since, useful output can be computed as a certain fraction of the electrical capability, depending on the output of the machine, the electrical capability is by no means of merely theoretical interest. The fraction of the electrical capability which represents the output of a machine, may be called the *coefficient of reduction from capability to output*, and varies with the

size of the generator, and the details of its structure. For example, in a particular series of bipolar generators, of different sizes, this fraction is 0.15 for generators of one K. W. capacity, and reduces to 0.034 for generators of 100 K. W. capacity. This reduction in the coefficient is partly due to the fact that as the size of the machine increases, the active surface of the armature, offered for the dissipation of heat, increases less rapidly than the mass in which the heat is developed, thus necessitating a relatively diminished output.

The electrical capability of a generator is independent of the character of the winding, provided the amount of winding space remains constant. This is true, however, only so long as the proportion of winding space, devoted to insulation, remains constant through all sizes of wire; thus, if the number of turns in the armature be doubled, the E. M. F. will be doubled, but the resistance will be quadrupled, since there will be twice as great a length of wire of half the cross section; hence the ratio of E^2 to r, remains the same.

137. Since the E. M. F. of a continuous current generator is proportional to $\Phi\, n\, w$, and its resistance is proportional to $\frac{w\, l}{a\, p^2}$, where l, is the length of one turn of conductor, a, its cross-section, and p, the number of poles, the electrical capability of a machine is proportional to $\Phi^2\, n^2\, w^2\, \frac{a\, p^2}{w\, l} = \Phi^2\, n^2\, p^2\, \frac{w\, a}{l}$, that is, proportional to $C\left(\frac{\Phi\, n\, p}{l}\right)^2$; where C, is the weight of copper on the armature. Consequently, for a given weight of

copper conductor on the armature, and a given cross-section of armature core, the output of the machine increases as the square of the speed of rotation, and as the square of the number of poles in the field frame. Various considerations, however, incidentally limit the range over which this rule can apply. Thus an increased capability and output may be attended by increased heating or sparking at the brushes, so that the coefficient of output may be lowered. A doubled speed of rotation would double the E. M. F. of the machine, and would quadruple the electric capability, enable twice the current strength to be delivered at full load at the same electrical efficiency, thus quadrupling the output; but, if, at the doubled speed, and with the doubled current, the armature unduly heated, the safe load, and coefficient of reduction, would require to be lowered.

138. In the design of a dynamo, the problem which presents itself is to produce the desired electromotive force at a given speed of rotation of the armature, determined by mechanical considerations, and to maintain a given current strength at that E. M. F. The problem, therefore, resolves itself into the proper proportioning of the amount of flux, the number of turns and size of conductor, and the number of poles in the machine. The value of the output which it is desired to produce, will, in reality, determine whether the machine is to be bipolar, or multipolar, since large sizes of bipolar machines are usually objectionable, partly owing to the large dimensions required. Having determined upon the number of poles, the total flux and the number of turns on the armature only remain to be determined.

The proper resistance of the armature for the type of machine required, is known by reference to tables of coefficients for the electrical capability and output, and, from this resistance one relation between the total flux, the length and the cross-section of wire is given. By trial the size of armature is found upon which the amount of wire of the necessary cross-section and number of turns is arrived at.

139. The magnetic flux requisite under a given condition of speed and armature turns, now remains to be provided for. The first step is to provide a path of sufficient cross-sectional area through the iron field frame and armature. In order to assign the proper flux density in the magnet cores, it is necessary to assume a certain quantity of leakage. The proportion of leakage is generally taken from observations made on machines of similar type, and is the principal source of uncertainty in the design of any given generator. The ratio of total flux to the useful flux passing through the armature varies from 1.2 to 2.1 in different types of machine. When this ratio is known, the total flux passing through the field cores is known, and the area of cross-section necessary for a given flux density is arrived at.

140. It will be seen that the reluctivity of the iron employed in the framework of the machine is a very important consideration, since on this depends the area of cross-section that must be employed for a given total flux. The core of the armature is always constructed of a given quality of laminated soft iron, with the flux passing parallel to the laminations. The field magnets are sometimes entirely constructed of cast iron, sometimes of cast steel, and sometimes of wrought iron in

the core, united with cast iron in the yoke. The choice of materials depends upon the character of the work the generator is to perform. In the case of cast iron, a flux density of 7.5 kilogausses is approximately the practical limit, while in wrought iron, or cast open hearth steel, a density as high as 17 kilogausses can be employed. The balance of advantage lies between cost of materials and limitations of size and weight.

141. Slight impurities in wrought iron or soft cast steel have a marked influence upon their reluctivity. The most common impurities in iron are carbon, silicon, sulphur, phosphorus and manganese. Of these, carbon produces the greatest influence on reluctivity, and taking the reluctivity of pure wrought iron at an intensity of 7.5 kilogausses as 0.0005, the influence of small quantities of these impurities on the reluctivity appear to be expressed as follows: Carbon, 0.25; silicon, 0.11; manganese, 0.06; phosphorus, 0.04, and sulphur, inappreciable.

Thus, one per cent. of carbon added to pure wrought iron might be expected to increase its reluctivity at 7.5 kilogausses from 0.0005 to $0.0005 + \frac{0.25 \times 1}{100} = 0.003$; *i.e.*, to three millioersteds in a cubic centimetre of material. These values can only be regarded as approximations. They appear to vary in different qualities of steel.

SYLLABUS.

The electromotive force produced by a dynamo, expressed in C. G. S. units, is the product of the flux, the revolutions per second and the number of conductors counted once around the armature, divided by the number of poles. The electrical capability of a dynamo, in watts, is equal to the square of its E. M. F. in volts, divided by its resistance in ohms.

The ratio of the total to useful flux varies in different machines between 1.2 to 2.1.

The flux density that can be employed without unduly increasing the reluctivity of the circuit is about 7.5 kilogausses in cast iron, and up to 17 kilogausses in wrought iron or soft cast steel.

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THE DYNAMO.

142. The commercial efficiency of a dynamo-electric machine, like that of any other electric source, is the ratio between the output and the intake, and varies, in different sizes of generators, between, say, 0.5 for a one KW. machine, to 0.98 for a generator of, say, 3,750 KW. In other words, the commercial efficiency of a machine is $\frac{\text{Output}}{\text{Intake}} = \frac{\text{Intake—Losses}}{\text{Intake}}$.

In order, therefore, to determine the efficiency of a machine, the intake being known, it only remains to determine the losses.

These are of three kinds; namely,

(*1.*) Mechanical losses, such air churning, brush friction, and journal friction.

(*2.*) Electrical losses of the type $i^2 r$; namely, losses in the armature winding, and in the field magnet winding; and losses, due to eddy currents set up in the metal by variations of flux.

(*3.*) Magnetic losses in the iron due to *hysteresis*.

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143. The mechanical losses, as a rule, are readily estimated. The loss by air churning is usually small, and, in large, slowly revolving armatures, may be neglected. Indeed, a small expenditure of energy in this direction, is, to a certain extent, advantageous, and is sometimes designedly incurred for the purpose of ventilating the armature, and, by consequent cooling, increasing the possible output. Brush friction is usually a large item of loss in very small machines, and an insignificantly small item in large machines. Journal friction can be readily estimated by the ordinary rules of mechanics, when the gravitational and magnetic forces, together with belt pulls, when such exist, are taken into account, together with the size of the shaft and length of bearings.

144. The electrical losses of the type $i^2 r$, are of three kinds:

First, those due to the passage of the armature current through its resistance, and of the field exciting current through the resistance of the field magnets.

Second, those due to the energy expended by wasteful currents in loops of conducting wire on the armature, when short-circuited by the action of the brushes on the commutator; and,

Third, those due to energy expended in the iron of the armature or pole-pieces, or in the copper wire on the armature, in setting up induction currents in them.

145. If the resistance of the armature of a 100 KW. machine be 0.05 ohm, and the E. M. F. of the machine be 500 volts, the current delivered by the machine will be 200 amperes. The electrical loss of energy in

the armature will, therefore, be 40,000 × 0.05 = 2,000 watts or 2 KW. = 2 per cent. of the output.

146. If the armature core were a solid mass of soft iron, then, from the variations of the magnetic flux produced in this mass during its revolution through the field, E. M. F.'s would be induced in it, which, acting through the very low resistance of so large a mass of metal, would generate powerful and wasteful currents. By laminating the substance of the core, in planes parallel to the magnetic flux, the E. M. F. in each lamina is reduced, and also the available cross-section for the action of the E. M. F. By thus building up the armature core of sheets of thin iron, the eddy current loss is brought down to very small limits. For the same reason the conducting wire on the surface of the armature, when of comparatively large size, needs also to be laminated by stranding. It is not usually necessary to insulate the separate strands from each other, as the E. M. F. in any particular cross-section of the wire is so small that the superficial layer of oxide of copper will interpose an effectual barrier to the passage of eddy currents. When the conductors are buried below the surface of the armature, as in grooved or toothed armatures, lamination of the conductor is not necessary, since the flux is almost entirely carried by the iron on one side or other of the conductor, and the transition is effected without cutting the substance of the conductor.

147. The third source of loss in the generator is purely magnetic, and is termed loss by hystere'sis. Hysteresis, meaning *a lagging behind*, is the lagging of the magnetism in a magnetic metal behind the magnetiz-

ing flux which produces it. Thus, on the reversal of the magnetizing flux exerted on a piece of iron, the zero of magnetism is reached at an instant of time sensibly later than the zero of magnetizing flux. This entails an expenditure of energy in the iron which takes the form of heat.

148. If an electric current be sent through a conducting loop, magnetic flux is produced through the loop, and, during the time the current strength is rising to its full value the rate at which flux is entering the loop will induce around the loop an E. M. F. of the type *e* volts. This E. M. F. is oppositely directed to the current *i*, which establishes it, and consequently the current does work upon the E. M. F. with an activity of *e i* watts. This energy is stored away in the air and the ether, as magnetic energy of the type $\frac{\mathcal{B}^2}{8\,\pi}$ ergs per cubic centimetre. On withdrawing the current and emptying the loop of flux, which occurs during the time the current is waning, an opposite E. M. F. is produced, aiding the current, doing work on the current and restoring the energy from the magnetic flux into the circuit. This interchange of energy from the circuit to the ether surrounding it, and thence back to the circuit is, so far as is known, apart from electromagnetic radiation unaccompanied by loss of energy. If, however, a bar of iron, or other magnetic material, be introduced into the loop, then the magnetizing flux due to the current passing through the loop, produces, as before, a magnetic flux through the loop, but this magnetizing flux acting on the iron, produces by the alignment of its molecules a powerful M. M. F. and flux in its

own direction. As before, the prime flux produces a counter-electromotive force, e, in the loop absorbing energy of the type $e\ i$, joules per second. The flux passing through the iron and loop also produces a more powerful E. M. F., E, volts, absorbing energy from the current at the rate $E\ i$, watts. This energy is stored in the magnetic circuit of the iron. The total counter-electromotive force produced will be $E + e$, volts, and the work expended

$$\int_0^T (E + e)\ i\ dt \text{ joules.}$$

On the withdrawal of the magnetizing current, the prime flux is withdrawn at the same rate as before, but the magnetic flux in the bar lags behind; *i.e.*, tends to persist, so that, although the current in the loop may be made to disappear entirely, the flux in the æro-ferric circuit does not totally disappear. Consequently, the rate at which the flux is poured out is less than that at which it was poured in, and the smaller E. M. F. thereby induced, restores to the circuit only a portion of the energy stored in the iron. That is to say, the lagging behind, or hysteresis, of the magnetism in the bar has caused energy to disappear from the electric circuit, or to be lost to it.

149. Let us now enquire what has become of the energy thus lost to the circuit. When, under the influence of the prime magnetizing flux, an alignment of the molecules of the iron has been produced, energy is stored in them. On the withdrawal of the prime magnetizing flux, the aligned molecules do not immediately break up or lose their alignment, but tend to remain fixed until the prime magnetic flux is sufficiently

far withdrawn to render their position untenable. They then suddenly break up their alignment and fall swiftly into new groups, but without uniform alignment. In

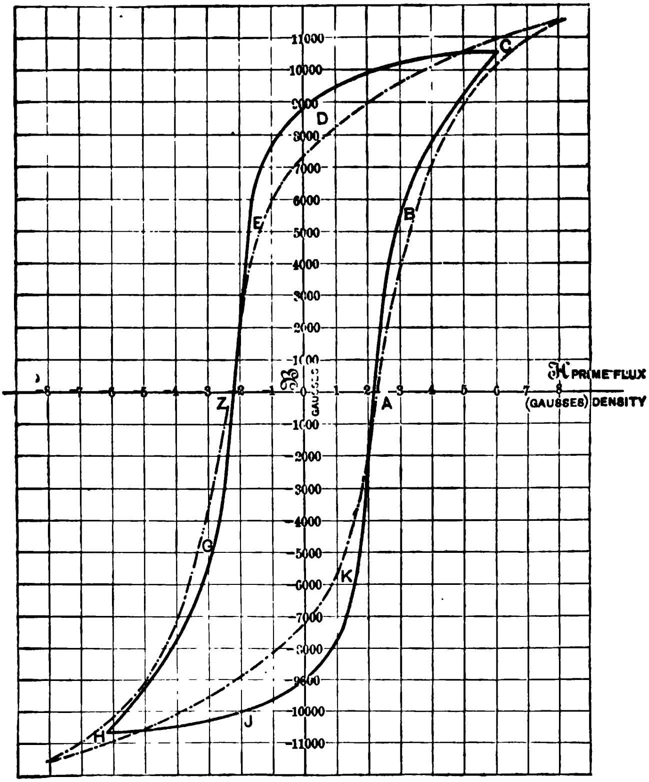

FIG. 59.

Hysteretic Diagram of Charcoal Iron Rings and of Hard Cast Steel.

Charcoal Iron:—Full line to indicated scale. From observations of Kennelly. $\mathfrak{H} \pm 6$, $\mathfrak{B} \pm 10{,}600$.

Hard Cast Steel:—Broken line, to 10 times indicated scale. From observations of Steinmetz. $\mathfrak{H} \pm 82$, $\mathfrak{B} \pm 11{,}500$.

this sudden relapse to the unmagnetized state, the molecules, in swinging around, acquire momentum which carries them past their new positions of equilibrium, thus causing them to oscillate to-and-fro about that position, and to dissipate their energy in the form of heat.

150. The magnetic changes which take place in their relation to magnetizing flux may be diagrammatically represented as in Fig. 59, which shows a hysteretic cycle for a soft iron ring.

151. At every reversal of the magnetization of the iron there is, therefore, an expenditure of energy in the iron. This is proportional in amount to the area of the hysteretic loop or the energy expended in the cycle. As the limits of flux density during the reversal are increased, the energy expended in the iron increases. If the iron be carried from an intensity of $\mathfrak{B} = + 5{,}000$ gausses to $\mathfrak{B} = - 5{,}000$ gausses, the range of reversal will be 10,000 gausses. If now the range be doubled, or increased to 20 kilogausses, the energy expended in the iron per cycle will be approximately trebled. The energy expended in a cubic centimetre of iron undergoing periodical reversals of magnetism is approximately expressed by the equation $W = \eta \, \mathfrak{B}^{1.6}$ ergs per c. c., where η, is a coefficient which varies from 0.002 for very soft iron to 0.080 in the hardest steels.

When, therefore, the core of an armature of soft iron having a total volume of, say, 8,000 c. c. makes 12 revolutions per second in a bipolar flux from $\mathfrak{B} = + 5{,}000$ to $- 5{,}000$ gausses, it will undergo one complete cycle or double reversal for each revolution. Consequently the expenditure of energy in the core by hysteresis, per revolution of the armature, will be $0.002 \times 5{,}000^{1.6} = 0.002 \times 828{,}600 = 1{,}657$ ergs per c. c.; or $8{,}000 \times 1{,}657 = 13{,}256{,}000$ ergs $= 1.3256$ joules per revolution, and at 12 revolutions per second a total hysteretic activity in the armature of 15.907 joules per second = 15.907 watts. For this reason the intensity in the armature is preferably kept much below saturation, in order to avoid the rapid increase in hysteretic loss at high densities according to the above rule.

152. If the three classes of loss of energy in a generator be summed, their total subtracted from the intake, is equal to the output, and this divided by the intake gives the commercial efficiency of the machine.

SYLLABUS.

The losses in a dynamo-electric machine are three; namely, mechanical, electrical and magnetic.

By hysteresis is meant the lagging of the magnetization behind the magnetizing force.

The loss of energy in a magnetic metal undergoing reversals increases approximately as the 1.6th power of the limiting flux density.

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ADVANCED GRADE.

THE DYNAMO.

153. The limitations to the output of a dynamo are of three kinds; namely, limitations by excessive drop or fall of pressure in the armature; limitations by excessive heating; and, limitations by excessive sparking.

When a powerful current passes through the armature of a generator, the fall of pressure, or drop in the resistance of the machine, may be so great that the limiting E. M. F. developed by the machine may not be capable of supplying at its terminals, the pressure required to operate the external circuit. This limitation exists only in the case of small machines; for, provided that their normal E. M. F. has been correctly apportioned, large machines find their limitations in other directions.

Limitations due to excessive heating are reached when the temperature of the machine acquires a certain limiting or critical value. The heat is chiefly developed in the armature, where friction, hysteresis, eddy currents

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and resistance losses, that is, $I^2 R$, losses in the winding, due to the load current, are all active.

154. In all properly drawn contracts for installing generators, the specifications require a certain limiting temperature elevation, which the machine shall not exceed after a certain duration of continuous, full load. The object of this limitation is mainly to prevent such a rise of temperature, on any part of the machine, as may endanger the insulation of its winding.

Cotton begins to char, or undergoes slow thermolysis, at a temperature slightly above the boiling point of water. It would, therefore, be theoretically safe to operate a generator at continuous full load with the copper wire at 100° C. Taking 25° C. as the normal temperature of the external air, this would represent a temperature elevation of 75° C. Supposing the limiting current of a generator were adopted so as to produce, after continuous full load run, a temperature elevation of 75° C. in the factory; in cases where the dynamo happened to operate in a hotter room, say, at 40° C., the same full load current would, probably, raise its temperature to 115° C., and an overload under such circumstances of, say, 10 per cent. might raise it to 125° C. It is evident, therefore, that a due regard to the safety of the insulation of a dynamo requires that the temperature elevation should be considerably less than 75° C. The temperature elevation frequently met in conservative specifications is 40° C.; in special cases where the generator has to work in a hot room, as low as 30° C.

155. The heat developed in the armature of a generator is dissipated by conduction, radiation and convection. The usual allowance of free surface in

armatures is 0.15 watt per square centimetre, but when the armature is specially ventilated, so that air passes through its substance as well as over its surface, this may be increased to as much as 0.45 watt per square centimetre.

156. Sparking at the brushes is in all cases the result of inductance. That is to say, to the effect of an E. M. F. produced in that coil or section of winding which is leaving contact, through its commutator segment, with the brush. If, as is represented in Fig. 60, the brush H, be in contact with the commutator bar *b*, the current $2I$, flowing through the brush, will be made up of two currents each equal to I, flowing through the adjacent coils, as represented by the arrows. If the armature revolves counter-clockwise, as shown by the arrow, then the relative motion of the brush is clockwise, or in the opposite direction. If the width of the brush be w cms., and the width of the gap between the adjacent bars be g

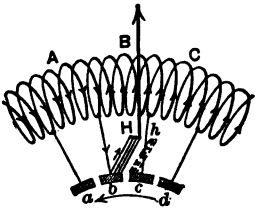

Fig. 60.

Commutation of segments on the Gramme-ring armature.

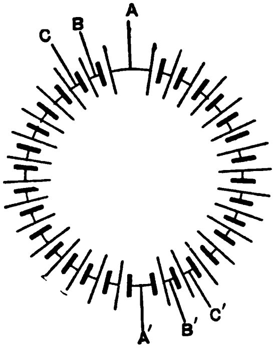

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Fig. 61.

Voltaic circuit made up of single-volt cells. Analogous to dynamo armature. Brushes at A A′, 15 volts ; B B′, 13 volts ; C C′, 11 volts.

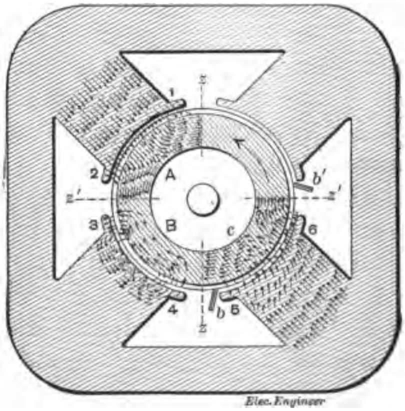
cms., then the distance through which short-circuit will be maintained between two bars will be $w - g$ cms. If a be the radius of the commutator, its circumference will be $2\,\pi\,a$, and the time occupied in the transfer of the brush from any bar to the next will be $\frac{w-g}{2\,\pi\,n\,a}$ seconds. It is evident, therefore, that the current in the winding section, H, must be stopped and reversed in this fraction of time if there is to be no E. M. F. between b and H, when H is transferred to c, and taken up the position h. If the current I, in the segment, B, produces independently of the flux from the field magnets a flux Φ, through its convolutions, and if there are v, convolutions in this section, the total flux linked with the section due to its own current will be $v\ \Phi$, and when the current is reversed to $-I$ the total linked flux will be reversed to $-v\ \Phi$; the total change will be $2\,v\ \Phi$, and the time in which this change is effected $\frac{w-g}{2\,\pi\,n\,a}$ seconds, so that the average E. M. F. established in the coil during the change is $\frac{4\,\pi\,n\,a\,v\ \Phi}{w-g} \times 10^{-8}$ volts. If there are 20 turns of wire in the section and $\Phi = 10$ kilowebers, $a = 15$ cms., $n = 12$ revolutions per second, $w - g = 1$ cm., the average E. M. F. would be:

$$12.57 \times 12 \times 15 \times 20 \times 10{,}000 \times 10^{-8} \text{ volts} = 4.525 \text{ volts.}$$

If the change were not at a uniform rate during the period of transfer, the E. M. F. in the last stages would be augmented, and might rise to 50 volts or more.

157. It is evident that Φ, depends upon the output, while v, depends, for a given machine, upon the number of commutator bars, so that $v\ \Phi$, is reduced by

increasing the number of bars in the commutator. Increasing this number up to a certain limit diminishes the E. M. F., or sectional E. M. F., and, therefore, diminishes the tendency to spark. On the other hand, after a certain limit is reached, the cost of construction and connection of the commutator increases rapidly with the number of bars. In order to supply the E. M. F. necessary to comply with the relations indicated, and to reverse the current in the short-circuited segment, it is usual to give the brushes a *lead*, that is to say, to move them forward in the direction in which the commutator is rotating. The lead has to be increased as the load increases. By this means the coil under commutation is brought within a portion of the flux from the field magnets whose variation through the coil supplies the E. M. F. required to reverse the current during the period of short-circuiting.



Elec. Engineer

FIG. 62.

Two consequences follow a lead of the brushes:

First, the reduction of the E. M. F. of the armature owing to the position and consequent neutralization of some of the E. M. F. generated, as shown in Fig. 61.

Second, a tendency to oppose and neutralize the controlling flux through the field magnets at the pole corner nearest the brush, as shown in Fig. 62.

In this figure the normal condition of the flux through the pole-pieces of the armature of a four-pole generator is shown at the quadrant A, when no current flows through the armature, the direction of rotation being indicated by the large arrow. At the quadrant, B, the figure represents, diagrammatically, the flux set up by the M. M. F. of the armature winding, in the quadrant under A, when no current flows through the same. This M. M. F. increases with the load.

At the quadrant *c*, the effect of combining or superposing these two conditions is similarly represented, and indicates the consequences of what is called *armature reaction* in the generator under load. It will be seen that the flux is crowded together, *i.e.*, its intensity is increased at the edge of the pole edge 6, and diminished under the edge 5.

158. The magnitude of the armature M. M. F. will be $1.257\ I\ N$ gilberts, where N is the number of conductors covered by a pole-face, and I, the current in the winding, and this will be almost entirely distributed in the two air-gaps, 3 and 4, or 5 and 6, since the path through the iron of the pole-pieces and armature is comparatively short. The difference of maximum difference of magnetic potential across each of these air-gaps will be $\frac{1.257\ I\ N}{2}$ gilberts; and, if the *entrefer* or length of

path between the iron and iron, be f cms., the maximum possible flux density, due to this difference of magnetic potential will be $\frac{1.257\ I\ N}{2\ f}$ gausses. This intensity is opposed to the controlling intensity in the *entrefer* from the field magnets at the edge 5, and added to it at the edge 6. When this armature intensity is equal to the intensity from the field magnets, they will neutralize and leave no intensity at the edge 5, while the intensity under the edge 6, will be doubled. When the neutralization is effected, no amount of lead can be of any service in checking sparking, since the flux whose variation should induce a controlling E. M. F. in the short-circuited segment, has been removed. The load current which, in the case of smooth-cored armatures, can be sustained without sparking, is, therefore, less than that which makes $\frac{1.257\,I\,N}{2\,f}$ equal to the intensity in the gap, when no current flows through the armature, and, in practice, only about half this limiting current strength can be allowed.

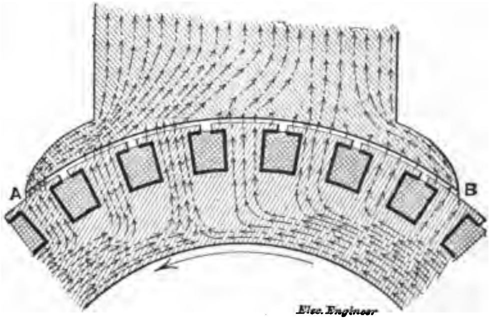


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FIG. 63.

Section of one Quadrant of a 4-pole Generator with Tooth-cored Armature.

159. In the case of toothed core armatures, such as shown diagrammatically in one quadrant by Fig. 63, the same general results occur, but if the cross-section of the teeth be properly designed, they will suffice to carry a normal intensity as at A, in Fig. 62, without saturation of the iron, although the intensity in them under this action may be high. When, however, owing to the effect of the M. M. F. in the armature, a crowding of the flux takes place towards the edge, A, this tendency to increase the intensity saturates the iron, and enormously increases its reluctivity in the teeth, thereby interposing a barrier to the distortion of the flux, and bringing about a more uniform distribution with less reduction of flux at the corner B.

For this reason toothed-core armatures can be made to sustain greater loads than prescribed by the sparking limitations of smooth-core armatures.

SYLLABUS.

The limitations of a dynamo arise from the drop in its armature, from excessive heating, or from excessive sparking.

The limitation of temperature in the armature of a dynamo is imposed in order to prevent the possibility of endangering the insulation of the armature by excessive heating.

Laboratory of Houston & Kennelly,
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—BY—

Prof. E. J. Houston, Ph. D.
AND
A. E. Kennelly, F. R. A. S.

ADVANCED GRADE.

The Regulation of the Dynamo.

160. In all cases where more than a single receptive device is actuated by a generator, some means are necessary to accommodate, automatically, the output of the machine to changes in the load. If the receptive devices are connected to the circuit in series, and are operated by a constant current, it is necessary to increase the E. M. F. of the generator in proportion to the number of devices thrown into the circuit. Such are series-arc-light generators (see Fig. 64). If the devices are connected in parallel, and are operated by constant current, the voltage at each device must be maintained uniform, and the current and pressure of the machine varied to suit this requirement. Most continuous current incandescent generators, are of this type (see Fig. 65).

161. When a series-arc-light generator is running at a constant speed, and with a fixed number of turns on its armature, the E. M. F. developed by the machine can only be varied either by altering the quantity of

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flux passing through the armature, or by varying the position of the brushes on the commutator. Fig. 61 indicates how the E. M. F. of a generator can be varied by changing the lead of the brushes; except that, whereas, in the voltaic analogue represented, the coils are indicated as having the same E. M. F., the coils on the armature of the generator have, in reality, different E. M. F.'s existing on opposite sides in pairs.

162. In most arc-light dynamos, the field magnets are connected in series with the armature which supplies a practically constant current, and, therefore,

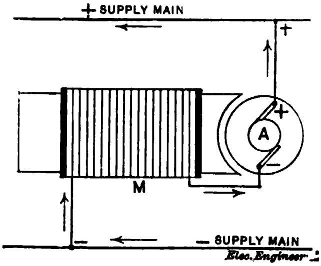

FIG. 64.
Series-wound generator.

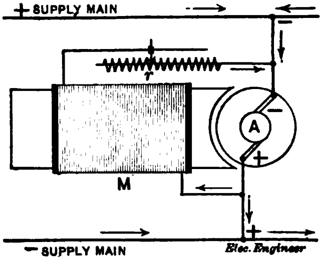

FIG. 65.
Shunt-wound generator.

the M. M. F. in the main magnetic circuit is practically constant, and the entire variation of E. M. F., say, from 50 to 3,000 volts, is provided for by varying the position of the brushes on the diameter of commutation from the diameter of minimum E. M. F. to the diameter of maximum E. M. F. The machines are so designed that the M. M. F. from the armature winding is sufficiently powerful to neutralize, by the flux it produces, the field flux through that portion of the air-gap and armature-surface in which the coils undergoing commutation are situated, in the manner already described in Section 156, so that the commutation is practically sparkless in all positions of the brushes within regulating limits. If this balance between armature and field M. M. F.'s and flux, were not maintained throughout this range, violent sparking would occur, especially as arc light machines reach such high pressures, and the number of volts per bar in the commutator is large.

163. When a generator running at a constant speed has to supply a varying current under an E. M. F. which is automatically maintained constant, either at its own terminals, or at the terminals of delivery, a comparatively large range of current variation has to be provided for, with a definite but much smaller range of E. M. F. variation. Thus a 100 KW. generator, intended to supply 125 volts at its terminals, must automatically maintain a pressure of 125 volts practically constant under all conditions of load, from no current to 800 amperes, the E. M. F. which the machine must generate being in one case 125 volts, and in the other, say, 130. This variation of E. M. F. of five volts must accompany the increase in output in due proportion. In such cases the variation of E. M. F. is obtained, not by changing the position of the brushes on the commutator, but by changing the M. M. F. of the field magnets.

164. If the magnetic circuits of a dynamo, including the leakage- or air-paths, be considered as analogous to a corresponding system of voltaic circuits, as represented, for example, in Figs. 35, 42, 43 and 44, each branch reluctance in the system has a value of the type $\frac{l}{s}(a + b\,\mathcal{H})$, where l, is the virtual or real length of the

branch in cms.; s, its virtual or real cross-section in sq. cms.; and $(a + b\,\mathcal{H})$ its reluctivity at the prime flux density $\mathcal{H}$, which in its turn may be considered as the average magnetic potential difference per cm. of length, through the branch, or, if $\mathcal{M}_b$, be the total magnetic P. D. between branch terminals $\frac{\mathcal{M}_b}{l}$ = mean $\mathcal{H}$. Under these conditions it can be shown that the flux Φ_a, passing through the armature will be

$$\Phi_a = \frac{\mathcal{M}}{\mathcal{R}_a} = \frac{\mathcal{M}}{a_1 + b_1\,\mathcal{M}};$$

so that the apparent reluctance of the magnetic circuit, considered as having no leakage or shunt paths, is a linear function of the M. M. F. in which a_1 and b_1, are constants depending for their magnitude upon the qualities of the iron in the dynamo, and on the configuration of the magnetic system. This relation is known as Frölich's law, and is a consequence of the experimentally observed fact that the reluctance of any branch is constant for a path through air, and of the form

$$\frac{l}{s}(a + b\,\mathcal{H})$$

for a path through iron.

In computing the reluctance in the magnetic circuit of a dynamo, a slight addition, strictly speaking, is necessary on account of the reluctance of such joints as may exist. It has been found by measurement that the reluctance of a joint between two smooth surfaces of wrought iron is equal to the reluctance of an air-gap varying from 0.0026 to 0.0043 cm. according to the intensity in the iron and other circumstances. The reluctance of a well fitted joint between smooth surfaces of soft iron, may, therefore, be estimated as 0.003 oersted divided by the area of the joint in sq. cms. This reluctance will usually be a negligibly small fraction of the total reluctance of the circuit. The reluctance of a badly fitted joint between soft iron surfaces may however be considerable.

165. When a generator has its load increased from no load to full current load of I, amperes, the pressure at its terminals under constant M. M. F., diminishes by $I\,R$ volts, R, being the resistance of the generator. If, in addition to this, a lead has to be given to the brushes to maintain sparklessness at the commutator, the E. M. F. of the armature will be diminished by an amount e, volts, which can be estimated from the distribution of E. M. F. in the coils around the armature, but which is almost impossible to compute accurately. The lead of the brushes also introduces a certain small counter M. M. F., from the armature acting through the main magnetic circuit of the field coils, thus reducing the main circuit flux and the armature E. M. F. by a certain small amount e_1, volts. The problem in designing an automatic constant potential generator, is, therefore, to cause the increase of current I, amperes, which tends to diminish the terminal pressure by the amount $(I\,R + e + e_1)$ volts, increase the M. M. F. of the field magnets to the extent necessary to increase the armature flux and generated E. M. F. by this amount.

If the constants a_1 and b_1, in the Frölich equation

$$\Phi_a = \frac{\mathcal{M}}{a_1 + b_1\,\mathcal{M}}$$

were known, the change, $\Delta\,\mathcal{M}$, necessary to introduce in

$\mathcal{M}$, for the required change in Φ_a, would be immediately determined, and this change could be effected by causing the load current I, to make such a number of turns t, around the field magnets in aiding the shunt-winding, as would make $1.257\ I\ t = \Delta\,\mathcal{M}$. The machine would then be compound-wound and self-regulating; *i.e.*, would maintain a constant M. M. F. through a shunt-winding connected to its constant potential terminals, and would develop a M. M. F. through a short stout winding in series with its armature. This auxiliary M. M. F. would be zero, at no load, and at full load, would reach a maximum capable of compensating for the tendency of the E. M. F. to fall.

166. In practical dynamo magnetic circuits, however, owing to the complexity of the means for computing the value of the constants a_1 and b_1, it is preferable to arrive at the same result by a synthetic process, as follows: Having given a total fall of pressure at terminals due to full load under a constant M. M. F., the increase over the flux through the armature at no load, necessary to recoup this loss, is immediately determined. The intensity in the armature will, consequently, be increased thereby from $\mathcal{B}_a$ to, say, $\mathcal{B}_A$, and the reluctance of the armature from

$$\mathcal{R}_a = \frac{l_a}{s_a}\left(\frac{a}{1 - b\,\mathcal{B}_a}\right), \text{ to } \mathcal{R}_A = \frac{l_a}{s_a}\left(\frac{a}{1 - b\,\mathcal{B}_A}\right) \text{ oersteds,}$$

a and b, being constants for the quality of the iron in the armature. The constant reluctance $\mathcal{R}_K$, of the leakage paths, placed in parallel with the armature, is supposed to be known from the type of machine, from previous data, or from direct computation, and the joint reluctance of the armature and leakage paths will be

$$\mathcal{R}_{\text{J}} = \frac{\mathcal{R}_{\text{A}}\,\mathcal{R}_{\text{K}}}{\mathcal{R}_{\text{A}} + \mathcal{R}_{\text{K}}} \text{ oersteds.}$$

The total flux to be supplied through the field coil or coils must, therefore, be

$$\Phi_{\text{M}} = \Phi_{\text{A}} \left(\frac{\mathcal{R}_{\text{A}} + \mathcal{R}_{\text{K}}}{\mathcal{R}_{\text{K}}}\right),$$

and the density in the iron of the field magnets becomes

$$\mathcal{B}_{\text{M}} = \frac{\Phi_{\text{M}}}{s_{\text{M}}},$$

so that the reluctance of the field cores will be

$$\mathcal{R}_{\text{M}} = \frac{l_{\text{M}}}{s_{\text{M}}} \left(\frac{a_{\text{M}}}{1 - b_{\text{M}}\,\mathcal{B}_{\text{M}}}\right),$$

the total reluctance in the circuit will, therefore, be

$$\mathcal{R} = \mathcal{R}_{\text{M}} + \mathcal{R}_{\text{J}},$$

and the M. M. F. required will be

$$\mathfrak{M} = \Phi_{\text{M}}\,\mathcal{R}.$$

In order to economize the extra M. M. F. required to maintain the pressure of the generator, it is necessary to keep the densities $\mathcal{B}_{\text{A}}$ and $\mathcal{B}_{\text{M}}$, well below the limits of saturation.

167. It is sometimes required to maintain the pressure constant, not at the terminals at the generator, but at the terminals of supply, which may be a mile or more distant, and situated at a real or virtual electrical distance r, from the generator, so that the drop in the armature and supply circuit together will be $[I\ (R + r) + e + e_1]$ volts. This case falls immedi-

ately into the preceding treatment by supposing the resistance of the armature increased by r, except that the shunt-winding is no longer maintained at uniform excitation, but slightly increases in excitation with the load.

In order to make up for any deficiencies in design of compound-wound generators so as to permit of over-compounding, resistances called *compensating coils* are sometimes connected, or left ready to connect, in parallel with the series-winding on the magnets, so as to increase or diminish their effect.

SYLLABUS.

Automatic regulation in generators is employed in practice to maintain a uniform current under varying E. M. F. or a uniform E. M. F. under varying current.

Constant-current generators are usually regulated by shifting the brushes.

Constant-potential generators are usually regulated by compound-winding, that is, by automatically varying the flux.

Over-compounded generators, are generators whose compound winding is designed to maintain automatically a constant pressure at the terminals of delivery, instead of at the terminals of the generator.

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Prof. E. J. Houston, Ph. D.
AND
A. E. Kennelly, F. R. A. S.

ADVANCED GRADE.

ELECTRODYNAMICS.

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168. Electrodynamics is that branch of electromagnetic science which treats of the apparent mutual attractions and repulsions between electric currents, or between electric currents and magnets. The apparent mutual attraction or repulsion between magnets, although frequently classed under electrodynamics more properly belongs to the separate branch of *magnetodynamics*.

When a conductor, situated in a magnetic flux, has a current passed through it, the conductor tends to move. This tendency to motion is the result of the mutual interaction between the flux in which the wire is situated, and the flux produced by the current in the wire.

169. When a wire of length l, cms., Fig. 66, situated in a uniform flux of intensity $\mathfrak{B}$ gausses, carries a current of strength i, C. G. S. units, the flux surrounding the wire will no longer be represented by a uniform field, but the flux above the wire will be increased in intensity, and the flux below the wire decreased. Under these circum-

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stances the wire will be forced mechanically towards the side on which the flux is weakest, and, since the difference of intensity above and beneath the wire depends upon the current strength through the wire, the electrodynamic force with which the wire is acted upon will depend upon the current strength. The value of the force exerted on the wire will be $f = i \mathcal{B} l$ dynes. It is evident that if in obedience to this force, the wire is moved downwards through the flux with a velocity v cms. per second, the E. M. F. generated in the wire will be $e = v \mathcal{B} l$ C. G. S. units, as already shown in Sec. 124; and, since the current will be opposed to this E. M. F., the current will do work upon it, and energy from the source of current will be expended in the wire at the rate $e\,i$ ergs per second. But the mechanical activity expended on the wire, in dynes, will be the force on the wire multiplied by its velocity in cms. per second, so that if f, be the force, then

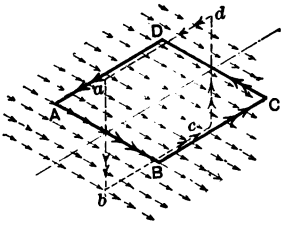

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FIG. 66.

Diagram of rectangle of active conductor A B C D, situated in a uniform magnetic flux.

FIG. 67.

$$f\,v = e\,i \qquad \text{ergs per second,}$$

$$\text{or, } f = \frac{e\,i}{v} \quad \text{dynes} = \frac{v\,\mathcal{B}\,l\,i}{v} = i\,\mathcal{B}\,l \qquad \text{dynes.}$$

The force exerted upon the wire will, therefore, depend upon the current strength through the wire in C. G. S. units, (of 10 amperes each), on the intensity of the flux, and on the length of wire in that intensity. It will be moved oppositely to the direction of motion which would be necessary to give to the wire, in order to set up a current in the direction of i, by induction from the flux.

170. Fleming's hand rule is frequently useful for determining the direction of the force in such cases. But it must be remembered that the left hand is employed in the case of motors, and the right hand in the case of dynamos. The motor rule may be phrased as follows. If the hand be held as shown in Fig. 67, then the *f*orefinger represents the direction of the *f*lux, the m*i*ddle finger represents the direction of the current i, and the thu*m*b points out the direction of the *m*otion, or tendency to motion, produced by the force.

171. We have seen that the mechanical force exerted upon an active conductor in a flux is a consequence of the law of the induction of E. M. F. in such a wire, together with the law of the conservation of energy. In order that a conductor may carry a current, it must form part of a complete loop or circuit, and the total mechanical force exerted on a conducting loop is the sum of all the elementary mechanical forces exerted around the loop in different positions, and, perhaps, in different intensities of flux. If a loop carrying a current of strength i, C. G. S. units, moves under mechanical forces in any manner, so that the flux linked with it is Φ, at any moment, the E. M. F. generated in the loop will be $\frac{d\Phi}{dt} = e$,

C. G. S. units, and the work done on that E. M. F. will be $e\,i$ ergs per second. This will be the mechanical activity expended upon the loop by the system of forces at work upon it. The total work done on the loop from the initial position it occupied, to its final position will be

$$\int_0 e\,i\,dt = \int_0 \frac{d\,\Phi}{dt}\,i\,dt = \int_\varphi i\,d\,\Phi = i\,\Phi \text{ ergs.}$$

That is to say, the total work will be the total increase of flux linked with the circuit multiplied by the current strength in that circuit, or the total increase of linked current-flux; and this work is independent of the velocity at any stage of the process, or of the manner in which the motion has been brought about. The capability, therefore, of a loop to do work by motion, depends only on the total flux it can add to its contents, and on the strength of the current it carries, assuming the current strength to have remained constant.

172. It is evident that an active loop always endeavors to move so as to add as much flux to its contents as possible, and will remain in stable electromagnetic equilibrium only when no excursion that it can make will increase the flux that it contains. The fundamental reason, therefore, for the mechanical force exerted upon an active loop is, that the electromagnetic energy in the ether within the loop tends to a maximum. That energy is expressed as we have seen in Sec. 93 by $\frac{\mathfrak{B}^2}{8\,\pi}$ ergs per cubic centimetre, and this energy in the space within the loop can only become a maximum by making $\mathfrak{B}$ a maximum, and, therefore, by uniting the direction of the external or prime flux, with the direction

of the flux produced in the local magnetic circuit of the active loop, so that an active loop will tend to move until its own flux is parallel to the external or prime flux, i.e., the flux producing the motion.

173. The key to all the phenomena of electrodynamics may be expressed as follows: the tendency of the electromagnetic energy in the ether to a maximum, and, consequently the tendency of fluxes to become parallel. For example, consider a single loop of wire A B C D, such as might be wound on a drum armature of the bi-polar electromagnetic motor shown in Fig. 66. Here the loop contains a flux of its own, owing to the driving current supplied to the motor, and this flux passes upwards through the loop as shown by the small arrows. The loop immediately tends to move into the vertical position in which it will hold the maximum possible amount of flux.

The amount of work which will be performed by the loop electrodynamically, during any small angular movement $d\,\beta$, is $i\,d\,\Phi$ ergs, where $d\,\Phi$ is the small increase of flux admitted into the loop during the angular movement $d\,\beta$. This work can also be expressed as $\tau\,d\,\beta$ where τ is the torque exerted on the loop about its axis, so that, equating the two expressions for the same amount of work, $\tau\,d\,\beta = i\,d\,\Phi$, or $\tau = i\,\frac{d\,\Phi}{d\,\beta}$. The torque exerted by the loop is therefore proportional to the rate at which a small angular movement will increase the bi-polar flux enclosed by the loop, and, if this loop existed on an armature alone, the motion would cease as soon as the loop reached the vertical, where the flux linked with the loop is a maximum. If we assume that at some point in the plane of the loop A B C D, the flux due to the M. M. F. of the loop is represented in magnitude and direction by the line B C, Fig. 68, and the external flux, by the line A B; then at this point the resultant flux is A C, or A K. When, however, the loop revolves into the vertical position, B C is brought to B D, in line with A B, and the resultant flux density at the point is A D. The voluminal energy in a cubic centimetre of space at this point being proportional to the square of the density, the increase owing to the revolution of the loop is proportional to the excess of the area A F G D over the area A E H K.

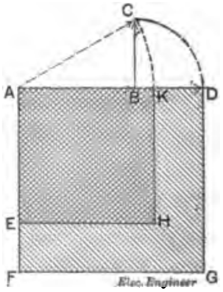

Fig. 68.

Diagram representing the increase of voluminal magnetic energy in space effected by the alignment of two independent magnetic fluxes.

If now another loop be added to the armature at right angles to the former, and also traversed by the driving current, this loop will be at its position of maximum torque when the first loop is devoid of torque, and, if, as in practice, the number of loops occupying various angular positions be placed on the armature, a continuous rotation will be effected. Any loop reaching the vertical position *a b c d*, has the maximum flux linked with it, and would tend to resist onward movement in the same direction, that is, it would oppose any decrease in the amount of flux it embraces. Where a number of loops are placed on an armature, in order to render the motion continuous in the case just supposed, it is necessary to reverse the direction of the current in each of the loops when they have reached the vertical position. This is effected by means of a commutator.

174. The torque of a motor is the moment of the rotating forces about the axis, and is equal to the virtual tangential force exerted at unit radius. Thus, if the force of F dynes, has to be exerted tangentially at a radius of d cms. from the axis of a motor, in order to just keep it from moving, the starting torque of the motor is Fd cm.-dynes; if also during rotation the motor exerts a tangential force, say, at its pulley, of 500 lbs. weight, and the radius of the pulley is 1.5 feet, the running torque of the motor will be 750 foot-pounds.

Since the torque exerted by a motor armature depends upon the intensity of the flux passing through its loops, it is evident that doubling the intensity of the flux will double the torque; consequently the object of iron in the armature, is to enable a powerful flux and flux intensity to be maintained through the armature, from the M. M. F. in the magnetic circuit of the field magnets.

SYLLABUS.

The cause of the movement, or tendency to movement, between active neighboring conductors, or between active neighboring conductors and magnets, or between neighboring magnets, is a tendency of voluminal electromagnetic energy in ether, of the type $\frac{\mathcal{B}^2}{8\pi}$ ergs per cubic centimetre to a maximum, within the substance of magnets, and within the interior of coils.

In order to produce a maximum voluminal electromagnetic energy in ether, fluxes tend to associate and unite in direction, and the motion or tendency to motion; i.e., the force, ceases, when parallelism of flux is attained.

An active conductor of any shape tends to alter its shape in such a manner as to increase the flux linked with it, i.e. the magnetic energy of its environment, and the force so exerted ceases when the flux linked with the circuit is a maximum.

Loops tend to become circles, and coils tend to shorten, magnets to approach or recede, and magnet systems to align themselves, in obedience to these laws.

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—BY—

Prof. E. J. Houston, Ph. D.
AND
A. E. Kennelly, F. R. A. S.

ADVANCED GRADE.

THE ELECTRIC MOTOR.

(CONTINUOUS CURRENT TYPE.)

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175. We have seen that when loops of wire on an armature are moved through a magnetic flux, supplied, for example, by field magnets, there will be E. M. F. generated in the loops of wire.

We have also seen that when the same loops of wire carrying a current are placed in a magnetic flux, there will be electrodynamic forces exerted upon the loops.

In all dynamo-electric machines, whether dynamos or motors, both these conditions necessarily coexist; that is, in the dynamo, the generation of E. M. F. is necessarily accompanied by the generation of electro-dynamic forces as soon as the E. M. F. drives a current through its circuit; and, in the motor, the generation of electrodynamic forces is necessarily accompanied by the generation of electromotive forces. In the dynamo, the direction of the electrodynamic force is opposed to the direction of the force mechanically exerted on the machine in producing its E. M. F., while in the motor, the

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direction of the E. M. F., generated by the motion of the armature, is opposed to the direction of the driving current. The E. M. F. of the motor is, therefore, called its *counter* E. M. F. (abbreviated C. E. M. F.); and, in the same way, the electrodynamic force of a dynamo may be called its *counter electrodynamic* force.

176. The direct E. M. F. (abbreviated D. E. M. F.) of a generator and the C. E. M. F. of a motor, being similarly produced, are subject to the same laws; consequently the C. E. M. F. of a motor is expressed by,

$$e = \Phi\, n\, w \qquad \text{C. G. S. units,}$$

the same equation as in the case of a generator, where Φ, is the total useful flux passing through one pole into the armature in webers; n, is the number of revolutions of the armature per second; and w, is the number of conductors lying on the surface of the armature, counted once entirely around.

From the identity of this E. M. F. equation for motors and dynamos, it is evident that if a motor were driven by externally applied force at its speed n, revolutions per second, but without receiving any driving current, its E. M. F. as a dynamo, would be equal to its C. E. M. F.

177. The work absorbed by a motor from the driving circuit will be $e\, i$, ergs per second, where i, the strength of the driving current in C. G. S. units of 10 amperes each $= i\, \Phi\, n\, w$, so that the work absorbed by the armature during each revolution will be $i\, \Phi\, w$, ergs.

178. The preceding considerations render manifest the reversibility of all continuous-current dynamo electric machines, in regard to their generation of current by motion, or their generation of motion by current.

It is in fact well known, that any motor, when driven, will act as a generator, and produce current, provided its field magnets are capable of self-excitation, or are separately excited; and, similarly, that any dynamo, when properly supplied by a current, will act as a motor and exert mechanical force.

179. All motors are required to produce a certain amount of mechanical activity, but the circumstances under which this activity is required will vary greatly in different cases. These different cases may be classed as follows; namely, where the requisites are,

(*1.*) Constant torque and constant speed.
(*2.*) Constant torque and variable speed.
(*3.*) Variable torque and constant speed.
(*4.*) Variable torque and variable speed.

The torque and the speed of a motor are its essential features, its activity being the product of the two. It will be necessary, therefore, to examine the conditions which determine both of these quantities. If τ, be the torque of the motor, and ω, its angular velocity, the activity of the motor will be

$$\tau \, \omega \quad \text{ergs per second.}$$

The angular velocity is the number of unit angles, or radians, described by the armature per second, and since there are 2π, radians in each complete revolution,

$$\omega = 2 \pi n;$$

so that, the activity of the motor is, $2 \pi n \tau$ ergs per second.

180. We have already pointed out that the torque is the mechanical couple exerted by or on a motor, or, its tangential force at unit radius, and in C. G. S. units at a

radius of one centimetre, so that if τ, dynes be exerted at the periphery of a motor shaft, whose radius is 1 cm., the work done in one complete revolution of the shaft will be $2 \pi \tau$, ergs, and if the shaft make n, revolutions per second, the work per second, or activity, will be $2 \pi n \tau$, ergs per second, as before.

For example, if the 4-pole shunt-wound machine represented in Fig. 58, be employed as a motor, instead of as a generator, its activity will be, say 100 KW., and its commercial efficiency, 0.9. Then the electrical activity, absorbed by the machine at its terminals, will be $\frac{100}{0.9} =$ 111.1 KW. at, say, 500 volts pressure and 222.2 amperes, of which, say, 218 amperes pass through the armature, and 4.2 amperes through the field. If the useful flux through each pole be 55 megawebers, the number of wires on the surface of the armature 200, and the resistance of the armature 0.05 ohm, the drop of pressure in the armature at full load will be $i\,r = 218 \times 0.05 =$ 10.9 volts, so that the C. E. M. F. generated in the armature will be $500 - 10.9 = 489.1$ volts. Since $e = \Phi\, n\, w$, the speed $n = \frac{e}{\Phi\, w} = \frac{489.1 \times 10^8}{55 \times 10^6 \times 2 \times 10^2} = 4.445$ revs. per second, or 266.7 revs. per minute, or an angular velocity of $4.445 \times 6.283 = 27.93$ radians per second. The torque of the motor at full load will be

$$\tau = \frac{P}{\omega},$$

where $P =$ the mechanical activity of the motor in ergs per second. In this case $P =$ 100,000 watts; and one watt being 10^7 ergs per second, $\tau = \frac{10^{12}}{27.93} = 3.58 \times 10^{10}$ dyne-cms. $= 3.58 \times 1.0203 \times 10^{10} = 3.653 \times 10^{10}$

milligrammes weight (at Washington) 3.653×10^4 kilogrammes weight, at a radius of one centimetre.

If the diameter of the pulley were 36″, and the thickness of the belt driven by the pulley $\frac{3}{8}''$, the effective radius of transmission $= 18\frac{3}{16}''$ or 46.2 cms., so that the effective pull exerted by the belt

$$= \frac{3.653 \times 10^4}{46.2} = 790.7 \text{ kilogrammes} = 1743 \text{ lbs. weight.}$$

181. When a continuous-current shunt-motor of resistance r, ohms with separately-excited field, and whose armature is ready to move, is connected to a source of constant E. M. F., E, volts, a current will flow through the armature of the motor on closing its circuit. The strength of this current will be determined not merely by Ohm's law, that is by the resistance and the E. M. F. in the circuit, but also by the inductance of the circuit, which tends to check the first rush of current. The initial strength of current, will, therefore be, either equal to, or less than

$$I = \frac{E}{r}.$$

The torque set up in the motor by this current will be

$$\tau = \frac{I \, \Phi \, w}{2 \, \pi},$$

and if this torque is sufficient to set the armature in motion against its load, the armature will immediately start, and the C. E. M. F., generated by its motion will reduce the current to some value i, expressed by

$$I = \frac{E - e}{r}.$$

Under the influence of the driving current, the motor will continue to accelerate until the working speed is arrived at, which satisfies both equations of energy and of E. M. F.; for, it is necessary, that:

First, the intake is equal to the total work performed, or that $e\ i$ = speed × torque. This torque includes not only the mechanical torque usefully exerted, but also the frictional torque due to eddy currents, hysteresis and journal friction.

In well designed armatures, employing properly laminated and insulated cores, the loss of power in eddy currents, and the torque exerted against their electrodynamic force, are comparatively small. The torque exerted against journal and brush frictions may be approximately determined, when the motor is disconnected from its circuit, by ascertaining the smallest weight which suspended by a cord over the pulley, will maintain the armature in motion. This weight in pounds, multiplied by the effective radius of the pulley, gives the observed frictional torque in pounds-feet. A torque of 1 pound-foot = 13,825 gramme-cms. = 13,550,000 dyne-cms. (Washington.)

As soon as the field magnets of the motor are separately excited, the torque resisting motion, observed in this way, will be found to have considerably increased. If v, be the volume of iron in the armature core in cubic centimetres and η, its hysteresis coefficient, (Sec. 151), $\mathcal{B}$, its maximum intensity, p, the number of field magnet poles, the energy expended in hysteresis per revolution of the armature will be approximately $\frac{v\ p\ \eta\ \mathcal{B}^{1.6}}{2}$ ergs, and the torque due to hysteresis corresponding to this expenditure of work will be $\frac{v\ p\ \eta\ \mathcal{B}^{1.6}}{4\ \pi}$ dyne-cms. Thus, an armature of a 4-pole machine (generator or motor) containing 120,000 c.c. of soft iron of which the hysteresis coefficient is 0.002, magnetizing the armature at full load to a density of 10 kilogausses in each direction, would exert a hysteresis torque resisting motion, of

$$\frac{120{,}000 \times 4 \times 0.002 \times 10{,}000^{1.6}}{12.57} = 1.918 \times 10^8 \text{ dyne-cms.}$$

$$= 14.15 \text{ lbs.-feet.}$$

182. Second, the pressure at the terminals must be equal to the C. E. M. F. plus the drop in the armature; or, that $E = e + ir$. The speed and current, therefore, co-operate to satisfy these two conditions, and these will determine the normal condition of operation in the motor for constant excitation, constant pressure, and constant load, the total activity absorbed by the armature being $E\,i$, watts. If now, the load on the motor, i.e., the mechanical torque, be increased, the speed will diminish and with it the C. E. M. F. until the current strength increases to a value, which satisfies both the energy and pressure equations.

The speed at which the motor runs is, in conformity with these conditions, always expressed by the formula

$$n = \frac{E}{\Phi\, w} - \frac{2\,\pi\, r\, \tau}{\Phi^2\, w^2} \qquad \text{revs. per second.}$$

The first term gives the speed at which the armature would run if it had no drop, and the second term gives the reluctance due to drop.

183. It is evident, therefore, that when a motor is prevented from moving by excessive torque, it can perform no useful work, because its C. E. M. F. would be zero. On the other hand, if all torque could be re-

moved from the machine its speed would be a maximum, because the current it would take would be zero, the maximum activity of the motor existing midway between these two conditions; namely, when its counter E. M. F. is half that of the pressure at terminals or equal to the drop. This would be represented by a commercial efficiency of less than 0.5. In practice, however, the activity of all motors of any considerable size must be considerably greater than 0.5, for the reason that if they were to expend internally half the energy they receive, they would become violently overheated.

SYLLABUS.

In all continuous current dynamo-electric machines, whether dynamos or motors, E. M. F.'s and electrodynamic forces are developed. In dynamos there is a counter-electrodynamic force and a direct E. M. F. In motors there is a counter E. M. F. and a direct electrodynamic force.

Dynamos and motors are reversible machines when the field magnets are capable of self-excitation. In dynamos the E. M. F. is greater than the pressure at the terminals, and in motors, the C. E. M. F. is less than the pressure at the terminals, by the amount of the drop in the machine.

The controlling factors in the activity of motors are the torque and the speed.

A torque of one pound-foot is 0.13825 kgm.-metre, or 13.55 megadyne-cms.

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ADVANCED GRADE.

THE ELECTRIC MOTOR.

(CONTINUOUS CURRENT TYPE.)

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184. When a motor, connected to constant potential mains, is loaded with a constant torque, there are three possible ways of varying its speed; viz.,

(*1.*) By shifting the brushes on the commutator, thus altering the amount of C. E. M. F. available in the motor circuit.

(*2.*) By inserting a resistance in the armature circuit, thus producing a drop of pressure in the circuit of the motor, and thereby lowering the pressure at its terminals.

(*3.*) By varying the M. M. F. of the field magnets of the motor, so as to induce a varying C. E. M. F. in the armature, forcing it to alter its speed in order to maintain a constant C. E. M. F.

185. The method of varying the speed of a motor by shifting its brushes is not practically employed; since, unless efficiency be intentionally sacrificed in the design of the motor, for the purpose of permitting such

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shifting, violent sparking would be produced at the brushes.

The method of inserting a resistance in the armature circuit is frequently adopted, especially with small motors. It is, however, a wasteful process.

We have seen that the torque exerted by a motor is $\frac{i \, \Phi \, w}{2 \pi}$ cm.-dynes, (including torque against friction) so that, the torque remaining constant, the value of the driving current is determined. If now, the speed required of the motor is such that the C. E. M. F. ($\Phi \, n \, w$) is small, the difference between this C. E. M. F., and the pressure in the mains must be made up of drop in resistance $i \, r$, and this drop, when considerable, will have to be almost entirely produced in external resistance. Calling this drop e, the activity lost in heating the external resistance will be $e \, i$ watts, which will be large when the motor is running slowly, and small when it runs at nearly full speed. Moreover, when the current i, is powerful, and the drop e, is large, the resistance must be constructed in such a manner as to liberate a large amount of energy without overheating, and this necessitates bulky, cumbersome and expensive rheostats.

186. The third method for varying the speed of a motor under constant torque, namely, that of varying the M. M. F. of the field magnets of the motor, though frequently employed, is necessarily limited in range. Shunt motors can have their M. M. F. controlled by means of resistance inserted in the circuit of their magnets. The range of variation of speed, obtained in this way, generally amounts to about 25 per cent. If the M. M. F. of the magnets be reduced beyond a

certain point, the armature current, which has to be increased in order to maintain a constant torque, will so far increase the M. M. F. of the armature, as to destroy largely or even to overpower the field flux, and thus give rise to violent sparking at the brushes.

187. In series motors the variation of M. M. F., required for varying the speed, is usually obtained by commuting the field coils; that is, by arranging the field-winding into a certain convenient number of coils, and connecting these coils, by a suitable device, in series or parallel. When the coils are all in parallel, their united M. M. F. is a minimum, and the speed of the armature is consequently highest. While, when the coils are all in series, their M. M. F. is a maximum, and the speed of the armature consequently least. It is undesirable to commute the field coils of shunt machines, owing to their large inductance, and the high pressures that may be excited in them when the current strength is suddenly altered by commutation.

188. It is possible, under extraordinary conditions, to obtain a shunt motor, of say, 30 KW. capacity, which by inserting resistance in the field circuit, and thus varying the M. M. F., will vary its speed under constant torque and constant terminal pressure in the ratio of 3 to 1. Under usual conditions, however, the speed cannot be altered in this way in a higher ratio than 1.25. Series-wound motors can be controlled in speed by commuting their field-coils in a ratio of from 1.25 to 2.0, depending upon the amount of torque exerted by the motor; that is, upon the current through the machine, and the reluctivity of the iron at the existing M. M. F.

189. The condition of variable torque and constant speed is commonly met with in operating machinery, especially machine tools, where varying loads have to be encountered at constant speeds. A series motor, unless specially controlled, is unable to maintain these conditions, since the M. M. F. is constantly varying with changes in the load.

A shunt motor would maintain a constant speed under variable torque, up to full load, disregarding the modifications induced into the magnetic circuit by armature reactions, if there were no drop in the armature resistance. In motors of between 3 and 50 KW. capacity, the armature drop averages about 4 per cent. at full load, and the speed will, therefore, alter in the ratio of about 1.04 between light and full loads. This is usually a sufficiently close regulation of speed, but it is possible to employ a compound-wound motor, having the same connections as a compound-wound generator, and, therefore, so arranged that the armature current slightly weakens the M. M. F. of the magnets, thus forcing upon the armature a constant speed under all loads.

190. The fourth condition, viz., that of variable torque and variable speed, is best exemplified in the case of the electric railroad motor. This condition is practically met both by the insertion of resistance in the armature circuit, and by varying the M. M. F., of the field magnets, which are usually of the series wound type. Such methods, however, while they may satisfy ordinary requirements, are far from being a complete solution of the problem, and no method has yet been introduced, which can accurately control, within a wide range, under variable torque, the speed of a motor on a constant po-

tential circuit. In this direction the capabilities of the electromagnetic motor appear to least advantage.

191. When a shunt-wound motor has to be started from rest, and, therefore, from a condition of zero C. E. M. F., it is necessary, since the resistance of a large motor armature is very small, that the terminals of the armature should not be directly connected to the mains, inasmuch as such an armature connection would practically constitute a short circuit to the mains, for the first rush of current passing through the armature before its inertia can be overcome, and its C. E. M. F. generated, may be sufficient to destroy the armature winding, or to injure seriously its mechanical construction.

In order to avoid this danger it is usual to introduce a resistance called a *starting resistance* into the armature circuit, by the drop of pressure in which, the pressure at the armature terminals may be correspondingly reduced. As soon as the armature is sufficiently accelerated to produce a suitable C. E. M. F., this resistance is gradually cut out of circuit, thereby causing the motor to still further accelerate, until its full speed and C. E. M. F. are attained, when the resistance is entirely removed.

192. In consequence of the reversibility of a generator and motor, it might be supposed that the output of a dynamo-electric machine would be the same, whether employed as a dynamo or as a motor. This, however, is not the case, since in a generator the friction losses are supplied directly from the engine, while in a motor they have to be supplied from the driving current. Suppose, for example, that a generator, supplies 100 amperes at 100 volts terminal pressure, or 10

KW. When running at full load, its loss, expended in friction, of say 2 KW., is supplied from the engine, which delivers 12 KW. to the generator pulley. If the machine be driven by the same full load current of 100 amperes as a motor, and with the same terminal pressure of 100 volts, its intake will be 10 KW. and its output say 8 KW.

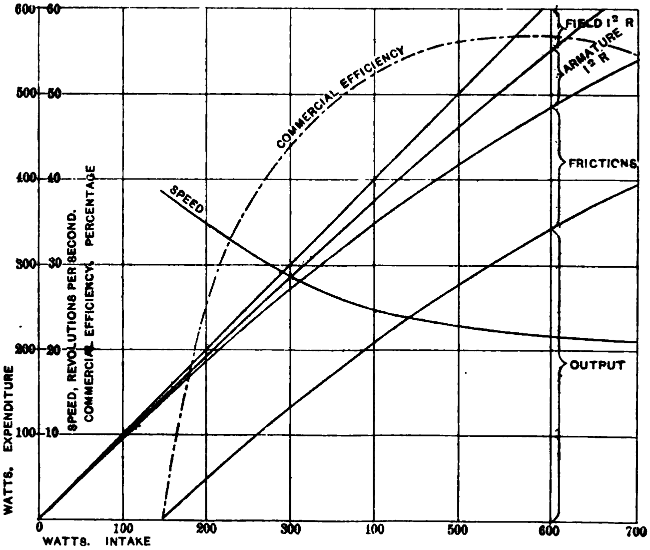

FIG. 69.

Curves showing Expenditure of Power in a Half-Horse-Power Series Wound Motor wound for 500 volts.

Owing to this cause, a one KW. generator will be, say, a 0.75 KW. motor, or practically one H. P., but a large generator of, say, 175 KW., might be a 160 KW. motor.

193. Figs. 69 and 70 give curves taken from actual tests of a well known type of motor; Fig. 69, being a test of a series-wound ½ H. P. motor, and Fig. 70

a corresponding test of a shunt-wound machine of the same make and power. The characteristic properties of shunt and series-winding, in regard to speed and efficiency, are clearly shown. The loss of energy taking place at all activities up to full load is shown, for the field as magnetizing energy ($i^2 r$), for the armature as drop ($i^2 r$) and for friction of eddy-current, hysteretic

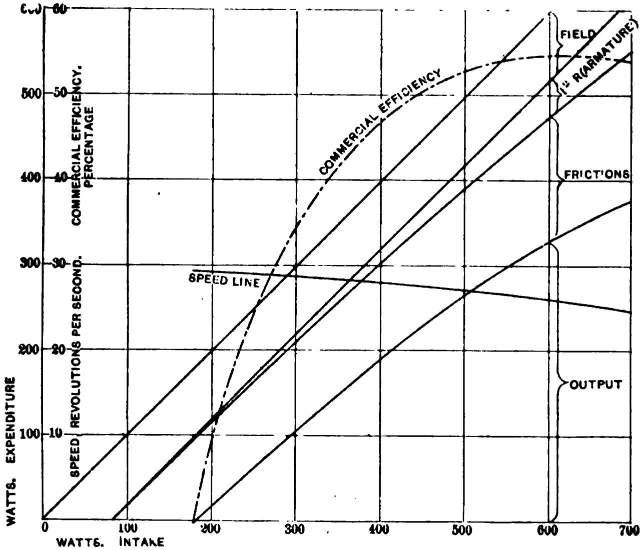

FIG. 80

Curves showing Expenditure of Power in a Half-Horse-Power, Shunt-Wound Motor wound for 500 volts.

and mechanical types combined. Thus, at an output of 350 watts, the shunt motor is seen to have absorbed 640 watts, of which 82 were expended in magnetizing the field, 50 in heating the armature, and the remaining 158 in frictions, representing a commercial efficiency of 54.6 per cent.; while, at the same output with the series machine, the intake was 620 watts, with 57 in the field magnets, 70 in the armature, and the remainder of 143 in frictions, representing a commercial efficiency of 56.4 per cent. The speed of the series machine drops from 38.5 revolutions per second at no load, to 21 revolutions per second at full load, while the speed of the shunt machine drops from 29.2 revolutions per second at no load, to 25 at full load. The series machine is somewhat cheaper to construct, since its field magnets are wound with a few turns of coarse wire instead of many turns of finer and more expensive wire, but the regulation in speed of the shunt motor is much closer.

SYLLABUS.

The regulation of speed in a motor under constant torque, connected to constant potential mains may be obtained either by inserting resistance in the armature circuit, or by altering the M. M. F. of the field magnets. This variation of speed is practically limited in range, and constitutes the principal disadvantage of the electric motor.

The uniform regulation of speed in a motor under variable torque, when connected to constant potential mains, is readily obtained either by shunt winding, or still more closely, by compound winding.

The regulation in speed in a motor under variable torque, when a wide range of speed and torque have to be maintained, is accomplished by the insertion of resistance in the armature circuit and by varying the M. M. F. of the field magnets, which are usually series-wound.

Dynamo-electric machines have, other things being equal, a greater output when employed as generators than as motors.

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ADVANCED GRADE.

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(CONTINUOUS CURRENT TYPE.)

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194. Motor armatures, like dynamo armatures, are either smooth-cored or toothed-cored. The toothed-cored armature was one of the earliest forms devised. Latterly, however, owing to its mechanical and electrical advantages, the toothed-cored armature has again come into almost universal favor. In a smooth-cored armature the electrodynamic force is mainly exerted upon the wires on its surface and, therefore, unless these wires are very carefully bound and secured, they are liable to be dislodged. In the toothed-cored armature, not only are the wires more completely protected from injury and in a position more favorable to complete insulation, but the electrodynamic force is no longer exerted upon the substance of the copper, but on the mass of the iron in the teeth. The M. M. F. of the current in the wires, affects the distribution of flux from the M. M. F. of the field magnets through the armature core, and produces a distortion of flux density which serves to

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rotate the armature according to the law of tractive force, $\left(\frac{\mathcal{B}^2}{8\pi}\text{ dynes per sq. cm.}\right)$ as already explained. Moreover, eddy currents are avoided in the substance of the conductor; for the flux no longer penetrates the wires themselves, but is deflected by the surrounding iron either to one side or to the other. This effect does not alter the E. M. F. produced in such imbedded wires, since the rate of linking flux with them remains equally effective; and, although the electrodynamic force set up by currents in the wires changes its point of application, yet its amount is unaltered.

195. The effect of armature reaction, in a motor, is the same as in a dynamo, except that its relative direction is reversed; that is to say, the polar edge which is weakened is the *trailing pole*, or the pole that is left, and the pole which is strengthened, is the *leading pole* or the pole that is approached. This is evidently owing to the fact that, other things being equal, the current in the armature is in the reverse direction to that produced when it is operating as a dynamo, and, consequently, the direction of the armature M. M. F. is reversed.

Since the rotation of a motor is produced by electrodynamic force, the leading pole requires to have its flux density strengthened by armature reaction, and the following, or trailing pole, must be correspondingly weakened. That is to say, the distribution of $\frac{\mathcal{B}^2}{8\pi}$, is such as to increase at the leading polar edge, in accordance with the principles described in Section 116, so that the armature is pulled around in the direction of the denser flux. In a generator, however, the armature has to be moved by mechanical force away from the denser flux, at the strengthened pole-piece where the distribution of $\frac{\mathcal{B}^2}{8\pi}$ is greater, and, therefore, the trailing polar edge is strengthened by armature reaction. Consequently, the fact that the direction of both armature reaction and armature M. M. F. must be opposite in a motor to that which exists in a dynamo, is the fundamental law underlying all considerations of direction of relative rotation in motors and generators.

196. As a consequence of the preceding fundamental law, it will be seen, that in order to preserve the same direction of rotation of the armature as a motor that it possesses as a generator, the direction of current through the armature must be reversed, unless the direction of the current in the field magnets is also reversed. That is to say, the relative direction of M. M. F. between field magnets and armature must be reversed.

(*1.*) Shunt-wound machines will preserve their direction of rotation as motors, either when the current through them retains the same direction, or when the E. M. F. at their terminals retains the same direction, as in their condition as generators.

(*2.*) Series-wound machines will reverse their direction of rotation as motors, either when the current through them retains the same direction, or when the E. M. F. at their terminals retains the same direction, as in their condition as generators.

(*3.*) In order to reverse the direction of rotation of a motor it is necessary to change the M. M. F. in either field or armature; i.e., to reverse the direction of either the field or armature. Merely reversing the direction of pressure at the motor terminals; or, what is the same thing, reversing the direction of current through the entire motor,

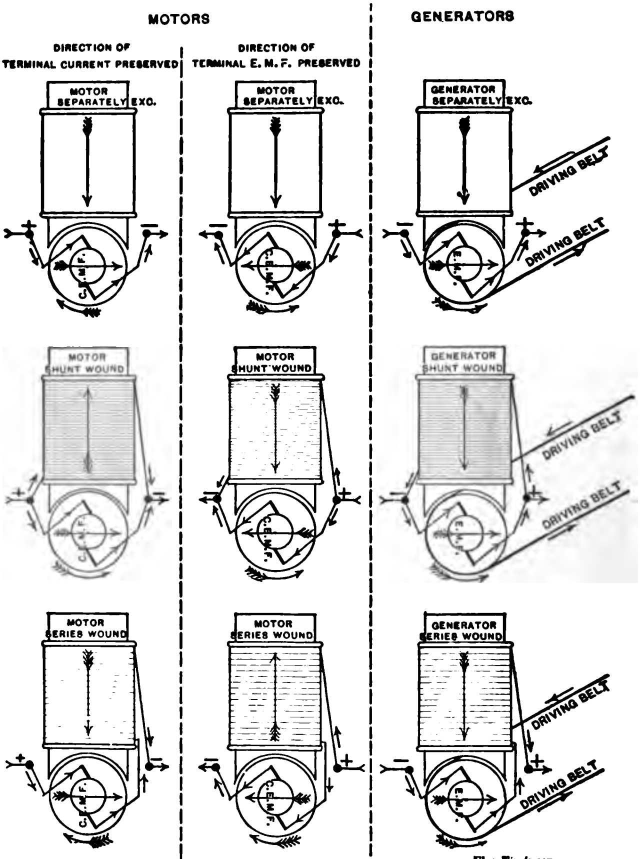

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FIG. 71.

Showing Relative Direction of Rotation in Generators and Motors.

does not change its direction of rotation unless the machine be separately-excited. These relations are indicated diagrammatically in Fig. 71, where the uppermost row of machines are separately excited, the middle row are shunt-wound, and the lowest row series-wound. The large arrows point out the directions of M. M. F. in field and armature, and the curved arrows the direction of armature rotation. It is evident that in order to retain as a motor the direction of rotation possessed as a dynamo, a relative reversal of M. M. F.'s in field and armature must be effected.

197. When a motor is connected with an E. M. F., current flows through the motor, and electrodynamic force is set up, as we have seen, between the armature and field fluxes. Under the action of this force, the motor accelerates until its C. E. M. F. is sufficient to limit the current strength it receives to the amount required for the performance of the total work expended in and by the motor at the speed which it must maintain to develop that C. E. M. F. When, however, two motors are connected in series, they will tend to accelerate, until, by their united C. E. M. F.'s, the current they receive is limited to the total work they absorb; but since by varying their relative speeds the same amount of C. E. M. F., and the same amount of work, may be distributed between them in an indefinitely great number of ways, it is clear that their relative speeds will be indeterminate. For, as an example of such instability, consider two similar, separately-excited motors A and B, to be connected in series, and each loaded by independent, equal and uniform torques, such as by weights suspended over their pulleys. Then, for a given current strength passing through the armatures, by symmetry, the two motors will run at equal speeds, dividing the

total voltage equally between them, and exerting equal activities. But any slight accidental increase in the torque imposed on one motor, say A, instead of automatically causing an increased current strength from the mains to overcome the extra load, might be met by the absolute stoppage of A, with a doubled speed on the part of its neighbor B. The same current strength would continue to flow through the armatures, but one motor would do all the work and generate the entire C. E. M. F.

198. For the same reason, motors which are operated in series arc circuits, are difficult to control in speed unless their torque increases with the speed, as in the case of fan motors, or unless some speed governing mechanism is employed to vary the torque in relation to the speed. Few motors of any considerable size are, therefore, operated upon series circuits.

It is often necessary in practice to reverse the direction of a motor. For this purpose it is only necessary to reverse the M. M. F. either of the field or of the armature. It is customary in such cases to reverse the connections of the armature. Care has to be taken, however, not to apply too powerful an E. M. F. at the brushes, immediately after such reversal; for, the C. E. M. F. of the armature, which will be still revolving by its momentum in the original direction, will now be an E. M. F. in the same direction as the driving current, and will, therefore, aid in producing a very powerful current through the armature, which may act as a short circuit on the mains.

199. Whatever may be the importance of small weight in the case of stationary electric motors, there can be no doubt that, in the case of electric loco-

motors, it is desirable to reduce their weight for a given output as much as possible. For a given output the torque required may vary within wide limits, and is inversely as the maximum speed the motor has to maintain. When a motor can produce a given output, it is evident that any torque can be theoretically obtained from it by sufficiently increasing or reducing the speed of rotation through the necessary gearing. In practice, however, such gearing is frequently objectionable from the friction, noise and wearing introduced by it. Thus, street-car motors as first employed, reduced their speed of rotation by double gearing from 12 to 25 times, according to the type and power of motor employed. They now usually reduce their speed by single gearing from four to five times, requiring, however, a slower armature speed and a greater corresponding torque for the same output; or, in other words, a more powerful motor. By employing cast steel, multipolar, field magnets, and by economy in weight, street car motors are built which develop at their armature shafts a torque of 133,000 dyne-cms., per ampere, per kilogramme, that is 0.00448 or $\frac{1}{223}$ pound-foot, per ampere, per pound of total motor weight, not including the weight of gearing; so that at this rate, a 500-volt motor weighing 223 pounds, and supplied with one ampere, would exert a torque of one pound-foot. A 500-volt stationary motor of about the same size (15 H. P.) usually exerts a torque at its armature shaft, of about 0.001 to 0.0015 pound-foot per ampere, per pound of weight, so that street-car motors are usually about four times more powerful than stationary motors in reference to their weight.

SYLLABUS.

In smooth-cored armatures, the electrodynamic force is largely exerted upon the substance of the conductors wound upon its surface, but in toothed-cored armatures, the armature is sheltered from both eddy currents and from electrodynamic force by the surrounding iron.

In a generator, the leading polar edge is weakened, while in a motor it is strengthened by armature M. M. F. and reaction; consequently, the M. M. F. in a motor armature must, for the same direction of rotation be reversed in direction to that which existed in a generator.

Motors operated in series, are unstable in speed unless their torques are either maintained in uniformity, or increased at a greater ratio than their speeds.

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Prof. E. J. Houston, Ph. D.
AND
A. E. Kennelly, F. R. A. S.

ADVANCED GRADE.

ELECTRIC HEATING.

200. When an electric current of strength i, expressed in C. G. S. units, passes steadily through a resistance of r C. G. S. units, a C. E. M. F. of $e = i\,r$ C. G. S. units is developed in the resistance, while energy is expended by the current against this C. E. M. F., at the rate of $e\,i = i^2\,r$ ergs per second, and appears in the resistance as heat. Transformed into practical units, a current of i amperes, passing steadily through a resistance of r ohms, develops a C. E. M. F. of $i\,r$ volts, and does work at the rate of one joule per second (10 megergs), or with an activity of one watt, as heat in the resistance.

201. The scientific unit of heat generally employed is the amount of heat required to raise the temperature of a gramme of water from 3° to 4° C. This unit is indifferently called the *lesser calorie*, the *therm*, the *gramme-calorie*, or the *water-gramme-degree-centigrade.*

Since the calorie is not a C. G. S. unit, it is more con-

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venient, in electrical engineering, to employ as the practical unit of heat, its mechanical equivalent; namely, the joule, or 10 megergs. The joule is equivalent to 0.239 therm; or, in other words, 4.18 joules approximately are required to be expended in heat to raise the temperature of one gramme of water one degree C. (One British Thermal Unit, or B. T. U., that is, 1 pound of water raised from 68° to 69° F., requires an expenditure of 1053 joules, so that, roughly, 1 B. T. U. = 1 kilojoule).

Heat produced in resistance by electrical currents is either purposely developed, as in electric heaters or electric furnaces, or incidentally and unavoidably developed, as in dynamo machinery and wires conveying currents.

202. The flow of heat through a conductor follows the same law as that which determines the flow of electricity through a conductor, i. e., Ohm's law. If θ, be the difference of temperature in degrees Centigrade, between two parallel plane surfaces of the conductor, and S, the thermal resistance of this portion of the conductor, then H, the strength of the thermal current, in joules per second, will be

$$H = \frac{\theta}{S}.$$

S, is determined as follows; viz., if y, be *the thermal resistivity*, l, the length of the conductor in cms., and a, its cross-sectional area in square centimetres, then

$$S = \frac{l\,y}{a}.$$

The thermal resistivity of a substance is the reciprocal of its thermal conductivity, and may be defined as being equal to the reciprocal of the amount of heat, expressed in joules, which will traverse a cube of the material one cm. in length of edge, in one second of time, with 1° C. difference of temperature between two opposed faces.

Thus, if a wire had a resistance when heated to 100° C. of 1.41 ohms, and was enclosed in a cubical box whose internal edge was 10 cms. in length, with walls composed of felt and 1 cm. thick, the external surface being zinc lined, and maintained by immersion in water, at a temperature of 20° C., then if the wire were so disposed within the interior that its temperature was immediately communicated to the internal surface of the walls, these would each have a cross-section of 100 sq. cms. and a difference of temperature of 80° C. between the inner and outer surfaces. The thermal resistivity of felt, expressed in C. G. S. units, according to the above notation is about 2750, so that the resistance of each wall would be $\frac{1 \times 2750}{100} = 27.5$, and since the box has six walls, the total thermal resistance would be $\frac{27.5}{6} = 4.583$. The flow of heat would, therefore, be $\frac{80}{4.583} = 17.46$ joules, and the current strength which would have to be sent through the wire to maintain its temperature, with that of the interior walls, at 100° C., would be $1.41 \times i^2 = 17.46$, or $i = 3.52$ amperes.

The preceding relations form the basis for determining the amount of energy required to be expended in obtaining a fixed temperature in a closed electric stove of given dimensions and material, after due allowance has been made for the thermal capacity of the contents; i.e., of the amount of heat required to be expended in such contents in order to raise them initially to the required temperature.

203. The following is a list of thermal resistivities for a few substances. These values can only be regarded as approximations. Comparatively few observations have been made, and the thermal resistivity of a substance, like its electric resistivity, varies both with its physical condition and with its temperature.

Like electric resistivities, it would seem that good thermal conductors conduct better, and good thermal insulators insulate better at low temperatures.

THERMAL RESISTIVITIES IN JOULEAN UNITS.

Thermal Conductors.	
Silver	0.17
Copper	0.225
Zinc	0.81
Brass	0.83
Iron	1.52
Lead	1.95
German Silver	2.29
Thermal Insulators.	
Stone	50
Chalk	100
Glass	100
Sand	300
Gutta-percha	500
Caoutchouc	600
Clay	800
Sawdust	2000
Wool	2100
Paper	2200
Vulcanized Indiarubber	2700
Felt	2750

204. In general, heat developed in a conductor by the passage of an electric current is dissipated by conduction, radiation and convection. The conduction losses, as we have seen, depend both upon the dimensions and thermal resistivity of the conducting substance, and the difference of temperature at opposing surfaces. The loss of heat by radiation follows less simple laws, and the loss of heat by convection is still more complex.

205. Radiant heat is believed to be a purely electromagnetic phenomenon, and its laws are not yet accurately known. The rule commonly employed in computing the amount of radiation from a hot body is an empirical rule determined by Dulong and Petit from a large number of practical observations: The loss by radiation is proportional to the surface of the heated body, to the nature of the surfaces of surrounding bodies, and is in geometrical proportion to the absolute temperature of the surfaces. The loss of heat from a body by convection depends upon the temperature of the surface of the body, the nature, and density of the surrounding medium, the normal amount of motion in the medium (for instance, wind in the case of air) and the form of the body, with the friction which its surface offers to the motion of the medium. The result is a complex thermodynamical and hydrodynamical problem which has only been reduced to quantitative results in a very few cases.

206. Although radiation from the surface of the hot wire takes place in geometrical proportion to its temperature elevation, yet it is usually sufficient, within the range of ordinary temperatures, to take a mean value of the radiation in direct proportion to the rise of temperature.

One sq. cm. of bright copper radiates 0.0006 watt per C° temp. elevation (approximately).

One sq. cm. of blackened copper radiates 0.0014 watt per C° temp. elevation (approximately).

For practical purposes the convective loss of heat from a wire supported horizontally in still air, may be taken as independent of the diameter, and as equal to

0.00175 watt per linear cm. of the wire per °C. of temp. elevation (0.0533 watt per foot). In moving air, as for example, in ordinary weather out of doors, the convective loss is usually many times greater.

207. The temperature elevation of a wire, for a given current strength, depends upon its resistivity, diameter, covering and environment. A bare wire is best cooled by supporting it on insulators in the open air, where any breeze or other motion of the air that may exist, will carry off its heat convectively. A covering of, say, cotton, rubber, or other electric non-conductor will, up to a certain thickness, serve to cool the wire by increasing its surface, even although the thermal resistivity of such materials is very high. A buried, insulated wire is usually kept much cooler, by conduction through the substance of the soil, than the same wire suspended in quiescent air; while an insulated wire, submerged in water, is maintained still cooler, by reason of rapid convection of heat through the water together with its large thermal capacity.

The following table gives the diameter of copper wire, required to carry the various current strengths, with an elevation of 20° C. in temperature, as deduced from actual measurements of the heating of wire under different conditions. If the normal temperature of a wire be 30° C., the continued passage of the tabulated current strength will cause the wire to approximately attain the temperature of 50° C., which will enable the wire to be held in the hand without pain, and such a temperature may be considered as a safe limiting temperature. Fire insurance rules both in the United States and in Great Britain require a lower temperature elevation and limiting current-strength in order to provide a margin of safety, namely, what is equivalent to an elevation of 10° C. at full load, or about 33 per cent. less current strength.

TABLE OF DIAMETERS OF COPPER WIRE, OF CONDUCTIVITY 98 PER CENT. MATTHIESSEN'S STANDARD, ELEVATED 20° C. BY VARIOUS CURRENT STRENGTHS IN AMPERES (ALTERNATING OR CONTINUOUS).

Effective Current Strength Amperes.	Covered Wire in Wooden Moulding.	Bare Wire Suspended Horizontally in Still Air Within Doors.		Bare Wire Suspended Horizontally in *Calm* Weather Out of Doors.	
		Bright.	Blackened.	Bright.	Blackened.
	Inches.	Inches.	Inches.	Inches.	Inches.
5	0.020	0.015	0.014	0.011	0.010
10	0.036	0.030	0.028	0.022	0.020
15	0.052	0.045	0.042	0.032	0.030
20	0.069	0.060	0.057	0.042	0.039
25	0.085	0.075	0.068	0.052	0.049
30	0.100	0.090	0.080	0.061	0.058
35	0.114	0.103	0.092	0.070	0.066
40	0.127	0.115	0.105	0.079	0.074
45	0.140	0.128	0.117	0.087	0.082
50	0.152	0.140	0.130	0.094	0.089
60	0.175	0.168	0.152	0.108	0.103
70	0.197	0.190	0.171	0.122	0.116
80	0.218	0.212	0.192	0.134	0.128
90	0.236	0.235	0.210	0.146	0.140
100	0.254	0.257	0.227	0.157	0.151
125	0.292	0.307	0.265	0.183	0.175
150	0.326	0.365	0.308	0.210	0.202
175	0.357	0.410	0.347	0.234	0.227
200	0.386	0.450	0.385	0.256	0.248
250	0.440	0.520	0.455	0.299	0.290
300	....	0.615	0.518	0.339	0.330
400	....	0.765	0.640	0.418	0.406
500	....	0.910	0.750	0.488	0.471
600	....	....	0.857	0.550	0.533
700	....	....	0.958	0.611	0.593
800	....	....	....	0.671	0.650
900	....	....	....	0.717	0.693
1000	....	....	....	0.782	0.745

208. The rapid elevation of temperature in an overloaded conductor is practically employed in safety fuses which are formed of high resistivity conductors with a small surface per unit length and a low melting point, so that an excess of current strength above their rated capacity readily fuses them. The heat developed in a safety fuse depends upon its resistivity at the working temperature, on the current strength, and on the length of the fuse. As the current strength, and consequently the temperature, increases, the resistivity of the metal increases, the energy expended in unit length increases, and the rate of dissipation of the energy by conduction, radiation and convection also increase until the melting temperature is attained. Except in fuse wires of very small diameter, the predetermination of their melting current becomes very complex and different under different circumstances, such as the inclination of the fuse, its surface, condition, etc. With large fuses the capacity for heat may be such that an enormous overload may be safely carried for a very brief interval, the energy being expended in raising the temperature of the metal while a much smaller steady increase of load would certainly melt them.

SYLLABUS.

Heat escapes from a hot body by conduction, radiation and convection. Conduction follows a law similar to Ohm's law. Radiation is believed to be a purely electromagnetic function of the ether and follows laws not yet fully ascertained. Convection is a still more complex function depending upon the material environment of the body.

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Prof. E. J. Houston, Ph. D.
AND
A. E. Kennelly, F. R. A. S.

ADVANCED GRADE.

INCANDESCENT LIGHTING.

209. When a substance is heated to the temperature of incandescence, it imparts energy, by wave motion or radiation, to the surrounding ether. This wave motion comprises a great variety of vibration frequencies. All waves of frequencies lying between the limits of approximately 390 trillions and 760 trillions per second, are capable of affecting the eye as light. All radiations whose frequencies lie outside these limits, since they fail to affect the eye are called *non-luminous* or *obscure* radiations. Of the radiant energy emitted by a body, only a certain quantity consists therefore of luminous energy.

210. If w, be the activity of radiation per unit difference of frequency, the total luminous activity can be expressed as,

$$P = \int_{390,000,000,000,000}^{760,000,000,000,000} w \, dn \qquad \text{watts},$$

where n, is the frequency. This is shown in Fig. 72,

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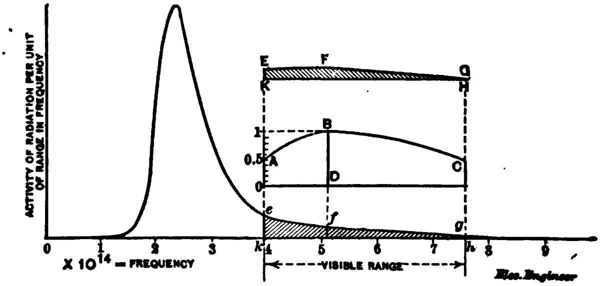
which roughly represents the distribution of radiant activity with reference to the frequency of vibration in an ordinary incandescent lamp. The frequencies comprised between the ordinates *e k* and *g h*, are luminous frequencies. The shaded area *e f g h k*, comprised between the curve and base, between these limits, represents the number of watts expended by the lamp in luminous radiation. The unshaded areas represent the non-luminous activity. The ratio of the shaded to the unshaded area is about 0.03.



Elec. Engineer

FIG. 72

Distribution of radiant energy from incandescent filament with respect to vibration frequency.
Area, *e f g h k* = luminous activity in watts.
Curve, A B C = physiological coefficient of illuminating power referred to standard frequency as unity.
Area, E F G H K = effective physiological illuminating power.

The total flux of radiation activity emitted by a filament of surface-area S sq. cms. at an absolute temperature T, is expressed by

$$P = k\, S\, T\, e^{aT} \qquad \text{watts,}$$

where k, is a constant depending upon the nature of the filament, and $a = 0.0043$.

The luminous effect produced on the average normal eye by a given quantity of radiation activity, say one watt, is not the same in different parts of the spectrum;

that is, at different frequencies. For the average eye, the maximum effect is produced in the yellow, at a frequency of about 500 trillions per second. The amount of illuminating power in a given source of light cannot therefore be determined from the total activity of radiation. It becomes necessary to determine the illuminating values of one watt of activity at all frequencies within visual limits. If this *physiological coefficient of illumination* be expressed by z, in suitably chosen units, or, as in Fig. 72, by reference to the physiological effect B D, at some standard frequency, taken as unity, then the illuminating value of any quantity of energy $w\ dn$, covering a small range of frequency dn, will be $z\ w\ dn$, and we obtain by the application of the coefficient z, a new curve E F G H K, whose area is

$$Q = \int_{3.9 \times 10^{14}}^{7.6 \times 10^{14}} z\ w\ dn$$

units of *physiologically effective illumination.* For this reason it is impossible to compare accurately the illuminating power of two different sources of light, such as a candle and an arc light, unless the physiological coefficient z, at present undetermined, be known for all parts of the spectrum, as well as the distribution of activity in the spectra of the two sources.

211. The most efficient source of light, if it could be produced, would be that in which all the energy radiated possessed a frequency within visual limits. Considering illuminating power alone, that particular frequency near which z, is a maximum, that is somewhere near the yellow of the spectrum, would be the most advantageous frequency the source could possess,

but, considered with reference to fitness for agreeable illumination and the distinction of colors, that distribution of frequencies would be the most desirable which best agreed with the distribution in sunlight.

212. The frequencies which are predominant in the radiation of bodies heated to incandescence, are non-luminous frequencies. Consequently, in all artifical sources of illumination, the larger proportion of the energy radiated is of a useless character. It has been found that, in the neighborhood of 1,000° C., as the temperature of a luminous body increases, the luminous radiation rapidly increases, so that the attainment of a very high temperature is essential for a successful artificial illuminant. A high refractory power is necessary, therefore, to sustain the high temperature required, and carbon is the only common substance which has yet fully met this requirement. An illustration of the importance of a high temperature, and the efficiency of luminous radiation, is seen in the case of the arc and incandescent lamps, each of which employ incandescent carbon as a source of radiant energy, but in the arc lamp the temperature attained, being that of the volatilization of carbon, is higher than that which the incandescent filament can safely and continuously sustain. The amount of energy expended in an arc lamp is usually about 450 watts, and of this about eight per cent. is expended in luminous radiation, the balance being non-luminous, while in incandescent lamps the percentage of luminous radiation is about three. Moreover, the distribution of energy differs in these two sources of light, the average physiological coefficient being greater in the spectrum of the arc lamp, than in the spectrum of the incandescent lamp.

213. The object in the commercial incandescent lamp is to produce an electrically heated incandescent surface at the highest practical temperature; for, as we have seen, such a temperature will produce the best efficiency of luminous radiation and a fair approximation to the character of sunlight.

The high temperature necessary for the proper working of an incandescent lamp must be uniformly maintained over the entire surface of the filament, and, to insure this, the resistance of the filament must necessarily be uniform per unit of length, since, otherwise, on the passage of a steady current through it, different parts would glow with unequal brightness and the parts unduly heated would be rapidly destroyed, or, if preserved at the safe temperature, the rest of the filament would be insufficiently heated.

214. The standard of physiological effective luminous radiation, or, as it is ordinarily called, the *standard of light*, differs in different countries. In the United States and in Great Britain it is the *standard candle* burning 2 grains (0.1296 gramme) per minute; in France, a *Carcel lamp* of definite dimensions, burning 42 grammes of colza oil per hour; in Germany, a *Hefner-Alteneck lamp* of definite dimensions burning amyl-acetate with a flame four cms. high. The light emitted from one square centimetre of platinum at a definite high temperature is also employed as a standard in Germany under the name of the Reichsanstalt Unit. In France the *Violle lamp* of molten platinum was adopted by the International Paris Conference of 1881, but has not come into general use.

According to the best determinations one standard

British candle = 0.0506 Violle, = 0.1053 carcel, = 1.14 Hefner Alteneck.

215. The illumination received by any surface is the quantity of light (the *physiologically effective flux* of light) received by its surface per unit area. Thus, if the standard candle be regarded as the unit point-source of light, the total quantity of light it emits is 4 π, units of luminous flux, and one unit of luminous flux received per square centimetre would constitute unit illumination. No name or unit of illumination has, however, yet been adopted, but common expressions of illumination refer to the candle-foot, or the carcel-metre as unit, these being respectively the illumination produced, on a perpendicular surface by a candle at a distance of one foot, and by a carcel at a distance of one metre. These intensities of illumination are nearly equal, one candle-foot being greater than one carcel-metre in the approximate ratio of 1.133.

216. The proper lighting of a room depends upon its dimensions, and upon the character of its interior surface. Highly diffusive wall surfaces require a smaller amount of light to produce the same general degree of illumination. The character of the illumination will also depend upon the amount of light and upon its distribution. A single source of light will usually produce the greatest local and the lowest average illumination, while the same total quantity of light from numerous distributed sources will produce the opposite results. In the case of incandescent lamps, an illumination upon the surface of a book, equivalent to one carcel-metre, is sufficient for the purposes of easy reading. This is usually obtained in a room by allowing $\frac{1}{5}$ candle power to the

square foot of floor space, or one 16 C. P. lamp to 50 sq. feet, while rooms not devoted to reading purposes, unless darkly papered, will be amply illumined by one 16 C. P. lamp per 100 sq. feet of floor space. The intensity of illumination from a single point-source of light is inversely as the square of the distance from the source, so that a room with a high ceiling, lighted by incandescent lamps placed on the ceiling, would receive on a desk or table a lesser degree of illumination than if the lamp were lower, and, in any case, the illumination on the surface of the desk or table is ordinarily greater than on the surface of the floor. In determining, therefore, the number of incandescent lamps required for the proper illumination of a room, reference must be had not only to the character of the illumination but to the parts of the room where such illumination is specially required.

217. The number of watts that have to be supplied to an incandescent lamp per candle power that it yields is frequently called the *efficiency of the lamp*, but could more accurately be called the inefficiency of the lamp or its *specific activity*, since the greater the number of watts supplied per candle obtained, the lower the effective physiological efficiency of the lamp.

The *true efficiency* of the lamp, or its specific illuminating power, is the reciprocal of this, or the number of candles obtained from the lamp per watt supplied to it. The efficiency at which new lamps are usually operated, ranges between $\frac{1}{3}$ and $\frac{1}{4 \cdot 5}$ candles per watt.

218. The higher the temperature at which a lamp is operated the greater its efficiency, but the shorter its probable duration of life.

When an incandescent lamp is steadily operated at constant pressure, the light it emits steadily decreases, that is, its efficiency becomes reduced. This is owing to two causes consequent upon the disintegration of the filament. First, to the deposition of the disintegrated material as an opaque coating on the walls of the lamp globe, thereby reducing the amount of light emitted, and second, to the reduction in the cross-section of the filament by the disintegration and the consequent increase in resistance, whereby less energy is absorbed by the lamp.

SYLLABUS.

The physiological effect on the retina of different relative frequencies within visible limits is different; generally, therefore, the physiological effect of a given quantity of luminous activity varies with different sources of light.

The illumination required on a well lighted table in an ordinary room is about one carcel-metre, or usually, two sixteen candle power lamps for every 100 square feet of floor space.

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Prof. E. J. Houston, Ph. D.
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ADVANCED GRADE.

INCANDESCENT LIGHTING.

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219. When a new incandescent lamp is connected to mains which supply it uniformly with the pressure for which it was designed, say, for example, a pressure of 110 volts, the lamp having an initial resistance when hot of 252 ohms, the current through the lamp will be 0.4364 ampere, and the activity in the lamp will be 48 watts; or, if the lamp supplies 16 c. p., an efficiency of $\frac{1}{3}$ candle per watt. The first effect of the high temperature upon the filament, may be to reduce its resistance by a coking or carbonizing process sustained by heating in a vacuum. The current, therefore, which passes through the lamp, together with the activity of the lamp, will increase in corresponding measure, thereby increasing the temperature of the filament, and the candle-power as well as the efficiency of the lamp. This diminution in resistance, which, however, does not occur in all lamps, soon ceases, and, after say twenty hours, the resistance of the filament begins to increase.

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The candle-power of the lamp attains its maximum with the minimum resistance of the filament.

220. The temperature of the filament in incandescent lamps, operated at an efficiency of $\frac{1}{3}$ candle per watt, is estimated to be about 1345° C.; at $\frac{1}{4}$ candle per watt about 1310° C.; and at $\frac{1}{4.5}$ candle per watt about 1290° C. An increase of one per cent. in the activity of a glowing lamp, i.e., in the number of watts it absorbs, is believed to increase the temperature of its filament about 2° C. and its candle-power or its total flux of light about three per cent.

221. The progressive increase in the resistance of the filament during use, is due to the reduction in the diameter and cross-sectional area of the filament. This reduction in the diameter of the filament is brought about by one or all of the following causes; namely,

(*1.*) Mechanical, by the explosive evolution of occluded gases in the surface layers of the filament.

(*2.*) Chemical, by the removal of the surface layers of the filament through chemical combination with some of the constituents of the residual gases in the globe.

(*3.*) Physical, by electrical evaporation of the surface layers under the influence of high temperature and electrification.

222. Not only is the diameter of the filament decreased and its resistance thereby increased, but the emissivity of the surface is considerably increased.

The temperature of the filament is thus lowered during the use of the lamp for two reasons; first, because the emissivity of the surface increases by reason of the surface change, thus enabling the same quantity of activity per unit surface to be radiated at a lower temperature, and secondly, because the diminished conductance of the filament causes it to take less activity from the mains in the same proportion. There is thus less activity in the lamp and also less temperature elevation required to radiate the activity that remains.

223. The carbon which is thus removed from the surface of the filament is slowly deposited on the inside of the lamp globe in a dark semi-opaque layer, cutting off some of the light emitted by the filament and, therefore, tending to reduce the efficiency of the lamp. Each of these three causes; namely, increased resistance, increased emissivity, and increased opacity, decreases the efficiency of the lamp to approximately the same degree. As a consequence, new lamps starting at an efficiency of $\frac{1}{3}$ candle per watt, steadily decrease in their efficiency after the first few hours, until an efficiency even lower than $\frac{1}{6}$ candle per watt may be ultimately reached.

224. The physiologically effective luminous radiation from a lamp increases rapidly with the current, between the fifth and sixth powers of the current strength. Since at incandescent temperatures, the temperature coefficient of variation in the resistivity of the filament is small, and a small change of temperature is accompanied by a great change in candle power, it follows that the candle-power of a lamp varies with the terminal voltage between its fifth and sixth powers, and therefore approximately as the cube of the intake in watts.

Since the efficiency of a lamp steadily decreases with its continued use, there must come a time when even if the filament does not break, the light emitted becomes so small in proportion to the power consumed, that it may be more economical to destroy the lamp and replace it by a new one, than to continue its use at such low efficiency. The length of time during which it will be advantageous to continue the use of such a lamp will depend on the cost of electrical energy and on the cost of new lamps. Although this may be determined on a large scale of operation, as in central station lighting, it is practically impossible to lay down an inflexible rule for the economical breaking point of any lamp, since it is evidently economical to retain a lamp in employment so long as it supplies sufficient light to meet the purposes required of it.

225. In large cities in the United States, practical experience in central station work shows that the maximum load is approximately 50 per cent. of the total number of lamps connected with the system, that the average load is approximately 27 per cent. of the maximum load and the minimum load from 10 to 20 per cent. of the maximum load.

226. Attempts have been made at different times to produce a lamp capable of being regulated in candle power, when supplied from constant potential mains, thus corresponding to the gradual turning off at the key in a gas burner. This has been accomplished in the case of the incandescent lamp in two ways; first, by introducing additional resistance into the lamp circuit and, second, by reducing the time during which, in periodic contacts, the filament is in connection with the mains. Both methods result in a considerable reduction

of efficiency in the lamp, and a diminished temperature of the filament, so that the light is not only more expensively produced but also becomes duller in color.

227. In large installations where the number of lights required is great and the distance from the supply centre not excessive, incandescent lamps are almost invariably connected to the supply mains in parallel. The parallel connection method of distribution is both simple and economical. But where the district to be lighted is scattered, necessitating long circuits on which the density of lighting is not great, this method becomes very expensive in all cases where a comparatively small drop is to be maintained on the supply mains. In such cases it is more economical to employ a high tension system; that is, either a series-connected system; or, as is more common, an alternating current system in connection with transformers. In a series incandescent system, the lamps are connected to the circuit in series. The resistance of series incandescent lamps is usually comparatively small and the current they take greater than in multiple incandescent lamps. In many cases incandescent lamps are connected in series arc circuits, and, therefore, require to be operated by the current generally employed in such circuits, namely, about 10 amperes.

228. The rupture of a filament, which merely extinguishes the lamp in a multiple-connected circuit, in a series circuit extinguishes all the lamps in that circuit, unless a device be employed to cut out the imperfect lamp. This is frequently accomplished by means of a *film cut-out.* Fig. 73 shows a series incandescent lamp and a film cut-out arranged in the lamp base. This

cut-out consists of a film of paper which insulates perfectly at a pressure of 20 volts, but breaks down completely under a pressure approaching that of the full pressure in the circuit, so that the two contact points separated by the film become welded together as soon as the lamp breaks. This cut out is placed either in the base of the lamp or in the socket.

529. Since one per cent. change in the pressure supplied to the terminals of an incandescent lamp, above or below the normal pressure, produces about 5 per cent. change in the amount of light supplied by the lamp, it is necessary to ensure that the drop of pressure in the mains supplying different lamps shall not be excessive. Where a large number of lamps have to be supplied in parallel from a network of mains, the pressure will be lowest at the most distant lamps. If when all the lamps are lighted, the maximum drop of the most distant lamps amounts to, say, 10 per cent. of the pressure of the dynamos, then it is desirable to make the average pressure for the whole system, the normal

Fig 73.
Series Incandescent Lamp with Film Cut-Out.

pressure for which the lamps are designed. In this case the distant lamps will be operated at 5 per cent. below pressure, while those nearest to the dynamo will be operated at 5 per cent. above pressure. This will reduce the average life of the nearest lamps in a very marked degree, while the distant lamps, will be below candle-power by an amount which depends upon their normal efficiency. The usual range of drop permitted in the wiring of buildings supplied by their own dynamos is from 2 to 5 per cent. of the pressure at the dynamo terminals, according to the size of the building, i.e., from 1 to 2½ per cent. above or below the normal mean. So that, allowing 5 per cent. drop, if the normal voltage of the lamps be 110, the dynamo pressure would be 112.8 and the pressure at the lowest lamp 107.2 volts. The lamps nearest the dynamos would, therefore, give say 19 candles initially and the lamps furthest from the dynamo 13.5 candles.

230. The difficulty arising from drop experienced in the lighting of a single building, is greatly increased when the lighting has to be extended over a large area, in a city, from a single central station. In such cases excessive drop may be avoided by the use of suitably located and proportional *feeders*. A feeder is a conductor, one end of which is connected to the bus-bars at the station, and the other end is connected to some point on the mains, there being no lamps connected directly to the feeders, so that the mains supply the lamps, while the feeders supply the mains. In this way it is possible to maintain say 110 volts at a very distant lamp, with a drop of perhaps three volts in the mains, making 113 volts at the feeding point, but with a drop

of 17 volts in the feeder or feeders, and a pressure of 130 volts at the central station. In other words, it is possible to have 15 or 20 per cent. drop in the feeders and only a very small range of drop in the mains and house wires.

The number and size of feeders employed in distributing currents from a central station depends upon the cost of the power which has to be expended in the drop under existing conditions of load, upon the cost of the various sizes of feeder, upon the facility with which the pressure can be maintained uniform at feeder terminals and its effect upon the average life time and proper operation of lamps. Feeders have to be so selected that the total cost, including these items, together with depreciation shall be a minimum.

SYLLABUS.

The resistance of an incandescent lamp increases with its use.

This increase is due to a decrease in its diameter that may be produced by mechanical, chemical and physical causes.

The effect of the decrease of the diameter of the filament is not only accompanied by an increase in its resistance, but also by an increase in the emissivity of its filament and by a blackening of the globe.

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A. E. Kennelly, F. R. A. S.

ADVANCED GRADE.

ARC LIGHTING.

231. The voltaic carbon arc consists of a bow-shaped cloud of volatilized carbon, formed between the points of two carbon electrodes by the passage of an electric current. In order to produce this current, a pressure of from 30 to 55 volts has to be maintained between the carbon electrodes. This pressure is required by reason of resistances in the arc, and partly by a C. E. M. F. The C. E. M. F. depends upon the quality of the carbons employed and the temperature attained. When the arc is very short, the temperature is comparatively low, and the C. E. M. F. comparatively small, but for arcs of $\frac{1}{8}$ in. or more, the C. E. M. F. varies from 35 to 40 volts, and the drop due to resistances increases with the length of arc, being about 50 to 75 volts per inch, (20 to 30 volt per cm.), of distance between the electrodes. The current strength employed varies from 3 to 200 amperes, according to the size and nature of the carbons. Consequently, the activity in the carbon voltaic

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arc may have a very wide range. The ordinary commercial arc lamp takes about 10 amperes at 45 volts pressure between lamp terminals, and, therefore, has an activity of about 450 watts. Such an arc lamp is sometimes described as a 450-watt arc lamp, but is more precisely described as a 450-watt 45-volt arc lamp.

232. Of the E. M. F. in the arc lamp, about 38 to 40 volts are usually developed at the surface of the positive electrode, 4 to 6 volts in the arc proper, and the remainder in the carbon rods and the electromagnetic apparatus. The activity is, therefore, about 400 watts at the surface of the positive carbon, 40 to 60 watts in the arc proper, and the remainder in the substance of the carbons and conductors. So large a proportion of the activity at the positive carbon surface, necessitates the development there of an exceedingly high temperature, and, consequently, this is the principal source of light in the voltaic arc, about 85 per cent. of all the light being emitted from the glowing surface of the positive carbon, about 10 per cent. from the arc proper, and about five per cent. from the surface of the negative carbon.

233. The high value of the C. E. M. F., developed at the positive surface, is analagous to the C. E. M. F. developed at the surface of the electrode in an electrolytic cell. In this case, however, the work is not expended in freeing ions from molecular combinations, but in volatilizing carbon, and the high temperature is accompanied by intense radiation. Consequently, all the volatilization occurs at the end of the positive carbon, which is thereby hollowed out in the form of a minute *crater*, the principal source of light in the arc. The character of the lumin-

ous radiation will depend on the quality of the carbon electrodes and their distance apart; but, with the same material, the distance being maintained constant, the temperature at the surface of the positive carbon will be uniform if the current strength is adequately maintained. Carbon having, like other substances under fixed conditions, a fixed temperature of volatilization, (estimated at 3,500° C.), the temperature at the positive carbon, and consequently the intensity of radiation, are thereby determined. It has been found that the brightness of the positive crater amounts to about 16,000 British standard candles per square centimetre, or, roughly, 100,000 candles per square inch.

The negative carbon is at a temperature sufficiently below that of the positive to permit the volatilized carbon to be condensed upon its surface in the shape of a small mound or nipple of graphite.

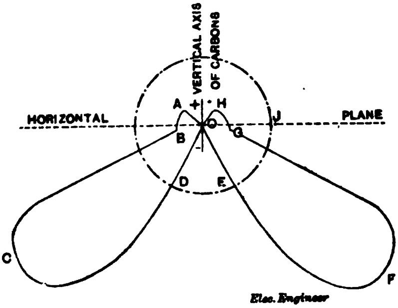
234. The temperature of volatilization of carbon being so much greater than that at which carbon monoxide forms, is probably above the dissociation temperature of carbon and oxygen, so that the carbon vapor can only oxydize at the external surface when the temperature suddenly falls. This accounts for the coating of flame which surrounds the arc itself, and for the comparatively slow rate of carbon consumption.

235. With the carbons arranged as is usual in street lamps with both carbons in the same vertical line the positive above the negative, the amount of light given off from the arc varies in different angular positions. This difference in the intensity of the emitted light is due to the following circumstances:

(1.) The positive crater, the main source of light, is not a plane surface, but is concave; the principal distribution of its radiant flux is, consequently, downwards.

(2.) The crater being surrounded by a wall of opaque carbon, the horizontal intensity of the emitted light is comparatively small, and, since the edges of the wall are irregular, the horizontal intensity varies in different azimuths.

(3.) However closely the axis of the two carbon electrodes may be aligned, the arc will rarely remain long at the centre, but tends to travel around the edges of the positive carbons, thereby causing the crater to appear on the side of the arc, and tending to increase the illumination on that side.



Elec. Engineer

Fig. 74.

Diagram Indicating Luminous Intensity of an Arc Lamp in Different Directions.

(4.) The arc is usually accompanied by some flame of a reddish color surrounding the arc proper, like a luminous cloud, and the light from this source is of an unstable flickering nature, shifting irregularly.

5. Even the smallest mechanical irregularity or chemical impurities, liable to be present in the best carbons, develop fluctuations in the intensity of the light.

Fig. 74 represents graphically, in polar co-ordinates, the relative distribution of luminous intensity from an ordinary 500-watt, 50-volt arc lamp. It will be seen that the maximum intensity is developed at an angle which is approximately 50° below the horizontal plane. The horizontal intensity is usually only about 10 to 20 per cent. of the maximum intensity, and, in the actual example here represented, O B, or O G is only about 11 per cent. of O C, or O F. The horizontal intensity is subject, owing to the causes above mentioned, to much greater fluctuations than the maximum intensity. The *mean spherical candle power* is the average intensity measured in candle-power for all directions. The mean spherical candle-power is usually about 30 to 40 per cent. of the maximum intensity and from 2 to 4 times greater than the horizontal intensity. In Fig. 74 the radius O J is, in length, 30 per cent. of the radius O C, or O F. The ordinary empirical formula which fairly expresses the observed relationship between the spherical and maximum intensities is,

$$\text{Mean spherical intensity} = \frac{\text{Mean horizontal intensity}}{2} + \frac{\text{Maximum intensity.}}{4}$$

It is evident that the total quantity of light emitted will be $4\pi \times$ mean spherical intensity in units of luminous flux.

236. The nominal intensity of an arc lamp, as for example, that of a 2,000 candle-power arc, means the maximum intensity of the arc under favorable conditions of carbons and operation. A preferable rating, however, would be either by the total luminous flux of the arc lamp or its mean spherical candle-power.

The greater the current strength through an arc lamp the greater the surface which becomes elevated in temperature. If the carbons are too small, this will be acaccompanied by flaming disintegration, and other disturbances, but if the carbons be suitably increased in diameter, the increase in total luminous flux will be safely obtained. For a given arc lamp of 48 volts pressure, it has been observed that an empirical relation exists between the intensity of the current strength fairly expressed by the formula,

$$\text{Maximum luminous intensity} = 190\, i + 4\, i^2,$$

where i = current strength in amperes.

So that a 480-watt, 10-ampere arc lamp gives under favorable conditions a maximum intensity of 2,300 candles.

237. A great difficulty exists in accurately measuring the candle power of an arc-lamp by the use of any of the ordinary standards of light. This difficulty is due not only to the rapid fluctuations constantly occurin the arc, but also to the difference in the character between the light of the arc and that of the ordinary standards with which it is compared. An arc lamp is particularly rich in waves of high frequency; namely, those near the violet end of the spectrum, and the eye is unable fairly to match intensities between lights of essentially different colors. This difficulty has led to the proposal of a special standard for arc lamp photometry, based on the relation which has been found to exist between the amount of light emitted per unit surface of the crater in the positive carbon.

The arc lamp whose luminous intensity is to be measured is compared in the photometer with what might be called *a unit-arc-light crater-intensity*, which consists of a standard arc lamp whose crater is exposed to the photometer through an opening of standard dimensions in an opaque and artificially cooled screen.

Most arc light carbons, in use in the United States, are provided with a thin metallic coating of copper, electrolytically deposited. The effect of this coating is not only to decrease the resistance of the rods, but also to prevent irregular burning and disintegration. Carbons so protected are more apt to burn with comparatively blunt ends, and, therefore, to last longer. Unprotected carbons are apt to burn in points, and thus seriously interfere with the proper feeding of the carbons, that is, their automatic adjustment as to distance.

238. During use, the carbons consume unequally.

The positive carbon consuming roughly twice as rapidly as the negative carbon. The more rapid consumption of the positive carbon is due not only to the higher temperature but also to the fact that it is volatilized. For this reason the positive carbon is generally made about twice as long as the negative carbon. Attempts have been made to prolong the duration of the carbons by increasing their diameter, but whenever the diameter of the carbon exceeds a certain limit, depending upon the strength of the current, the light becomes unsteady, owing to the tendency of the arc to travel around the edges of the larger cross section offered. Less objection is experienced to increasing the diameter of the negative carbon, alone, but even here, the increased duration is accompanied by increased fluctuations in the light.

The average length of the positive carbons employed in systems of street lighting does not usually exceed 12 inches, the length of the negative carbon being about seven inches. Such a pair of carbons will ordinarily last about 7 hours when $\frac{7}{16}$ in. in diameter, and about 9 hours when $\frac{1}{2}$ in. in diameter. Consequently, during prolonged runs, such as are necessitated during winter, a lamp provided with but a single pair of such carbons would require re-carboning during the night. In order to avoid this, various expedients have been adopted, such as an increase in the diameter of carbons already alluded to. The method in general use is that of employing two pairs of carbons side by side, so arranged that one pair of carbons is first consumed and the second pair is then automatically switched into the circuit. Such a lamp is commonly called a *double-carbon*, or *all-night lamp.*

SYLLABBUS.

The C. E. M. F. of a carbon voltaic arc is principally resident at the surface of the carbon electrode or crater.

It is commonly considered that a 450-watt 45-volt arc-lamp gives a maximum of 2,000 candle-power, but this is only true under the most favorable conditions.

An arc light differs principally from the light of candles, incandescent lamps and other luminous sources, in being richly provided with luminous waves of high frequency.

The mean horizontal intensity of an arc lamp is much more variable than its maximum intensity, but is commonly about 15 per cent. of the maximum intensity.

The mean spherical intensity of an arc lamp is about 35 per cent. of its maximum intensity.

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ADVANCED GRADE.

ARC LIGHTING.

239. On constant current circuits, arc lamps are generally operated in series and are then supplied by special generators known as arc-light dynamos. These generators are always series-wound. The number of lamps operated in series is commonly about 50, though occasionally it reaches about 100, and in rare instances 200. The largest arc generator yet constructed being for 200 lamps, a pressure of about 10,000 volts exists between machine terminals. Since the number of arc lights in a circuit is seldom constant, the generator must maintain a constant current under all conditions and must be able to vary the E. M. F. it generates.

When a series arc-light circuit, Fig. 75*a*, containing say 10 lamps, and having a total pressure at machine terminals of 500 volts, is perfectly insulated from the ground, there will, by symmetry, be a difference of potential between the positive brush and the ground of 250 volts, and a difference of potential between the negative brush

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and the ground of an equal amount, while a point B, in the circuit, situated electrically midway between the terminals, will be at zero potential, and could, therefore, be connected to the ground without in any way disturbing the pressures or current strengths in the circuit. If, however, instead of connecting the circuit to ground at its central point, the ground connection were made at any other point, such, for example, as at the positive terminal of the generator, D, Fig. 75*b*, that point would be reduced to zero potential, the symmetry of pressure at the dynamo terminals would be disturbed and the potential of the negative terminal would become 500 volts. Similarly, if the ground connection, instead of being made at D, were made at G, Fig. 75*c*, the potential at that point would be reduced to zero, producing the distribution of potential shown. There would, however, be no permanent flow of current from the circuit through the ground connection while the insulation of the rest of the

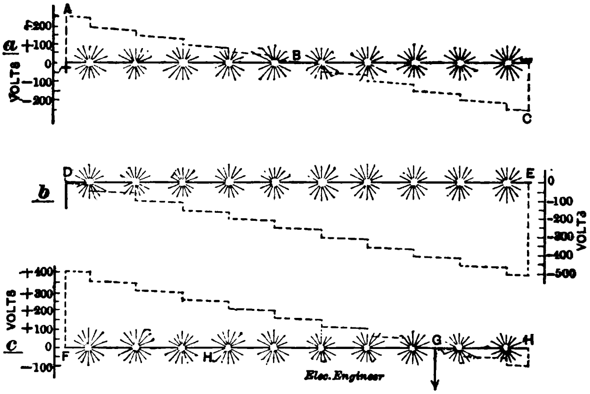

FIG. 75.
Distribution of Electric Potential in Continuous Current Series Arc Circuit.

circuit is preserved. If, however, a second ground connection occur, then a current would flow through both ground connections of a strength determined by the resistance of the ground connections and the difference of potential between the points of contact.

Thus if, Fig. 75*c*, a slight leak to ground were attached at the point H, the E. M. F. tending to send a current through the leak to ground would be the difference of potential between H and the ground, or 250 volts. A man standing on the ground at H and coming in contact with the wire would be subjected to a pressure of 250 volts.

240. Arc lights are sometimes operated on constant-potential circuits, usually at from 110 or 220 volts pressure. In Fig. 75 ten arc lamps are represented as being connected in series, and the extremities of the circuit are assumed to be connected to a generator directly. The advantage of the series arc-circuit is a high pressure and the small diameter of conductor which may be employed for conveying the total strength of current, usually 10 amperes, and this, in a district of street lighting covering an extended area, is a matter of considerable importance. The advantage of multiple-arc lighting is found in the combination of arc lamps with incandescent lamps, from the same circuit, without additional wires or generators. In many cases it is more economical to place arc lights on already existing incandescent circuits, rather than establish entirely separate series-circuits and a special generator, even though the amount of copper required in the circuits be considerably increased. This is partly on account of the increased simplicity of the system, the reduced space and cost of generators, and partly

on account of the greater efficiency of incandescent generators. But the conditions under which economy exists in the use of constant potential arc lamps are limited, and each case must be determined on its own merits.

Two lamps are generally operated in series on a 110 volt circuit, and four lamps in series on 220 volt circuit. A small resistance is usually inserted in the circuit of each lamp to assist in its regulation. The usual current strength employed in constant-potential lamps is from 4 to 10 amperes. The best results are obtained with a cored carbon, for the positive electrode. With the usual twelve inch positive and seven inch negative carbons, of $\frac{7}{16}$ inch diameter, a lamp with a current of nine amperes will last nearly nine hours.

241. Arc lamps are sometimes operated from special generators on alternating current circuits. In such cases, since the carbons are alternately positive and negative, neither crater nor nipple forms on the carbons, which burn with comparatively blunt points. In the alternating current arc, the temperature is evenly distributed; consequently, the horizontal candle-power does not differ so markedly in intensity from the maximum, and there are two directions of maxima, one upwards and one downwards, as shown in Fig. 76, which represents in polar co-ordinates the distribution of light in a particular case. The distribution varies considerably with different current strengths and characters of carbon. Alternating current arcs are usually operated from local step-down transformers, by which a pressure of from 28 to 35 volts alternating is supplied directly to the lamp terminals. The current in the main or primary cir-

cuit of the generator is usually about 30 amperes, instead of 9 or 10, as in the continuous-current circuit, by which means a lower total pressure for a given number of arcs can be obtained.

242. In the application of the voltaic arc to searchlights, exceedingly powerful currents are employed with suitably proportioned carbons, so that a very great intensity of light is obtained. In order to throw this into an approximately parallel beam, instead of diffusing it in all directions, the carbon arc is formed at the focus of a suitable projector. These projectors may be catoptric, i. e., reflecting; or, dioptric, that is, refracting.

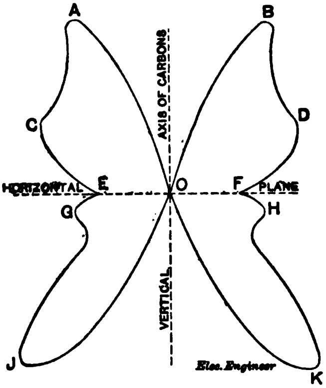

Fig. 76.

Distribution of Light from an Alternating Current Are as measured in a particular case.

The usual form is, however, a reflector of the parabolic or spherical type. Since large parabolic projectors are very expensive, a spherical reflector is generally employed. This consists of a lens-shaped mass of glass the two sides of which have different radii of curvature, the exterior surface being silvered. The light from the carbon arc, on entering the substance of the glass, suffers refraction, and it is then reflected from the silvering at the exterior surface again suffering refraction on issuing from the glass into the air. The curvatures are so chosen with regard to the index of refraction of the glass, that the light emerges in a sensibly parallel beam. The carbons are usually tilted at such an angle that the crater is most effectively exposed towards the reflecting mechanism. In many cases a small reflector is employed near the arc to throw its light on the lens.

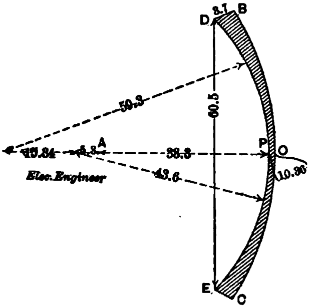

FIG. 77.
Speciment of Magnetic Reflector. Section through Axis. Dimensions in Centimetres.

The beam of light issuing from a search light projector being only approximately parallel, necessarily diverges and lessens in intensity with the distance from the apparatus. Beyond a distance of a few hundred feet, the illumination produced by the beam approximately varies inversely as the square of the distance. Neglecting, however, the absorption of light by dust and fog in the atmosphere, the total flux of light in the beam remains constant, so that the area of the beam increases after the first few hundred feet approximately as the square of the distance.

A Mangin reflector is represented in axial section in Fig. 77. The arc being placed at the principal focus A, which in this case is 38.3 cms. from the centre of the inner surface, throws a beam parallel to P A, of diameter D E, the surface B O C, being silvered.

243. Carbon arc lights for street lighting are generally surrounded by globes, which may be clear or ground. In either case a loss of light is thereby entailed, but a more uniform diffusion of light obtained. This is especially the case where the globe is ground or consists of translucent glass or porcelain. The loss of light may amount to as much as 60 per cent., but the general illumination produced is better and shadows are avoided.

SYLLABUS.

Arc lights are generally operated commercially on series circuits from specially designed dynamos, generating sufficient difference of potential to maintain the current constant under all conditions. Arc lamps are sometimes operated on constant-potential circuits, with either two or four lamps in series.

Arc lamps are sometimes operated on alternating-current circuits from transformers.

Alternating-current lamps have two maxima of light intensity, one upwards and one downwards, no marked crater being formed.

Reflectors are commonly used with large search lights.

Globes placed around arc lights, while useful for diffusing the light, cut off from 10 to 60 per cent of the total light emitted.

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ADVANCED GRADE.

Alternating Currents.

244. The comprehension of the following definitions is necessary to a clear understanding of alternatting currents.

An *alternating* E. M. F. or *current* is an E. M. F. or current which successively reverses its direction.

Fig. 78 is a graphical representation of an E. M. F. or current which is alternating, for it successively changes its sign, being, say, in the positive direction at *a*, and in the negative direction at *c*, while at *b d f h k*, it has zero value and no direction.

A *periodic alternating* E. M. F. or current, is an alternating E. M. F. or current which not only periodically reverses its direction, but also periodically repeats its changes in magnitude. Figs. 79 to 84 represent periodic alternating E. M. F.'s or currents.

The terms alternating E. M. F., or alternating current, as ordinarily employed, designate periodic alternating E. M. F. and current, respectively.

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Each reversal of a periodic alternating E. M. F. or current is called an *alternation* or *semi-period*. Thus in Fig. 79, *o a b c*, or *c d e f*, or *j k l m*, each represent one alternation or semi-period.

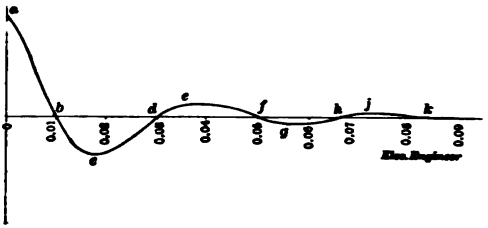

Fig. 78.
Graphical Representation of Non Periodically Alternating Current or E. M. F.

A complete double alternation or double reversal is called a *cycle*. Thus in Fig. 80, the curve *o a b c d e f g h*, represents one double reversal or cycle. It is not necessary, however, that the cycle commence at the zero point. Thus *c d e f g h j k l*, or *f g h j k l m n p*, each represent one cycle.

The time occupied in executing a cycle is called a *period*. Thus in Fig. 79, the period is 0.01 second. In Fig. 80 the period is 0.1 second, and in Fig. 82, 0.004 second.

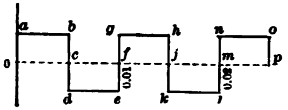

Fig. 79.
Periodic Alternating E. M. F. or Current.
Rectangular Type.

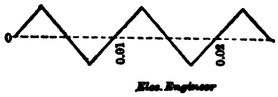

Fig. 81.
Periodic Alternating E. M. F. or Current.
Zig-zag Type.

The number of periods described in one second, or the reciprocal of the period, is called the *frequency*,

245. Thus in Fig. 79, the frequency is 100 ~ ; that is, 100 periods per second; in Fig. 80, 10 ~ and in Fig. 82, 250 ~.

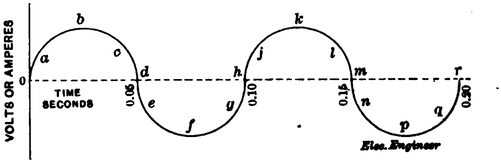

FIG. 80.
Periodic Alternating E. M. F. or Current. Circular Type.

When the E. M. F. or current periodically reverses, it may do so in an infinite variety of ways; namely,

(1.) It may change suddenly from the positive maximum value to the negative maximum value, and vice versa, as shown in Fig. 79. The graphical representation of these reversals is a rectangular curve.

(2.) It may change gradually from the positive maximum value to the negative maximum value and vice versa, as shown in Fig. 81.

(3.) It may change from the full positive value to the full negative value according to a definite law, more complex than either (1) or (2). Such changes are shown in Figs. 78, 82, 83 and 84. Thus, Fig. 78 represents graphically a circular variation or type of wave; Fig.

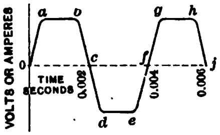

FIG. 82.
Periodic Alternating E.M.F, or Current.
Flat-Topped Curve.

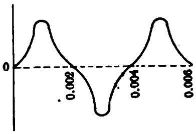

FIG. 83.
Periodic Alternating E.M.F. or Current.
Peaked Curve.

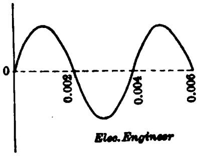

FIG. 84.
Periodic Alternating E.M F. or Current.
Sinusoidal Curve.

82, a flat-top type of wave; Fig. 83, a peaked type of wave and Fig. 84, a sinusoidal wave.

246. A *sinusoidal* E. M. F. or current is one whose successive magnitudes with respect to time vary as the sine of a quantity proportional to the time, or which alternates in time according to a simple harmonic law.

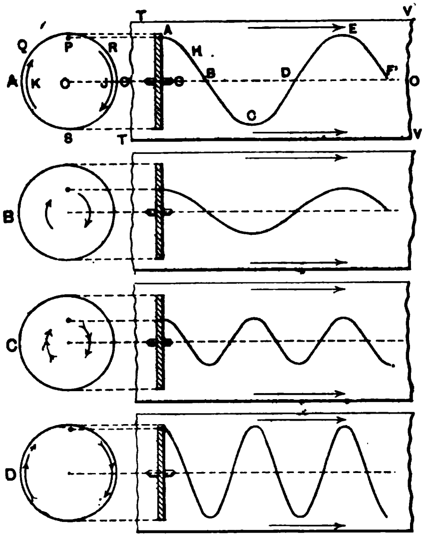

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FIG. 85.
Graphical Representations of Simple Harmonic E. M. F.'s or Currents.

An example of a simple-harmonic motion (abbreviated. S. H. M.) is seen in the motion of a vertically falling shadow projected upon a horizontal plane, from a point on a vertical disc which is uniformly rotating about a horizontal axis. Thus in Fig. 85, the disc Q R S, revolving uniformly about its horizontal axis at O, has a pin P, whose shadow falling vertically upon a band of paper T V, lying in a horizontal plane, produces an S. H. M. upon the paper. If, therefore, the paper be moved in the direction of the arrow in its own plane and in a direction at right angles to the plane of the disc, the shadow will trace on the paper a *simple-harmonic or periodic curve,* A B C D E F, and this curve is called *sinusoidal,* because any ordinate, such as O A, is proportional to the sine of the angle contained between the radius of the pin and the vertical plane through the axis of the disc. It is evident that the outline of the sinusoidal curve so traced depends upon the distance of the pin from the axis, and the velocities of disc-rotation and paper-progression. Thus, assuming the paper to move with the same velocity in the four cases represented at *A*, *B*, *C* and *D*, respectively, then the shape of the curve will only depend upon the position of the pin and on the velocity of its rotation. Thus, the pin is shown in the same position at *A*, and at *D*, namely, near the edge of the disc; but since the velocity of rotation at *D*, is twice that at *A*, the cycle is completed in half the time at *D*, and the frequency of the periodic motion is, therefore, twice as great at *D*, as at *A*. At *B* and *C*, the pin is shown half way between the centre and the edge of the disc, while *C*, has twice the rotary velocity of *B*. The amplitude or maximum ordinate in *B* and *C*, is half that in *A* and *D*. The frequencies in *C* and *D* are equal, and the frequencies in *A* and *B*, are equal. The waves represented at *A*, *B*, *C*, or *D* are, therefore, all sinusoidal waves in spite of their differences of appearance, and any E. M. F.'s or currents, whose successive changes in time are represented by such curves are sinusoidal E. M. F.'s and currents.

Calling t, the time in seconds dating from the initial or zero position, ω, the angular velocity of the disc in radians per second, and Y, the amplitude, the y—ordinate at any instant is $y = Y \sin \omega t$.

The angle contained between the radius vector O P, and the initial ascending vertical radius O K, is called the *phase* of the motion. Thus the phase of the point C, in the curve of A, is 270°, the phase of the point E, 90° and of B and F, zero.

247. A coil of insulated wire, rotated with uniform velocity about any diameter as axis in a uniform magnetic flux, has generated in it a sinusoidal E. M. F. In practice *alternators*, or dynamos for producing alternating E. M. F.'s, never produce strictly sinusoidal E. M. F.'s, although they frequently generate E. M. F.'s that are sufficiently nearly sinusoidal to be considered as true sinusoids for purposes of computation. The E. M. F. generated by any practical alternator may be regarded as lying between the flat-topped type of Fig. 82 and the peaked type of 83.

248. Since an E. M. F. or current which is changing its direction many times a second, has, at different instants of time, all values comprised between its maximum and zero, it becomes necessary to define conventionally the numerical value of such an E. M. F. or current. If this value were taken as being equal to the maximum, the E. M. F. or current could only attain its nominal value twice in each cycle. If the arithmetical mean value without regard to sign be taken, the value so obtained is called the *mean E. M. F.* or *mean current* but this value has very little practical application. The value of an alter-

nating E. M.F. or current is most conveniently defined by reference to its heating power and this is the method invariably adopted. If a given continuous-current pressure maintains a given thermal activity in a fixed resistance, then the alternating E. M. F., which will maintain the same thermal activity in the resistance, will have the same nominal value expressed in volts, so that this value, which is called the *effective value of the* E. M. F., may be regarded as the value of the continuous E. M. F. required to produce the same thermal activity. Since, according to Ohm's law, the thermal activity maintained in a resistance R, by an unvarying E. M. F. of e_1 volts, is $\frac{e_1^2}{R}$ watts, the activity developed by a periodically varying E. M. F., at any instant, will be similarly $\frac{e^2}{R}$ watts, where e, is the instantaneous value of the E. M. F. The value of e^2, if averaged over a sufficiently long time, will correspond to the square of the effective E. M. F. e_1; or in mathematical language, if E, be the effective E. M. F.,

$$E^2 = \frac{1}{\tau}\int_0^\tau e^2\,dt,$$

where τ, is the time occupied in any complete cycle. In the case of a sinusoidal or simple harmonic E. M. F., the effective E. M. F. is always the maximum E. M. F. in the cycle divided by $\sqrt{2}$, so that the effective E. M. F. = 0.7071 maximum E. M. F.

SYLLABUS.

A sinusoidal E. M. F. or current is one whose successive magnitudes with respect to time are represented by the ordinates of a sinusoidal or simple-harmonic curve.

No alternators generate strictly sinusoidal E. M. F.'s, but many generate E. M. F.'s which are sufficiently nearly sinusoidal to be regarded as such for purposes of computation.

The effective value of an alternating E. M. F. or current is the square root of the time average of its square.

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249. When two simple harmonic E. M. F.'s, of the same frequency, are simultaneously impressed on the same circuit in series, as, for example, when two similar alternators are driven on the same shaft, the effect produced will depend upon the relative phases of the two E. M. F.'s; for, if the two E. M. F.'s are in phase their combined effect or resultant will be equivalent to their arithmetical sum, or to that of a single source of double E. M. F. If, however, the two E. M. F.'s are in opposite phase, their resultant will be zero, and between these two conditions there will be an infinite number of possible intermediate resultants. In all cases, however, the sum of their E. M. F.'s will be their vector or geometrical sum.

250. In order to determine the value of the sum of two or more vectors in a plane we proceed as follows. Let A B, be a quantity whose magnitude and direction are represented by the line A B, Fig. 86, and let C D, be a

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quantity whose magnitude and direction are represented by the line C D; then the vector sum of A B and C D, will be found by laying off from B, a line parallel to and

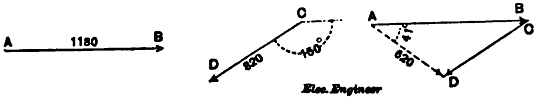

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FIG. 86.

equal to C D, so that the straight line A D, will be the vector sum. If now A B, represents at some instant of time the relative position of the sinusoidal E. M. F., of say 1180 volts effective, a second sinusoidal E. M. F. of say 820 volts effective, whose phase is 150° in rear of A B; i.e., $\frac{5}{12}$ of a complete revolution, or period, will be represented by C D. For if the lines A B and C D, be each set revolving counter-clockwise about the extremities A and C, respectively, with equal and uniform angular velocities in the plane of the paper, then if C D, lags 150° behind A B, when A B, reaches the position shown C D, will also occupy the position indicated. Their vector sum A D, will be the resultant or combined E. M. F. of these two generators when placed in series at this definite phase relationship and will be represented by a sinusoidal E. M. F. of 620 volts effective, 41° behind A B. If, as represented in Fig. 87, two equal sinusoidal

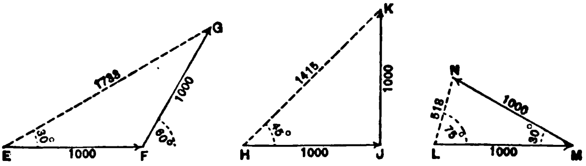

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FIG. 87.

E. M. F.'s, each 1000 volts effective, are connected in series at an angle of 60°, or $\frac{1}{6}$ period, their resultant will be a sinusoidal E. M. F. of 1733 volts, $\frac{1}{12}$ period ahead of E F, or $\frac{1}{12}$ period behind F G. If the angular displacement be $\frac{1}{4}$ period, their resultant will be 1415 volts effective, $\frac{1}{8}$ of a period later than J K; while, if the angular divergence be 150°, their resultant will be 518 volts effective, 75° in advance of L M.

Any number of co-periodic simple harmonic E. M. F.'s can be compounded into an equivalent single resultant by finding their vector sum.

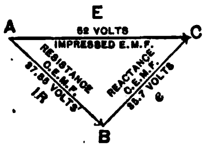

FIG. 88.

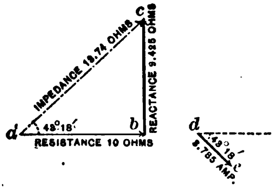

FIG 89.

The impressed E. M. F. at the terminals of an alternating current circuit is always equal to the geometrical sum of the component C. E. M. F.'s in that circuit, that is to say, if there be only resistance in the circuit, the impressed E. M. F. will be expressed by the C. E. M. F., IR; or, if there be additional C. E. M. F.'s of the type e, the impressed E. M. F. will be expressed by the vector sum $IR + e$.

This law applies to the continuous current circuit where however the sum is merely arithmatical.

If, therefore, the drop IR, in the resistance, be laid off on the line A B, Fig. 88 and the C. E. M. F., e, due to the variation of flux linkage be laid off by the line B C, at right angles to A B, and 90° in advance of it, then A C, will be the vector sum of the C. E. M. F.'s, and this must be equal to the impressed E. M. F.

251. Instead of considering the current produced in an alternating-current circuit as being numerically equal to the quotient of the resultant E. M. F. by the ohmic resistance of the circuit, in accordance with Ohm's law, it is frequently simpler to regard the impressed E. M. F. as acting alone in the circuit, and that the resistance of the circuit is altered to what is called the *impedance* of the circuit. For example, if the circuit possesses a resistance *a b*, Fig. 89 of say 10 ohms, and the frequency of the E. M. F. be 100 ~, its angular velocity will be 628.3 radians per second; i.e. the angular velocity of the revolving line in the plane of the paper will be $100 \times 2\pi$. If the inductance be 0.015 henry, the *reactance b c*, will be $0.015 \times 628.3 = 9.425$ ohms, 90° in advance of *a b*, and the impedance of the circuit will be the geometrical sum of these two components, or *a c*, 13.74 ohms, 43° 18′ in advance of *a b*. We have, therefore, in an alternating current circuit, the vector relationships,

(1) Reactance of an Inductance L henrys $= j\,2\,\pi\,n\,L$ ohms where n, is the frequency and j, is the symbol of direction or $\sqrt{-1}$, indicating that the reactance is set at right angles to the resistance.

(2) Impedance = Resistance + Reactance. Ohm's law applied to such a circuit becomes $I = \frac{E}{J}$ amperes, where J, is the vector impedance. In dividing vectors, their lengths are divided numerically and their angles subtracted. Thus, if P Q, and R S, Fig. 90 be two vectors P Q, being $0.8 \; \underline{/60^\circ}$, and R S, $0.5 \; \underline{/270^\circ}$; or,

0.5 $\overline{\diagdown 90^\circ}$, their arithmetical product will be 0.8 × 0.5 $\underline{/270^\circ + 60^\circ = 0.4\ /330^\circ} = 0.4\ \overline{\diagdown 30}$. While the quotient of $\frac{\text{P A}}{\text{R S}}$ = will be $\frac{0.8}{0.5}\ \underline{/60 - 270} = 1.6\ \underline{/-210}$ $= 1.6\ \overline{\diagdown 210} = 1.6\ \underline{/150^\circ}$ as shown at T V, and V W, Fig. 90. If then, the resistance coil last considered, have an impressed sinusoidal E. M. F. of say 52 volts, connected to its terminals, the current in the circuit will be

$$\frac{52}{13.74\ \underline{/43^\circ\ 18'}} = \frac{3.785}{\underline{/43^\circ\ 18'}} = 3.785\ \overline{\diagdown 43^\circ\ 18'}$$

as represented in Fig. 89, and the current will, therefore, lag 43° 18′ behind the E. M. F. in phase.

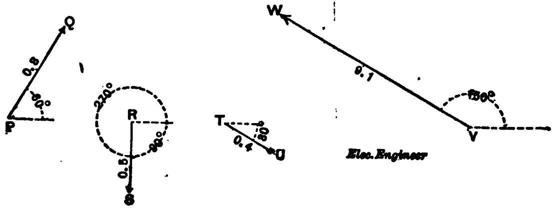

FIG. 90.

252. The reactance of a condenser is the reciprocal of the product of its capacity in farads and the angular velocity of the impressed E. M. F. Thus, if a condenser of 10 microfarads; that if, 10^{-5} farad, be connected directly with the terminals of the alternator of 100 ∼, and 1100 volts effective, as measured at terminals, the reactance of the condenser at this frequency will be $\frac{1}{10^{-5} \times 628.3} = 159.2$ ohms; but this reactance, while set off at right angles to ohmic resistance, is marked in the opposite direction to inductance-reactance as shown

at B G Fig. 91. This is for the reason that inductance and capacity in a circuit tend to neutralize each others influence, and, calling the inductance-reactance positive, capacity-reactance is reckoned as negative. The current through the condenser under these conditions will be $\frac{1100}{159.2 \ \overline{\backslash 90^\circ}} = 6.91 \ \underline{/90^\circ}$, so that the current in this case leads the E. M. F. by a quarter period. The reactance of a condenser of capacity c, farads is therefore $-j \frac{1}{c\,\omega}$ ohms, where $\omega = 2\,\pi\,n$, the angular velocity.

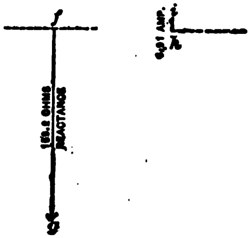

FIG. 91.

The drop in a resistance in a continuous current circuit is $I\,R$, volts. The drop in the impedance of an alternating current circuit is $I\,J$, volts.

Let for example, a coil of 50 ohms have an inductance of 0.02 henry (20 milli-henrys) and be connected in series with a condenser of 10 micro-farads capacity to the terminals of a sinusoidal alternator delivering 1100 volts effective at a frequency of 100 ~. The reactance of the condenser will be — 159.2 ohms as before. The reactance of the inductance in the coil will be $628.3 \times 0.02 = 125.7$ ohms, and the resultant reactance — 33.5 ohms as shown in Fig. 92. The impedance, or resultant of the reactance and resistance, will, therefore, be 60.2 ohms and the current in the circuit will be $\dfrac{1100}{60.2\ \overline{\backslash\ 33^\circ\ 50'}} = 18.27\ \underline{/33^\circ\ 50'}$ amperes, so that a current of 18.27 amperes effective, leads the E. M. F. by approximately 34°.

The drop at the terminals of the condenser will be $18.27\ \underline{/33^\circ\ 50'} \times 159.2\ \overline{\backslash\ 90^\circ} = 2909\ \overline{\backslash\ 56^\circ\ 10'}$ volts. The drop in the inductance if it could be isolated; that is, separated from the resistance in the coil would be $18.27\ \underline{/33^\circ\ 50'} \times 125.7\ \underline{/90^\circ} = 2296\ \underline{/123^\circ\ 50'}$ volts, and the drop in the resistance $18.27\ \underline{/33^\circ\ 50'} \times 50 = 913.5\ \underline{/33^\circ\ 50'}$ volts. As, however, the inductance and the resistance in a coil are inseparably united, the drop can only be measured on the impedance of the coil, which will be $50+125.7j=135.3$ ohms, as shown in Fig.

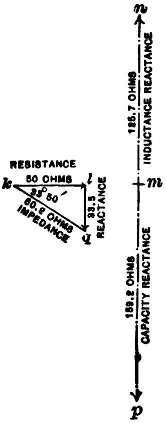

FIG. 92.

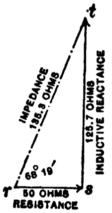

FIG. 93.

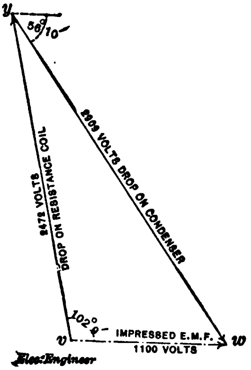

FIG. 94.

93, j, being the symbol of $\sqrt{-1}$ or an operator which rotates 125.7 through a right angle, counter-clockwise from the direction of the resistance. The drop at the terminals of the coil will, therefore be $18.27 \, \underline{/33^\circ \, 50'} \times 135.3 \, \underline{/68^\circ \, 19'} = 2472 \, \underline{/102^\circ \, 09'}$, and as shown in Fig. 94 $2472 \, \underline{/102^\circ \, 09'} + 2909 \, \overline{\setminus 56^\circ \, 10'} = 1100$.

It is evident that by the combination of an inductance with a condenser, the pressure at the terminals of a condenser may exceed that of the impressed E. M. F. The vector sum of the total C. E. M. F.'s or drops in the circuit must, however, be equal to the impressed E. M. F. as shown in Fig. 94.

If the reactance of a condenser is equal to the reactance of the inductance in a circuit, the impedance of the circuit is reduced to its simple resistance, so that an alternating-current circuit on a coil having a small resistance but large inductance, in circuit with a properly selected condenser may develop an exceedingly high pressure at the condenser and at the coil terminals. Such a circuit is said to be *resonant*.

SYLLABUS.

The sum or difference of two co-periodic, sinusoidal E. M. F.'s is their geometrical or vector sum, or difference.

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253. Since in an alternating-current circuit we have the vector equation $I = \frac{E}{J}$ amperes $= E \, . \, \frac{1}{J}$ amperes, we may write $I = E\,A$ amperes, where the *admittance* A, is the reciprocal of the impedance, and is expressed in mhos. In a continuous-current circuit, $I = \frac{E}{R} = E\,G$ amperes, where G, is the conductance in mhos (Sec. 24), and the admittance A, degrades into a simple conductance.

In a continuous-current circuit, the joint conductance of a number of separate conductances in multiple is their arithmetical sum.

In an alternating-current circuit, the joint admittance of a number of separate admittances in multiple, is their geometrical sum. For example, let two impedances be connected to the terminals of an alternator delivering 1100 volts at collector rings, with a frequency of 125 ∼,

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one of these impedances being offered by a coil of 30 ohms resistance and 0.2 henry inductance, and the other impedance being that of a condenser of 10 microfarads capacity; then the angular velocity of the E. M. F. is 785.4 radians per second. As shown in Fig. 95, the reactance of the inductance is $157.1 \; \underline{/90^{\circ}}$ ohms, and the impedance *C E*, $159.9 \; \underline{/79^{\circ}\,11'}$ ohms. The reactance

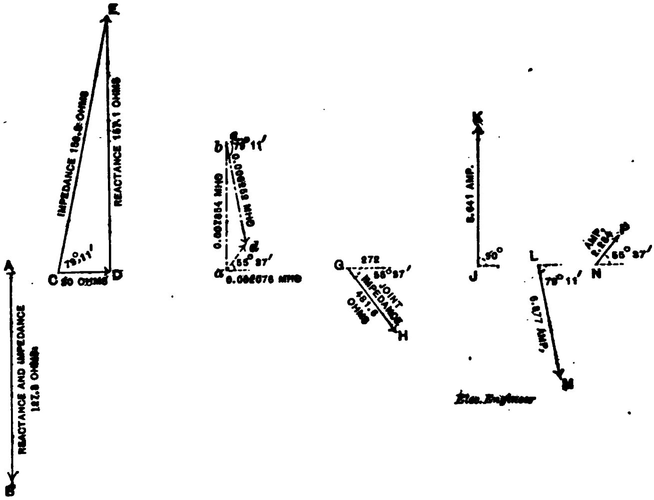

Fig. 95.

Illustrating the Combination of Impedances in Parallel.

of the condenser is $127.3 \; \overline{\backslash\, 90^{\circ}}$ ohms, and this, in the absence of resistance in series with it, becomes its impedance. The admittance of the coil is, therefore

$$\frac{1}{159.9 \; \underline{/79^{\circ}\,11'}} = 0.006252 \; \overline{\backslash\, 79^{\circ}\;\, 11'} \text{ mho},$$

as represented by the line *c d*, to a convenient scale; and the admittance of the condenser is similarly $\dfrac{1}{127.3 \; \overline{\backslash\, 90^{\circ}}}$

$= 0.007854\ \underline{/90^\circ}$ mho. The geometrical sum, or joint admittance of $\overline{a\ b}$ and $\overline{c\ d}$, is $\overline{a\ d}$, $0.002076\ \underline{/55^\circ\ 37'}$ mho. The joint impedance is, therefore

$$\frac{1}{0.002076\ \underline{/55^\circ\ 37'}} = 481.6\ \overline{\backslash 55^\circ\ 37'},$$

as represented by the line *G H*, to a suitable scale. The current supplied from the alternator will, consequently be $\dfrac{1100}{481.6\ \overline{\backslash 55^\circ\ 37'}} = 2.284\ \underline{/55^\circ\ 37'}$ amperes, as shown by the line N P. The current in the resistance coil will be $\dfrac{1100}{159.9\ \underline{/79^\circ\ 11'}} = 6.877\ \overline{\backslash 79^\circ\ 11'}$ amperes, as represented by the line L M, and the current in the condenser will be $\dfrac{1100}{127.3\ \overline{\backslash 90^\circ}} = 8.641\ \underline{/90^\circ}$ amperes, as shown by the line J K. The arithmetical sum of these two branch currents would be $6.877 + 8.641 = 15.518$ amperes, whereas the current actually supplied is only $2.284\ \underline{/55^\circ\ 37'}$ amperes, and this is the vector sum of $6.879\ \underline{/79^\circ\ 11'} + 8.641\ \underline{/90^\circ}$.

254. In a continuous-current circuit we have the condition that the sum of the currents arriving at a point in a network of conductors is equal to the sum of the currents leaving that point (Sec. 50). This is true in an alternating current circuit if we substitute the geometrical sum for the arithmetical sum.

The multiplying power of a shunt (Sec. 37) in a continuous current circuit is $\dfrac{G + S}{S}$, where G, is the resistance of the galvanometer or similar device, and S, the resistance of the shunt. In an alternating-current circuit, the same ratio holds when the computation is effected geometrically. Thus, suppose a galvanometer applicable to an alternating-current circuit; for instance, an electro-dynamometer is placed in a circuit whose frequency is 125 ~, and whose angular velocity is therefore, 785.4 radians per second, with a resistance of 6.19 ohms, and an inductance of 0.01 *H*. Its reactance, therefore will be 7.854 ohms, and its impedance, as shown in Fig. 96 of $10\ \underline{/51^\circ\ 45'}$ ohms. If this dynamometer is shunted by a simple non-inductive resistance *S*, of 10 ohms, and whose impedance is, therefore, $10\ \underline{/0^\circ}$ ohms, then the vector sum $G + S$, is shown to be $17.99\ \underline{/25^\circ\ 53'}$ ohms. This sum divided by *S*, is clearly $1.799\ \underline{/25^\circ\ 53'}$, so that the readings of the dynamometer would have to be multiplied by 1.799 in order to obtain the total current strength.

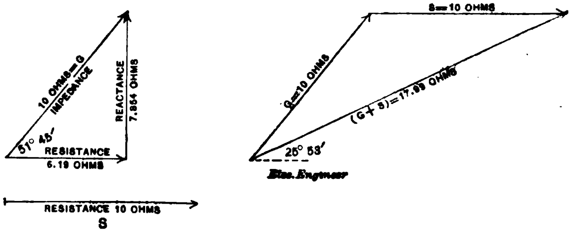

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FIG 96.
Illustrating the Multiplying Power of a Shunt.

255. In the same way it may be shown that all the formulæ, applying to continuous-current circuits, apply equally to sinusoidal-current circuits when the proper impedances are substituted for the various resistances, and the computation is carried out vectorially.

In a continuous-current circuit, the activity is *E I* watts, $E\,I$, being the numerical product. In a sinusoidal-current circuit, the activity is $E\,I$ watts, $E\,I$, being interpreted geometrically, and representing the co-directed product; or, if a, be the angle included between E, and I, the activity is $E\,I\cos a$ watts.

Thus, considering the case represented in Fig. 95, the activity supplied to the condenser by the alternator of 1100 volts, will be $1100 \times 8.641 \times \cos 90° = 0$, (see Fig. 97.) Such a current is sometimes called a *wattless* current.

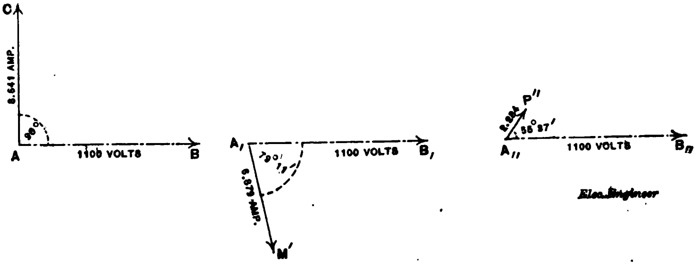

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Fig. 97.

Illustrating Activity Relations.

Considering next the activity in the circuit of the coil, we have $1100 \times 6.877 \times \cos 79° 11' = 1419$ watts. This energy is expended in heating the coil.

256. When the circuit embraces iron, there will be hysteretic expenditure of energy in the iron at each cycle (Sec. 148). The effect of the hysteresis will be to apparently increase the resistance in the circuit, to what may be termed its *equivalent* resistance. Thus, if an E. M. F. of 100 volts at 130 ∿ be impressed upon the terminals of a coil embracing iron, having a resistance of 10 ohms (Fig. 98) and an inductance of 0.1 henry (symbolically written 0.1 H), the angular velocity being

816.8 radians per second, the reactance of the coil will be 81.68 /90° ohms, and the impedance of the coil will be 82.31 /83° 01′ ohms. If, owing to the effect of hysteresis, the 100 volts is opposed by a C. E. M. F. 20 /150° volts, as shown by the line E F, the resultant E. M. F. will be D E + E F, or 83.28 /6° 54′ volts, and this resultant E. M. F., considered as acting on the impedance of the coil, will produce a current $\frac{83.28}{82.31\ \underline{/83^\circ\ 01'}} = 1.012\ \overline{\backslash\ 83^\circ\ 01'}$ amperes, represented by the line *d g*, which will be 76° 07′ behind the impressed E. M. F. *d l*, of 100 volts. The same result can, however, be obtained if we consider the impressed E. M. F. as acting directly on a circuit whose impedance is 98.8 /76° 07′ ohms, and whose equivalent resistance and reactance are 23.7 ohms, and 95.96 /9° ohms, respectively.

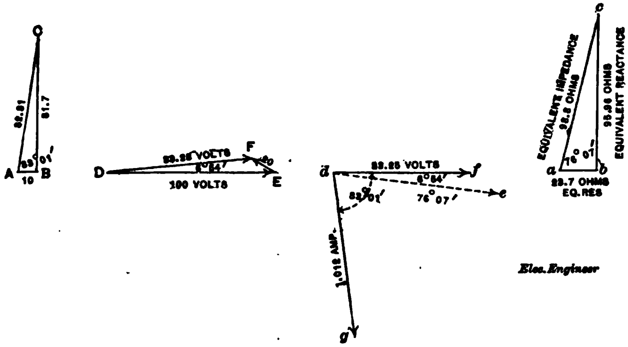

FIG. 98.

Illustrating Equivalent Resistance, Reactance and Impedance.

257. The ratio of the impedance to the ohmic resistance in a conductor or circuit is called its *impedance factor*. The ratio of the reactance to the ohmic

resistance in a conductor or circuit is called its *reactance factor*. The impedance factor is therefore the secant, and the reactance factor the tangent, of the angle of lag.

258. When an alternating current in a primary circuit is linked with a secondary circuit, through the medium of a mutual inductance of L_μ henrys, an E. M. F. is set up in the secondary circuit, and a C. E. M. F. is set up in the primary circuit under the influence of the current in the secondary circuit. Without, however, analyzing the direction and magnitude of these E. M. F.'s, it is sufficient to modify the impedance of the primary circuit in order to determine the results produced. If R_a, K_a, J_a, represent respectively the resistance, reactance and impedance of the primary circuit, r_b, k_b, and J_b, the corresponding quantities in the secondary circuit and

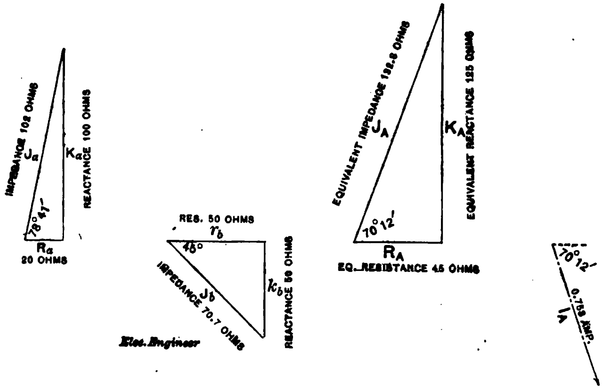

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FIG. 99.
Illustrating Equivalent Resistance, Reactance and Impedances of Mutually Inductive Circuits.

$$n = \frac{\omega L_\mu}{J_b};$$

then R_a, has to be increased by $n^2 r_b$ to R_A; and K_a, diminished by $n^2 K_b$ to K_A; when the impedance $J_A = R_A + K_A$, is the equivalent impedance of the primary circuit.

Thus, if a primary circuit of 20 ohms resistance and 100 ohms reactance have impressed upon its terminals a sinusoidal E. M. F. of 100 volts, whose angular velocity is 1000 radians per second, and is linked through a mutual inductance of 0.05 henry, with a secondary circuit of resistance r_b of 50 ohms, reactance k_b, of 50 $\overline{\diagdown 90^\circ}$ ohms, and, therefore, an impedance of 70.7 $\overline{\diagdown 45^\circ}$ ohms, as shown in Fig. 99,—then $n = \frac{1000 \times 0.05}{70.7} = 0.707$ and $n^2 = 0.5$, so that the primary resistance has to be increased by $0.5 \times 50 = 25$ ohms, and the secondary reactance diminished by $0.5 \times -50 = -25$, i. e., increased by 25 ohms, so that the equivalent resistance R_A, is 45 ohms, the equivalent reactance K_A 125 $\underline{/90^\circ}$ ohms, and the equivalent impedance 132.8 $\underline{/70^\circ\ 12'}$. The primary current is, therefore $\frac{100}{132.8\ \underline{/70^\circ\ 12'}} = 0.753\ \overline{\diagdown 70^\circ\ 12'}$. The secondary E. M. F. is expressed by $-j\ \omega\ L_\mu\ I_A = \omega\ L_\mu\ I_A\ \overline{\diagdown 90^\circ} = 50\ \overline{\diagdown 90^\circ} \times 0.753\ \overline{\diagdown 70^\circ}\ 12' = 37.65\ \overline{\diagdown 160^\circ\ 12'}$, and this secondary current I_b will be $\frac{37.65\ \overline{\diagdown 160^\circ\ 12'}}{70.7\ \overline{\diagdown 45^\circ}} = 0.532\ \overline{\diagdown 115^\circ\ 12'}$.

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ALTERNATORS.

259. The essential difference between an alternator and a continuous-current generator is that the alternator has to supply an alternating E. M. F. of definite frequency and of definite wave character, whereas the continuous-current generator has only to supply a constant E. M. F. If p, be the number of poles, or the number of magnetic circuits in the alternator field, and if n, be the number of revolutions made by the armature per second, the frequency will be $\frac{p\,n}{2}$ periods per second, except in the case of some *inductor alternators* whose frequency is $p\,n \sim$. Thus a bipolar machine, to produce a frequency of $100 \sim$, would have to make 100 revolutions per second, or 6000 revolutions per minute, a speed only attained by steam turbines. Consequently, almost all commercial alternators are multipolar machines, some having as many as 112 poles. The lower the frequency adopted on a circuit, the smaller the number of poles, or the fewer the number of revolutions per minute.

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260. The shape of the wave of E. M. F. produced by an alternator depends primarily upon the rate from instant to instant, at which flux is linked with the armature-winding during revolution. It will, therefore, vary with the shape of the poles and with the shape of the winding on the armature.

The simplest form of wave that can be produced by an alternator is a sinusoidal wave. As already mentioned, many alternators produce a close approximation to the sinusoidal wave form, but, no matter how far the wave may deviate from a strictly sinusoidal form, it may always be considered as a combination of a number of sinusoidal waves, or, in other words, as a *complex sinusoidal* wave. According to what is known as Fourier's theorem, every periodic and single-valued wave of whatever complexity, may be resolved into a combination of a single fundamental wave or simple sinusoid, having the frequency of the complex wave, and a number of shorter sinusoidal waves or *harmonics*, whose frequencies are all some integral multiple of the fundamental frequency.

261. The first harmonic has a frequency twice that of the fundamental. The second harmonic three times, and the nth harmonic, n—1 times that of the fundamental. In some forms of waves the complexity is so great as to necessitate the resolution into an indefinitely great number of harmonics superposed upon the fundamental, while for practical purposes, waves may usually be considered as simply composed of a fundamental together with the superposition of the second, fourth and sixth harmonics, since beyond the sixth, the effect of higher harmonics becomes practically negligible.

Fig. 100, represents at A, a fundamental wave of the sinusoidal type; B, its first harmonic; C, its second; D, its third, and E, its fourth. Fig. 101 represents at S, a fundamental sinusoidal E. M. F. of 800 volts amplitude, i. e., 565.6 volts effective and 100 ~; at P, its first harmonic of 400 volts amplitude, and at Q, its second harmonic of 500 volts amplitude. If these E. M. F.'s were

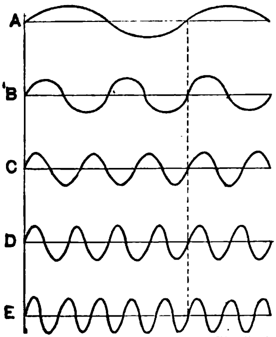

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FIG. 100.

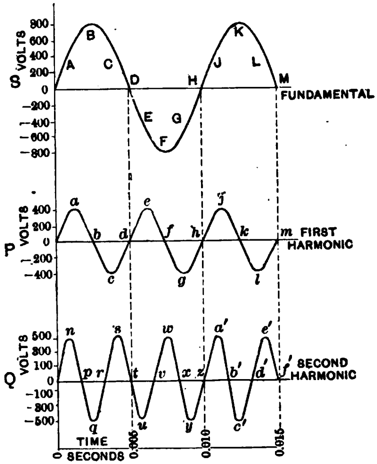

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FIG. 101.

independently produced by three separate alternators, which could be coupled rigidly together on the same shaft, then when alternators P, and Q, were so connected, starting together from the zero in the same direction, the E. M. F. which would be produced is shown in Fig. 102. If all three were connected together the resultant wave type of E. M. F. is represented in Fig. 103.

262. It is evident from an inspection of Figs. 102 and 103, that no ordinary alternator could produce such types of wave as are there represented, because the positive waves are not geometrical inversions of the negative waves, with respect to the zero line. Thus the negative wave D E F G H, should continue from D, to the point as far below the zero line as A, is above it, in order to represent the symmetry that must be developed in an alternator where the various outlines of a positive wave are repeated in due succession in the negative wave, with mere alteration of direction. The discrepancy is due to the introduction of the first harmonic. It can be shown, that in order to maintain the symmetry referred to, only the even harmonics, that is, second, fourth, sixth, etc., can be present; that is, those harmonics whose frequency is an odd number of times that of the fundamental frequency. Fig. 104 represents the combination of a fundamental sinusoidal wave of E. M. F. with its second harmonic of 200 volts amplitude in two different cases of phase relationship. At F + A, the fundamental F, and second harmonic A, are united, while at F + B, the same fundamental and B, are united. It is evident,

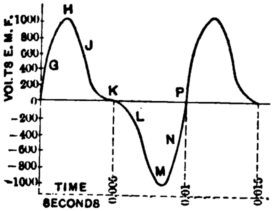

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FIG. 102.

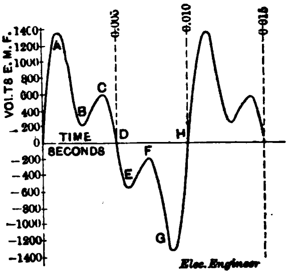

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FIG. 103.

therefore, that the marked presence of a second harmonic in an alternating-current wave may produce either a flat topped or a peaked form of wave, according to the phase of the harmonic, relatively to the fundamental.

Any alternating E. M. F. may, therefore, be expressed by the formula,

$$e = E_1 \sin(\omega t + a_1) + E_3 \sin(3\,\omega t + a_3) + E_5 \sin(5\,\omega t + a_5) + \ldots \text{ volts}$$

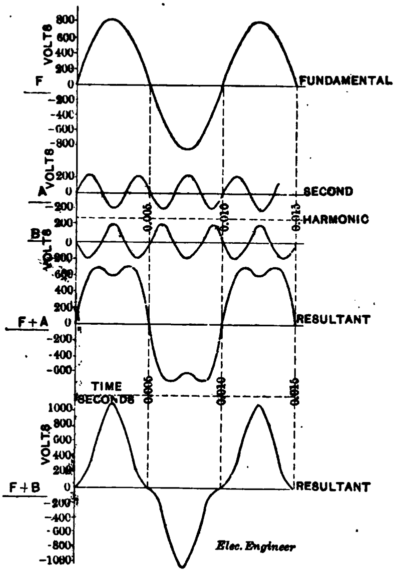

Fig. 104.

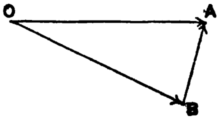

Fig. 105.

where e, is the instantaneous value of the E. M. F., the frequency, E_1, E_3, E_5 the amplitudes of the fundamental and the second and fourth harmonics, and a_1, a_3, a_5, the respective phase angles. If it be required to eliminate the phase angles, this may be done by resolving each successive wave into two sinusoidal components in quadrature, thus

$$e = E_{\prime} \sin \omega t + \mathrm{E}' \cos \omega t + E_{\prime\prime\prime} \sin 3\omega t + E'' \cos 3\,\omega t + E_{\mathrm{v}} \sin 5\,\omega t + E^{\mathrm{v}} \cos 5\,\omega\, t, +$$

This equation is one expression of Fourier's theorem. The same applies to any alternating current, if *i*, and *I*, be substituted for *e*, and *E*, respectively.

263. When a complex-harmonic E. M. F.; that is, an E. M. F. distinctly deviating from the simple-sinusoidal type, sends a current through a circuit containing inductance, the reactance of the circuit to the harmonics will be greater than to the fundamental E. M. F., and, therefore, the impedance will be greater to the harmonics. For this reason the components of harmonic currents are weakened relatively to the fundamental, and the current tends to approach the sinusoidal form more closely than the E. M. F. When, however, the circuit is linked with iron, the presence of hysteresis will usually introduce a greater amount of distorsion than the inductance can compensate, so that the current supplied to alternating current transformers, particularly on light loads, are usually much distorted.

264. When two alternators are connected in series and driven by independent sources of power, the armatures tend to take up such a position that the E. M. F. waves of one are in the opposite direction to the E. M. F. waves of the other, so that no resultant E. M. F. is developed. For this reason, when alternators have to be connected together, they are connected in parallel instead of in series unless rigidly connected on the same shaft.

When two alternators are connected in parallel, they must necessarily keep step, and their phase difference must, therefore, be constant, within certain limits. Suppose two alternators, of 1100 volts effective E. M. F., running independently, be suddenly connected together at a time when their phase difference is, for example, as represented in Fig. 105. The E. M. F. existing in the circuit connecting the two armatures, whose E. M. F.'s, are represented in magnitude and phase by O A, and O B, will, therefore, be represented by the line A B, and this E. M. F. tends to send a current through the armature, whose effect will be to accelerate the lagging armature and retard the leading armature, thus bringing the machines into phase. On the other hand, by armature reaction, the current will tend to produce a C. M. M. F. in the various magnetic circuits, tending to weaken the E. M. F. of the leading machine. The machines, will, therefore, rapidly fall into step, or out of step, according to which of these influences preponderates. The smaller the phase difference existing at the time of interconnection, the lesser the liability of derangement in operation. The lower the frequency and the smaller the armature reaction, the more readily the interconnection can be brought about. If this connection be made at an unfavorable moment, the current in the armatures may reach unduly great strengths, sufficient to blow the fuses, and the mechanical strains brought to bear upon the machines are liable to be excessive. For this reason with alternators at American frequencies in electric lighting, i. e., from 120 ~ to 135 ~ parallel workings has not come into use, although in Europe, where the frequencies are from 40 ~ to 100 ~, parallel working is the general practice. Instruments called *phase detectors* are frequently employed to ascertain the right moment at which to throw the switch connecting two machines in parallel.

SYLLABUS.

Any single-valued, periodic function may be analyzed into a sinusoidal wave, of the frequency of the function, and a series of superposed harmonics.

The *n*th harmonic has a frequency of *n*—1 times that of the fundamental.

A complex E. M. F. acting upon a circuit of considerable reactance tends to generate a current more nearly sinusoidal in type.

Alternators connected in series, unless rigidly connected together, tend to annul each other's E. M. F.

Alternators, when suddenly connected in parallel, may either fall into step, or short circuit one another, according to the nature of the machines, the armature reaction, frequency, and the phase difference between them.

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ALTERNATORS.

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265. A *multiphase alternator*, or *multiphaser*, as distinguished from a *uniphase alternator*, or *uniphaser*, is a machine which generates two or more alternating currents in definite phase relationship with each other. Multiphase alternators are *diphase* when they produce two separate alternating E. M. F.'s *in quadrature*, or separated by a quarter cycle, and *triphase* when they produce three separate alternating E. M. F.'s separated by one-third of a cycle. Multiphase generators are only required for the purpose of operating alternating current motors, which can start from rest at full torque; that is, induction motors.

266. Diphase generators employ on the armature two sets of coils or windings so arranged that the E. M. F.'s generated in them shall have the same magnitude, frequency and wave type, but shall differ in phase by 90°, or a quarter cycle, so that the diagram representing such effective E. M. F.'s will be shown in Fig. 106. The

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current produced by these two E. M. F.'s may be carried in two independent circuits, necessitating four wires and four collector rings, as in Fig. 107, or in two inter-connected circuits, with one common return requiring three wires and three collector rings, as in Fig. 108.

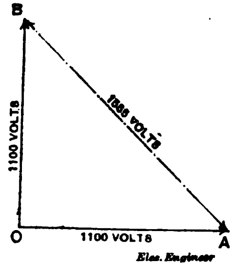

FIG. 106.
Diagram of Diphase E. M. F.'s.

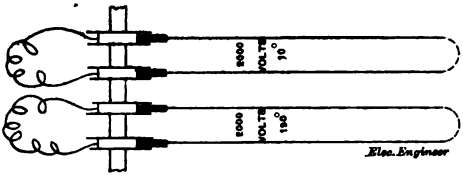

FIG. 107.
Diphase Connections, Separate Circuits.

The E. M. F. between the collector rings A and O, will in the latter case, be the same as between B and O, while the E. M. F. between the rings A and B, will be equal to the length of the line B A, joining the extremities of O B and O A, Fig. 106; or supposing O A and O B, to be each equal to e, the E. M. F., B A will be $e\sqrt{2}$. When the third wire is employed as a common return circuit, as for example, wire (2) in Fig. 108, the current it carries will be the sum of the two currents in circuits X and Y, which, being in quadrature, will be represented as in Fig. 109, and will be $i\sqrt{2}$, where i, is the current strength in each circuit.

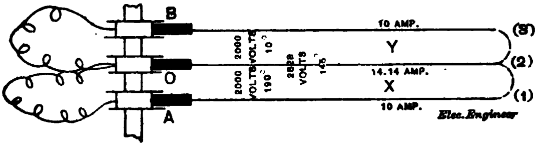

FIG. 108.
Diphase Connections, Interconnected Circuits.

The cross-section of wire (2) will, therefore, have to be 41 per cent. greater than that of either of the two other wires in order to have the same drop of pressure when the two circuits X and Y, are similar in all respects.

267. Triphase generators are wound in two ways; that is, the *star winding* and the *triangular winding*, as shown in Figs. 110 and 111. A combination of the two has also been introduced as shown in Fig. 112, but owing to its greater complication it is seldom employed.

The winding of a triphase alternator may, therefore, be effected by employing three separate sets of coils, each generating an E. M. F. of equal magnitude, frequency and wave type, but differing in phase by $\frac{1}{3}$ cycle. Or, they may be made to differ by $\frac{1}{6}$ cycle, so far as regards their production in the armature, and, by reversing the intermediate E. M. F. relatively to the external circuit, the spacing is converted into $\frac{1}{3}$ cycle, as shown in Fig. 113, where O A, O B and O C, represent three equal E. M. F.'s of $\frac{1}{6}$ cycle angular displacement. The E. M. F. of O B, being reversed, with respect to the external circuit, becomes O B′, when O A, O C, and O B′, are three triphase E. M. F.'s displaced by $\frac{1}{3}$ cycle.

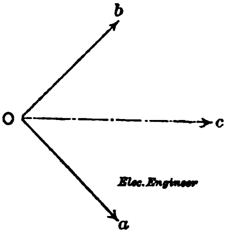

FIG. 109.
Diagram of sum of two equal Diphase Currents.

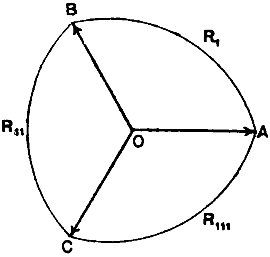

FIG. 110.
Star Triphase Winding.

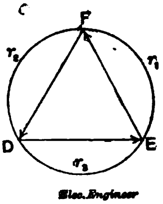

FIG. 111.
Triangle Triphase Winding.

The E. M. F. of a triphase alternator; that is, a *triphaser*, is not measured from the common connection O, to the terminals A B, or C, but between any pair of terminals A B, B C or C A. The E. M. F. so measured will be represented by the lengths of the line A B, B C or C A, and if the effective E. M. F. in each branch O A, O B or O C, be denoted by e, the effective E. M. F. between any pair of terminals will be $e\ \sqrt{3}$.

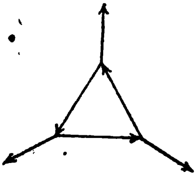

FIG. 112.
Combination Triphase Winding.

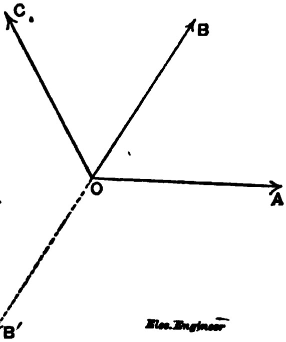

FIG. 113.
Diagram representing the transformation of ⅙ cycle E. M. F.'s into External Triphase E. M. F.'s.

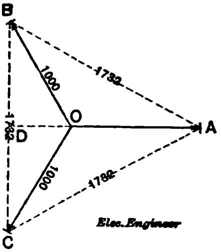

FIG. 114.
Triphase E. M. E. Diagram.

268. A triangular winding may be obtained in a bipolar machine by tapping the armature at three points 120° apart, as shown in Fig. 114, when a triphase E. M. F., not usually sinusoidal, will be generated.

In multipolar triphase machines, the armature is wound with three sets of coils which are subsequently connected either in series, that is, triangularly; or in parallel; that is, star-connected. If e be the effective E. M. F. of each winding, the E. M. F. of a triangularly connected machine, between any pair of terminals, will be e, and in a star connected machine the E. M. F. between any pair of terminals, will be $e\ \sqrt{3}$. The out-put in both cases will be the same; for, if a triangular triphaser have a load represented by a resistance r, connected to each pair of terminals as shown in Fig. 111, the current in each circuit will have the numerical strength $\frac{e}{r} = i$ amperes; e, being the pressure at machine terminals, so that the output of the machine will be $3\ e\ i$ watts. If, however, a star-triphaser has a resistance $R = 3\ r$, connected as a load to each pair of terminals, the current $i_{,}$ in $R_{,}$ will be

$$i_{,} = \frac{e + e\ \overline{\diagdown 60^\circ}}{3\ r} = \frac{1.732\ e\ \overline{\diagdown 30^\circ}}{3\ r} = \frac{i}{1.732}\overline{\diagdown\ 30^\circ}$$

similarly, the current in $R_{,,,}$ will be $i_{,,,} = \frac{e + e\ \underline{/60^\circ}}{3\ r}$ $= \frac{i\ \underline{/30^\circ}}{1.732}$. The current in the winding $O\ A$, will be the sum of $i_{,} + i_{,,,}$ or $\frac{i}{1.732}\underline{/30^\circ} + \frac{i}{1.732}\ \overline{\diagdown 30^\circ} = i$.

The external activity of this winding will, therefore, be $e\ i$ watts, and the total external activity of the machine $3\ e\ i$ watts, as before. In order that the star-triphaser shall have the same activity, as the triangular-triphaser, its resistance must be $3\ r$, on each circuit; that is, its load must be $\frac{1}{3}$ of the load on the triangular triphaser. The E. M. F. of the star triphaser is, therefore, $\sqrt{3}$ times the E. M. F. of a triangular triphaser of the same winding, but will carry $\frac{1}{3}$ of the current strength in the external circuits that a triangular triphaser will sustain. It is, therefore, necessary to reduce the number of turns in a star-triphaser by about 43 per cent. in order to produce the same E. M. F. at terminals as a triangular triphaser, and to utilize the winding space so gained for an increased conductivity, in the proportion of 1.732. The E. M. F. at the terminals of both machines will then be the same, and the drops in each machine the same.

269. Fig. 115 represents a star-triphaser A B C, connected to the star load A′, B′, C′, the load of which may consist either of loaded transformers, of diphase

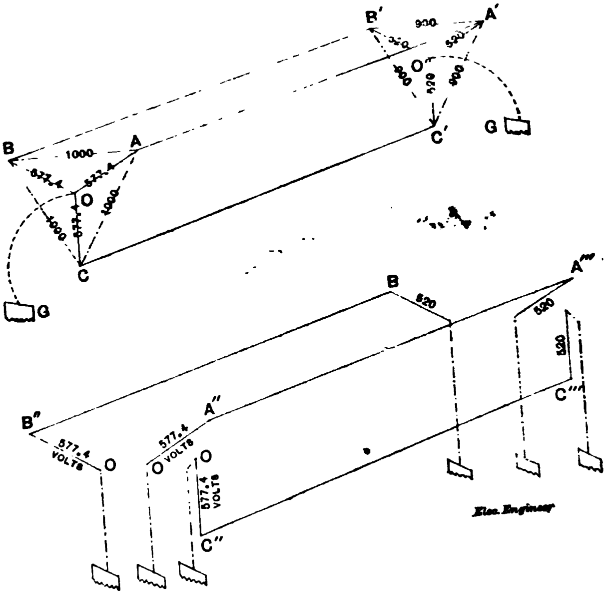

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Fig. 115.
Diagram and Analysis of Triphase Circuit.

motors, of lamps, etc. It is evident that the middle points O and O′, are, by symmetry, always at zero potential when the system is perfectly insulated, so that the ground plates G, G, may be connected to this point at each end of the line without altering the distribution of currents and potentials, and, therefore, without carrying any current through the ground ; for, if the loads on the three circuits be equally balanced, the current in O A, will be the vector sum of the currents in O B and O C. For purposes of transmission, therefore, the circuits may-be regarded as constituting three separate uniphase circuits O A″ A‴ O, O B″ B O, and O C″ C‴ O, each employing a ground return circuit of zero resistance ; so that a triphaser supplying 1000 volts at its terminals is equivalent, in regard to pressure of delivery and economy in conductor, to a uniphaser at 577.4 volts pressure with

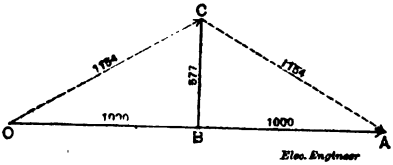

FIG. 116.
Monocyclic E. M. F. Diagram.

ground return, or, to a uniphaser of 1155 volts employing a simple return circuit.

270. A recent combination of uniphase and triphase systems is called the *monocyclic* system.

A monocyclic alternator is wound with two circuits, one of which is the principal circuit and has the principal E. M. F., O A, Fig. 116, for uniphase transmission, while the second circuit has a smaller wire of fewer turns with an E. M. F., B C, in quadrature with A B, and about ¼ of the E. M. F., O A. The second winding is connected to the middle of the main winding as shown. The terminals O, A, are connected to the incandescent lighting mains, and the E. M. F. between these terminals will be simply uniphase. Where, however, induction motors are required to be operated with full starting torque, a third or *power wire* connected to the terminal B, is brought into connection with the motor through two transformers O C and C A, connected, as shown in Fig. 117. The

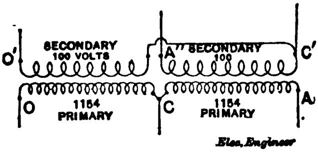

FIG. 117.

Monocyclic Triphase Transformer Connections.

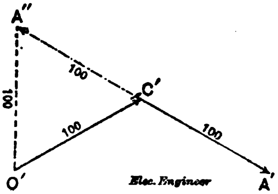

FIG. 118.

Combination of Secondary Monocyclic E. M. F. into Triphase System.

E. M. F.'s, suitably reduced by transformers, are O′ C′ and C′ A′, Fig. 118, differing in phase by 60°. By reversing the secondary connections of the E. M. F., C′ A′, the E. M. F.'s developed are O′ C′, C′ A′′, and their sum O′ A′′, which are three triphase E. M. F.'s, and which may be connected to the terminals of a triphase motor.

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ADVANCED GRADE.

Alternating Current Transformers.

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271. An alternating-current transformer is a device for inducing an E. M. F. in a secondary circuit magnetically linked with a primary circuit, under the influence of an alternating current in the primary.

A transformer enables a powerful alternating current to be developed in a local consumption circuit without the necessity of conveying the current over a long distance, thus avoiding either the heavy loss of energy, or the heavy expenditure in conductors, which would otherwise be necessary. The energy may be transmitted over the main circuit at high pressure with a small current strength, and is transformed locally to a lower pressure and correspondingly increased current strength. Such a transformer is called a *step-down* transformer. On the other hand, a transformer employed to raise the pressure and diminish the current, is called a *step-up* transformer. The *ratio of transformation* is the ratio of the effective E. M. F. at secondary terminals to the effective E. M. F. at

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primary terminals on open circuit. The ratio of transformation in a step-down transformer is, therefore, always less than unity, and in a step-up transformer always greater than unity. The transformers ordinarily employed in electric incandescent lighting usually step-down, from a pressure of 1000 or 2000 volts in the primary mains, to 50 or 100 volts in consumption circuits, and, therefore, have a ratio of transformation of from $\frac{1}{10}$ to $\frac{1}{40}$.

If a ring of iron wire *c c c*, Fig. 119, be wrapped with a primary coil P, of N turns, and connected to an alternating effective voltage E, the C. E. M. F. of self-induction added geometrically to the "drop" in the coil must be equal to E. Since the drop in the primary coil of a small transformer at full load would not exceed two per cent., while in a large transformer it would be less than one per cent., we may practically ignore the drop, and consider that the C. E. M. F. produced by the transformer must be equal and opposite to this impressed E. M. F. Consequently, whatever be the wave form of the impressed E. M. F., the wave form of the C. E. M. F. developed must be the same. The flux through the magnetic circuit of the primary coil must, therefore, be such that the rate of change in the flux-linkage, shall at every instant be equal to the impressed E. M. F., in C. G. S. units. Expressed in symbols, if Φ, considered as a variable dependent upon time, be the flux linkage in the primary circuit, expressed in weber-turns; then neglecting drop, $\frac{d\Phi}{dt} = e'$; where e', is the instantaneous value of the effective E. M. F. E, expressed in C. G. S. units.

The current-strength, which will pass through the primary coil at each instant, is such that the M. M. F. it develops in the coil shall maintain the flux linkage Φ.

As a simple example, suppose a primary coil of 500 turns, and 2 ohms resistance, to be wound on a magnetic circuit of 0.002 oersted reluctance, and connected to a sinusoidal E. M. F. of 1000 volts effective, with 800 radians per second angular velocity. Assuming the absence of hysteresis and eddy currents, and that all the flux passes through all the turns, the effective flux must be 250,000 webers, for the flux-linkage will be 250,000 × 500 = 125,000,000 weber turns, and the time variation of this will be 800 × 125,000,000 = 10^{11} C. G. S. units of E. M. F. = 1000 volts. The effective M. M. F. required to produce this flux will be 250,000 × 0.002 = 500 gilberts = 400 ampere-turns approximately, or 0.8 ampere effective through the coil. This is called the *exciting* or *magnetizing current.*

Under all conditions of load in a transformer, the resultant M. M. F. remains constant, when the drop in the primary coil may be neglected.

272. If now the secondary coil s, of n turns, Fig. 119, be wound on the ring, no change will take place in the primary circuit, so long as the secondary coil is open, but the flux pulsating through the magnetic circuit and the secondary coil, will set up an E. M. F. of $e = \frac{n\,E}{N}$, effective volts in the latter.

When the secondary coil has its circuit closed through a total impedance J, a current of i amperes $= \frac{e}{J}$ will pass through the secondary circuit, and a M. M. F. of $n\,i$ effective ampere-turns will then be imposed upon the magnetic circuit. If the secondary load be non-inductive, say, a series of incandescent lamps, and if there existed no leakage flux in the transformer magnetic circuit, this M. M. F. will be in step with the induced secondary E. M. F.; that is to say, it will be in step with the primary C. E. M. F. and, neglecting primary drop, it will be in opposition to the impressed E. M. F. It will therefore be opposed to the M. M. F. in the primary circuit. The resultant will be a M. M. F. at a phase intermediate between them. In order to maintain the resultant M. M. F. at the magnitude and phase of the M. M. F. of excitation, a greater current strength will enter the primary coil.

Consequently, as load is added to the secondary circuit, the primary current is compelled to vary both in magnitude and phase, so as to maintain the fixed M. M. F. of excitation as a resultant. The result is a stronger primary current, a greater primary power factor, and a greater primary activity. An automatic adjustment is thus maintained in the primary C. E. M. F., whereby electrical energy is transferred from the primary to the secondary circuit.

The foregoing operations might be readily observed and computed in any transformer, if they were not considerably complicated both by hysteresis and leakage.

Owing to hysteresis, the M. M. F. needed to establish the primary C. E. M. F. is not of the same wave type as the latter, and is different in the ascending and descending branches of each wave of flux, as is readily observed from an inspection of Fig. 59. The simplest existing means of predetermining the current wave, is by the graphical application of Fig. 59 to the C. E. M. F. wave required, so as to determine the primary M. M. F. and current strength at different points in the cycle. Energy is expended in the primary circuit and absorbed in the core of the transformer hysteretically, as explained in Sec. 151.

The effect of leakage is to unduly increase the drop at secondary terminals under load. If there were no leakage, that is, if all the flux passed through each primary and secondary turn, the secondary coil would act as through devoid of inductance and possessing only ohmic resistance, its inductive reaction being entirely expended against the primary circuit; but if some of its flux escapes the primary coil, that portion developes a C. E. M. F. of self-induction in the secondary coil, and an apparent inductance is added to the secondary circuit within the transformer, increasing the drop at load. In practice, there is always some leakage in a transformer, and the secondary drop due to leakage is frequently as great as the drop due to the ohmic resistance of both coils. One of the principal objects in the design of a transformer is the reduction of magnetic leakage to a minimum; for, although the presence of leakage does not give rise to waste of energy, yet it produces a drop in pressure which unduly limits the output of the apparatus for incandescent lighting.

When the load in the secondary circuit is non-inductive, the wave form of secondary current will be very nearly the same as the wave type of secondary E. M. F., and therefore of impressed E. M. F. When the secondary load is inductive, the secondary current and M. M. F. lag behind the secondary E. M. F. This necessitates a greater primary M. M. F. and current to maintain the resultant M. M. F. of excitation, and not only is the primary current strength greater, but it is forced into more nearly complete opposition with the secondary current. The former effect increases the ohmic drop and the latter effect increases the leakage drop, since the two opposing M. M. F.'s produce a greater magnetic difference of potential between the opposite surfaces of the magnetic circuit F F, Fig. 119.

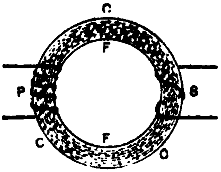

FIG. 119.

273. Let O E, Fig. 120 c, represent the effective C. E. M. F. in the primary circuit of a transformer of say 1000 volts, and O e, the effective E. M. F. in the secondary circuit, in step with O E. When the secondary

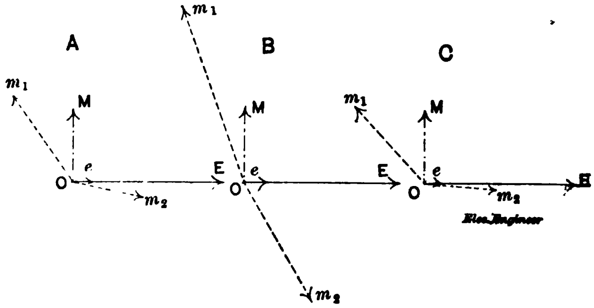

FIG. 120.

circuit is open, let O M, 400 $\underline{/90^\circ}$, be the effective M. M. F. in gilberts or ampere-turns required to maintain the C. E. M. F., O C; then the primary current must be in phase with O M. Now suppose a non-inductive load connected to the secondary circuit. The E. M. F., O *e*, will send a current through this circuit lagging in phase very little behind O *e*, and the M. M. F. of this current will be O m_2, say 400 \ 5°. In order to maintain the resultant M. M. F. O M, the primary M. M. F. must assume the position O *m*, or approximately 600 /140° and thus approach in phase the impressed E. M. F., which will be equal but opposite to O E.

If now, owing to magnetic leakage in the transformer, inductance appears in the secondary coil, the secondary current and M. M. F. may lag 15°, as shown at *A*,

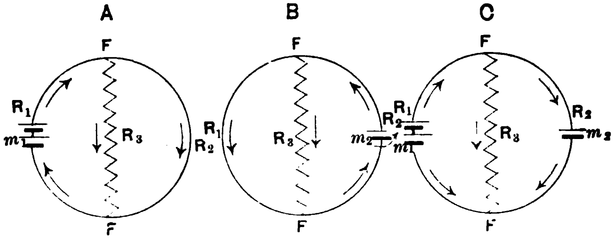

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FIG. 121.

and the primary M. M. F., (O M — O m_2), will become approximately 625 /125°. Again, if the same load in watts be applied inductively to the transformer, as, for example, an alternating-current motor, the power factor of the secondary circuit will be reduced, and a greater current strength will be required, which will lag considerably and the M. M. F. may become, as at *B*, 770 \ 60°. The primary component will be increased to approximately 1100 /20°, and the components will be nearly in opposition.

The effect of this opposition in M. M. F.'s upon the leakage through both coils is illustrated in Fig. 121. If o m_2 and o m_1 are in quadrature, each M. M. F. is at its maximum when its neighbor is zero, and when as at A, m is at its maximum, m_2, is absent, and the flux through the reluctance R_3, of the leakage path will be only that due to the magnetic drop in the reluctance R_2, of the iron occupied by the secondary coil. Similarly when m_2, is at its maximum the flux through R_3, is that due to the drop in R_1. When, however, the M. M. F.'s are forced into opposition, the leakage flux from each through R_3, is increased.

274. The power factor of transformers varies not only with their load, but also with the nature of their load, for, when the secondary circuit is loaded inductively, the current in the secondary circuit lags considerably behind the impressed secondary E. M. F. Consequently the C. M. M. F. is brought more nearly in opposition to the primary M. M. F. The drop in each circuit is, therefore, increased as will be evident from an inspection of Figs. 120 and 121. Large transformers, whose power factor may be 0.995 at full non-inductive load, with a full load drop at secondary terminals of 1 per cent., may have a power factor of 0.90 with an inductive load, and have their full load drop at the same time increased to 4 per cent. On the other hand a condenser connected to secondary terminals tends to diminish the secondary drop and increase the power factor.

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INDEX.

ERRATA.

Page 23, ¶ 23, 5 lines from bottom of page. For resistivity of a mile read resistance of a mile.

" 24, Syllabus, 9th line. For begohm read bicrohm.

" 34, ¶ 34. For $R = \sqrt{R_{\text{app.}}\, r_{\text{app.}}}\ \coth^{-1} \sqrt{\frac{r_{\text{app.}}}{R_{\text{app.}}}}$

read $R = \sqrt{R_{\text{app.}}\, r_{\text{app.}}}\ /\tanh^{-1} \sqrt{\frac{r_{\text{app}}}{R_{\text{app.}}}}$

" 34, last line. For $\coth^{-1}\sqrt{\frac{2656}{6024}}$ read $1 / \tanh^{-1}\sqrt{\frac{2656}{6024}}$

" 175, ¶ 182. For reluctance read retardation.

" 203, " 211. For yellow read green.

" 230, Fig. 77, caption. For Magnetic read Mangin.

" 258, ¶ 261. For $n-1$ read $n+1$.

" 264, Syllabus. For $n-1$ read $n+1$.